Supplementary Material: Descriptions of the Continouos-Discrete Time Extended Kim Filter (CDEKimF) Proposed in "Representing Sudden Shifts in Intensive Dyadic Interaction Data Using Differential Equation Models with Regime Switching"

As summarized in the paper titled "Representing Sudden Shifts in Intensive Dyadic Interaction Data Using Differential Equation Models with Regime Switching," the CDEKimF is composed of four key steps for the purposes of latent variable and regime probability estimation: (1) the CDEKF (for latent variable estimation); (2) the Hamilton filter (to estimate the probability of the latent regime indicator, $S_i(t_{i,j})$); (3) a collapsing procedure (to consolidate regime-specific estimates to reduce computational burden); and (4) a smoothing procedure to compute refined latent variable estimates by using data from all time points, $\mathbf{Y}_i(T_i)$. Here, we describe each of these four steps in turn. Additional procedures to perform parameter estimation and computation of information criterion measures using by-products of these steps have been described in the paper. To facilitate generalization of modeling terminology to scenarios involving individuals as opposed to dyads as the smallest independent unit of analysis, we describe i as indexing person in this section.

0.1 Step 1: The Continuous-Discrete Extended Kalman Filter (CDEKF)

With regime-specific dynamic and measurement functions, the prediction and update steps in the CDEKF-RS, as distinct from standard CDEKF procedures, are regime-dependent. Let $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m} = \mathrm{E}\left(\boldsymbol{\eta}_i(t_{i,j})|S_i(t_{i,j}) = m, S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1})\right)$; $\mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m} = \mathrm{Cov}\left(\boldsymbol{\eta}_i(t_{i,j})|S_i(t_{i,j}) = m, S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1})\right)$; $\boldsymbol{v}_i(t_{i,j})^{l,m}$ is the one-step-ahead prediction errors and $\mathbf{V}_i(t_{i,j})^{l,m}$ is the associated covariance matrix; l and m are indices for the previous regime and current regime, respectively.

The *prediction* step of the CDEKF now involves obtaining $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m}$ and $\mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m}$ by numerically solving the ODEs in Equations 8 and 9 using the dynamic functions of the mth regime, $\boldsymbol{f}_{S_i(t)}(.)$ to yield $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m}$ and $\mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m}$, and with initial conditions $\hat{\boldsymbol{\eta}}_i(t_{i,j-1}|t_{i,j-1})^{l} = \mathrm{E}\left(\boldsymbol{\eta}_i(t_{i,j-1})|S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1})\right)$ and $\mathbf{P}_i(t_{i,j-1}|t_{i,j-1})^{l} = \mathrm{Cov}\left(\boldsymbol{\eta}_i(t_{i,j-1})|S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1})\right)$. The Jacobian matrix $\frac{\partial \boldsymbol{f}_m(\hat{\boldsymbol{\eta}}_i(t),t,\boldsymbol{x}_i(t))}{\partial \hat{\boldsymbol{\eta}}_i(t)}$ shown in Equation (9) is now based on differentiating the dynamic functions from the mth regime evaluated at $\hat{\boldsymbol{\eta}}_i(t) = \hat{\boldsymbol{\eta}}_i(t_{i,j-1}|t_{i,j-1})^{l}$, and with the time-varying covariates in $\boldsymbol{x}_i(t)$ fixed at their observed values, $\mathbf{x}(t_{i,j})$; or specifically, $\frac{\partial \boldsymbol{f}_m(\hat{\boldsymbol{\eta}}_i(t),t,\boldsymbol{x}_i(t_{i,j}))}{\partial \hat{\boldsymbol{\eta}}_i(t)}|_{\hat{\boldsymbol{\eta}}_i(t)=\hat{\boldsymbol{\eta}}_i(t_{i,j-1}|t_{i,j-1})^{l},t=t_{i,j},\boldsymbol{x}_i(t)=\boldsymbol{x}_i(t_{i,j})}$. In particular, the gth row and hth column of this Jacobian matrix carries the partial derivative of the gth dynamic function characterizing regime m with respect to the hth latent variable, evaluated at $\hat{\boldsymbol{\eta}}_i(t_{i,j-1}|t_{i,j-1})^{l}$, with $\boldsymbol{x}_i(t) = \mathbf{x}(t_{i,j})$.

