# Supplementary Material:

# Observed meltwater-induced flexure and fracture at a doline on George VI Ice Shelf, Antarctica

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## ADDITIONAL MODEL DETAILS

### Governing Equations and Boundary Conditions

The model predicts ice velocity as a function of time in cylindrical components: $\mathbf{u}(r,z,t) = u_r\mathbf{n}_r + u_z\mathbf{n}_z$, where $\mathbf{n}_r$ and $\mathbf{n}_z$ are unit vectors that align with the radial and vertical $r$ and $z$ directions, respectively. These velocities are used, in turn, to predict strain rates (in cylindrical coordinates) and the deviatoric stress $\mathbf{T}'$,

$$\mathbf{T}' = 2\nu_{\text{eff}}\,\dot{\mathbf{e}} \tag{S.1}$$

where $\dot{\mathbf{e}}$ is the strain rate, and the effective viscosity $\nu_{\text{eff}}$ is defined using Glen's flow law,

$$\nu_{eff} = \frac{B(z)}{2}\,(\dot{e}_{II})^{\frac{1}{n}-1} \tag{S.2}$$

The equation which is solved to predict $\mathbf{u}(r,z,t)$, the Stokes equation, is

$$\nabla \cdot \mathbf{T}' - \nabla p - \rho_i g \mathbf{n}_z = \mathbf{0} \tag{S.3}$$

where $p(r,z,t)$ is pressure, $\rho_i(r,z,t)$ in units of kg m$^{-3}$ is ice density, and $g = 9.81$ m s$^{-2}$ is the acceleration of gravity. In addition to the balance of stresses (Eqn. S.3), the ice is assumed incompressible,

$$\nabla \cdot \mathbf{u} = 0 \tag{S.4}$$

To account for symmetry about the $z$ axis at $r = 0$, all variables are assumed to be independent of $\theta$, the azimuthal coordinate, and $u_\theta$, the azimuthal velocity, is assumed to be zero everywhere.

Time dependence is treated by four means. First, the geometry of the doline and surrounding ice shelf are translated according to the velocity of the ice $\mathbf{u}$ as determined by stress balance (Eqn. S.3). Second, the density of in a thin surface layer of the geometry is modified to account for surface ablation. Third, pressure is applied to the upper and lower surfaces to account for influx of meltwater and sea water pressure that change through time according to surface meltwater movement and changing $z$ of the ice base. Four, specification of a "far-field" boundary condition at $r = 2$ km, which represents the outer boundary of the numerical domain.

To implement the first treatment, the movement of the geometry, the representation of the geometry as a finite-element mesh is allowed to translate according to the velocity field. This is accomplished using the "moving mesh" capability of the finite-element code used to implement the model (see 'Numerical Implementation in COMSOL', below). To avoid complexity of additionally modifying the representation of the geometry by a moving mesh as a result of surface ablation, the ice density in upper 10 m of the geometry was altered to account for meltwater runoff, which is the second treatment described above. Melting which was not accompanied by meltwater runoff in the basin of the doline did not lead to a change in ice density. So, with $d$ being the thickness of the upper layer where density of ice is variable, and with $\dot{A}$ being the melting/meltwater-runoff rate, the rate of change of the ice density in the upper layer is specified to be,

$$\frac{\partial \rho_