The *update* step of the regime-specific CDEKF can be summarized as:

$$\boldsymbol{v}_i(t_{i,j})^{l,m} = \boldsymbol{y}_i(t_{i,j}) - \Big(\boldsymbol{\tau}_m + \boldsymbol{\Lambda}_m\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m} + \mathbf{A}_m\mathbf{x}(t_{i,j})\Big), \quad \text{(S.1)}$$

$$\mathbf{V}_i(t_{i,j})^{l,m} = \boldsymbol{\Lambda}_m\mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m}\boldsymbol{\Lambda}_m^T + \mathbf{R}_m, \quad \text{(S.2)}$$

$$\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m} = \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m} + \mathbf{K}_m(t_{i,j})\boldsymbol{v}_i(t_{i,j})^{l,m}, \quad \text{(S.3)}$$

$$\mathbf{P}_i(t_{i,j}|t_{i,j})^{l,m} = \mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m} - \mathbf{K}_m(t_{i,j})\boldsymbol{\Lambda}_m\mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m}, \quad \text{(S.4)}$$

where $\mathbf{K}_m(t_{i,j}) = \mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m}\boldsymbol{\Lambda}_m^T[\mathbf{V}_i(t_{i,j})^{l,m}]^{-1}$ is called the Kalman gain function. The regime-specific CDEKF algorithm works recursively (i.e., one time point at a time) from time 1 to T_i and $i = 1, \ldots n$ until $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m}$ and $\mathbf{P}_i(t_{i,j}|t_{i,j})^{l,m}$, have been computed for all time points and people.

0.2 Step 2: The Hamilton Filter

The Hamilton filter is also a recursive process and it can be expressed as:

$$\begin{aligned}
\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})] &= \Pr[S_i(t_{i,j}) = m|S_i(t_{i,j-1}) = l]\times \\
&\quad \Pr[S_i(t_{i,j-1}) = l|\mathbf{Y}_i(t_{i,j-1})], \\
f(\mathbf{Y}_i(t_{i,j})|\mathbf{Y}_i(t_{i,j-1})) &= \sum_{m=1}^{M}\sum_{l=1}^{M} f(\mathbf{Y}_i(t_{i,j})|S_i(t_{i,j}) = m, S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1}))\times \\
&\quad \Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})],
\end{aligned} \quad \text{(S.5)}$$

$$\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})] = \frac{f(\mathbf{Y}_i(t_{i,j})|S_i(t_{i,j}) = m, S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1}))\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})]}{f(\mathbf{Y}_i(t_{i,j})|\mathbf{Y}_i(t_{i,j-1}))}, \tag{S.6}$$

$$\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})] = \sum_{l=1}^{M}\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})], \tag{S.7}$$

$$\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})] = \sum_{l=1}^{M}\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})] \tag{S.8}$$

where $\Pr[S_i(t_{i,j}) = m|S_i(t_{i,j-1}) = l]$ is the transition probability of switching from regime l to regime m based on Equation (7). $f(\mathbf{Y}_i(t_{i,j})|S_i(t_{i,j}) = m, S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1}))$ is a multivariate normal likelihood function expressed as

$$f(\mathbf{Y}_i(t_{i,j})|S_i(t_{i,j}) = m, S_i(t_{i,j-1}) = l, \mathbf{Y}_i(t_{i,j-1})) = (2\boldsymbol{\pi})^{-p/2}|\mathbf{V}_i(t_{i,j})^{l,m}|^{-1/2}\exp\{-\frac{1}{2}(\boldsymbol{v}_i(t_{i,j})^{l,m})^T(\mathbf{V}_i(t_{i,j})^{l,m})^{-1}\boldsymbol{v}_i(t_{i,j})^{l,m}\}. \tag{S.9}$$

Since the prediction error decomposition function computed using Equation S.5 is essentially a raw data likelihood function, missing values can be readily accommodated by using only the non-missing observed elements of $\mathbf{Y}_i(t_{i,j})$ in computing the prediction errors, $\boldsymbol{v}_i(t_{i,j})^{l,m}$ and their associated covariance matrix. To handle missing data in the rest of the regime-specific CDEKF procedures, we used the approach suggested by Hamaker and Grasman (2012), that is, to only update the estimates in $\{\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m}$, $\mathbf{P}_i(t_{i,j}|t_{i,j})^{l,m}$, $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l}$, $\mathbf{P}_i(t_{i,j}|t_{i,j})^{l}$, $\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})]$, $\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})]\}$ using non-missing elements from each measurement occasion.

0.3 Step 3: The Collapsing Process

At each $t_{i,j}$, the regime-specific CDEKF procedures use only the marginal estimates, $\hat{\boldsymbol{\eta}}_i(t_{i,j-1}|t_{i,j-1})^l$ and $\mathbf{P}_i(t_{i,j-1}|t_{i,j-1})^l$, from the previous time point. This is because to ease computational burden, a collapsing procedure is performed on $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m}$ and $\mathbf{P}_i(t_{i,j}|t_{i,j})^{l,m}$ after each CDEKF step to yield $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^m$ and $\mathbf{P}_i(t_{i,j}|t_{i,j})^m$. Given a total of M regimes, if no collapsing is used, the M sets of computations involving $\hat{\boldsymbol{\eta}}_i(t_{i,j-1}|t_{i,j-1})^l$ and $\mathbf{P}_i(t_{i,j-1}|t_{i,j-1})^l$ in the prediction step of the CDEKF (Step 1) would have to be performed using $\hat{\boldsymbol{\eta}}_i(t_{i,j-1}|t_{i,j-1})^{l,m}$ and $\mathbf{P}_i(t_{i,j-1}|t_{i,j-1})^{l,m}$ for every possible value of l and m. As a result, the number of possible values of filtered estimates to store can quickly become unwieldy if the number of time points is large. To circumvent this computational issue, Kim and Nelson (1999) proposed collapsing the $M \times M$ sets of new $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m}$ and $\mathbf{P}_i(t_{i,j}|t_{i,j})^{l,m}$ at each t as

$$
\begin{aligned}
\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^m &= \sum_{l=1}^{M} \mathbf{W}^{l,m}(t_{i,j})\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m}, \\
\mathbf{P}_i(t_{i,j}|t_{i,j})^m &= \sum_{l=1}^{M} \mathbf{W}^{l,m}(t_{i,j})\Big[\mathbf{P}_i(t_{i,j}|t_{i,j})^{l,m} \\
&+ (\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^m - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m})(\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^m - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^{l,m})^T\Big], \\
\mathbf{W}^{l,m}(t_{i,j}) &= \frac{\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})]}{\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})]},
\end{aligned}
\tag{S.10}
$$

Collapsing the estimates across the current regime, m, yields the estimates:

$$
\begin{aligned}
&\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j}) = \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})]\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^m \text{ and} \\
&\mathbf{P}_i(t_{i,j}|t_{i,j}) = \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})] \\
&\quad \Big[\mathbf{P}_i(t_{i,j}|t_{i,j})^m + (\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j}) - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^m)(\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j}) - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^m)^\top\Big].
\end{aligned}
\tag{S.11}
$$

Although not needed for latent variable and parameter estimation purposes, it is sometimes of interest to obtain the predicted latent variable estimates and their associated covariance matrix, as well as the prediction errors and their associated covariance matrix for each $t_{i,j}$ as collapsed across the previous and current regimes l and m as:

$$\begin{aligned}
\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^m &= \sum_{l=1}^{M} \mathbf{W}^{l,m}(t_{i,j-1})\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m}, \\
\mathbf{P}_i(t_{i,j}|t_{i,j-1})^m &= \sum_{l=1}^{M} \mathbf{W}^{l,m}(t_{i,j-1})\Big[\mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m} \\
&\quad + (\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^m - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m})(\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^m - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m})^T\Big], \\
\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1}) &= \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})]\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^m \text{ and} \\
\mathbf{P}_i(t_{i,j}|t_{i,j-1}) &= \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})] \\
&\quad \Big[\mathbf{P}_i(t_{i,j}|t_{i,j-1})^m + (\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1}) - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^m)(\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1}) - \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^m)^\top\Big].
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{v}_i(t_{i,j})^m &= \sum_{l=1}^{M} \mathbf{W}^{l,m}(t_{i,j-1})\boldsymbol{v}_i(t_{i,j})^{l,m}, \\
\mathbf{V}_i(t_{i,j})^m &= \sum_{l=1}^{M} \mathbf{W}^{l,m}(t_{i,j-1})\Big[\mathbf{V}_i(t_{i,j})^{l,m} + (\boldsymbol{v}_i(t_{i,j})^m - \boldsymbol{v}_i(t_{i,j})^{l,m})(\boldsymbol{v}_i(t_{i,j})^m - \boldsymbol{v}_i(t_{i,j})^{l,m})^\top\Big], \\
\boldsymbol{v}_i(t_{i,j}) &= \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})]\boldsymbol{v}_i(t_{i,j})^m, \text{ and} \\
\mathbf{V}_i(t_{i,j}) &= \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})] \\
&\quad \Big[\mathbf{V}_i(t_{i,j})^m + (\boldsymbol{v}_i(t_{i,j}) - \boldsymbol{v}_i(t_{i,j})^m)(\boldsymbol{v}_i(t_{i,j}) - \boldsymbol{v}_i(t_{i,j})^m)^\top\Big],
\end{aligned} \tag{S.12}$$

where $\mathbf{W}^{l,m}(t_{i,j-1})$ is computed as:

$$\mathbf{W}^{l,m}(t_{i,j-1}) = \frac{\Pr[S_i(t_{i,j-1}) = l, S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})]}{\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})]}. \tag{S.13}$$

0.4 Step 4: The Smoothing Procedure

Given the latent variable and regime probability estimates from Steps 1-3, these estimates can be further refined using observed data from each individual's entire time series. Using $\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j-1})^{l,m}, \mathbf{P}_i(t_{i,j}|t_{i,j-1})^{l,m}, \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^l, \mathbf{P}_i(t_{i,j}|t_{i,j})^l$, $\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})]$ and $\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j-1})]$, the smoothing procedure can be implemented for $t_{i,j} = T_i - 1, \ldots 1$ and $i = 1, \ldots n$ as follows. First, smoothed estimates from regime m to the next regime o at time $t_{i,j+1}$ are obtained as

$$\begin{aligned}
\Pr[S_{i,t+1} = o, S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)] = \\
\frac{\Pr[S_{i,t+1} = o|\mathbf{Y}_i(T_i)]\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(t_{i,j})]\Pr[S_{i,t+1} = o|S_i(t_{i,j}) = m]}{\Pr[S_{i,t+1} = o|\mathbf{Y}_i(t_{i,j})]}, \\
\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)] = \sum_{o=1}^{M}\Pr[S_{i,t+1} = o, S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)], \\
\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^{m,o} = \hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^o + \widetilde{\mathbf{P}}(t_{i,j})^{m,o}(\hat{\boldsymbol{\eta}}_i(t_{i,j+1}|T_i)^o - \hat{\boldsymbol{\eta}}_i(t_{i,j+1}|t_{i,j})^{m,o}), \\
\mathbf{P}_i(t_{i,j}|T_i)^{m,o} = \mathbf{P}_i(t_{i,j}|t_{i,j})^o + \widetilde{\mathbf{P}}(t_{i,j})^{m,o}(\mathbf{P}_i(t_{i,j+1}|T_i)^o - \mathbf{P}_i(t_{i,j+1}|t_{i,j})^{m,o})\widetilde{\mathbf{P}}(t_{i,j})^{m,o},
\end{aligned} \tag{S.14}$$

where $\widetilde{\mathbf{P}}(t_{i,j})^{m,o} = \mathbf{P}_i(t_{i,j}|t_{i,j})^m \boldsymbol{B}_o^T[\mathbf{P}_i(t_{i,j+1}|t_{i,j})^{m,o}]^{-1}$, where $\boldsymbol{B}_o$ is the Jacobian matrix, $\frac{\partial \boldsymbol{f}_o(\hat{\boldsymbol{\eta}}_i(t),t,\boldsymbol{x}_i(t))}{\partial \hat{\boldsymbol{\eta}}_i(t)}|_{\hat{\boldsymbol{\eta}}_i(t)=\hat{\boldsymbol{\eta}}_i(t_{i,j}|t_{i,j})^o, t=t_{i,j}, \boldsymbol{x}_i(t)=\boldsymbol{x}_i(t_{i,j})}$. Similar to the collapsing procedure used in the regime-specific CDEKF, a collapsing process is implemented here as

$$\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^m = \sum_{o=1}^{M} \frac{\Pr[S_{i,t+1} = o, S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)]}{\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)]} \hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^{m,o},$$

$$\mathbf{P}_i(t_{i,j}|T_i)^m = \sum_{o=1}^{M} \frac{\Pr[S_{i,t+1} = o, S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)]}{\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)]}$$
$$\left[\mathbf{P}_i(t_{i,j}|T_i)^{m,o} + \left(\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^m - \hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^{m,o}\right)\left(\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^m - \hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^{m,o}\right)^T\right].$$

Finally, smoothed latent variable estimates and their associated covariance matrix are obtained by summing over the M regimes in effect to yield

$$\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i) = \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)]\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^m \text{ and}$$

$$\mathbf{P}_i(t_{i,j}|T_i) = \sum_{m=1}^{M} \Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)]$$
$$\left[\mathbf{P}_i(t_{i,j}|T_i)^m + (\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i) - \hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^m)(\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i) - \hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)^m)^T\right]. \tag{S.15}$$

Equations S.14—S.15 yield three sets of estimates: $\hat{\boldsymbol{\eta}}_i(t_{i,j}|T_i)$, the smoothed latent variable estimates conditional on all observations, the smoothed covariance matrix, $\mathbf{P}_i(t_{i,j}|T_i)$, and $\Pr[S_i(t_{i,j}) = m|\mathbf{Y}_i(T_i)]$, the smoothed probability for person i to be in regime m at time $t_{i,j}$.

References

Hamaker, E. L. and Grasman, R. P. P. P. (2012). Regime switching state-space models applied to psychological processes: Handling missing data and making inferences. *Psychometrika*, 77(2):400–422.

Kim, C.-J. and Nelson, C. R. (1999). *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*. MIT Press,

Cambridge, MA.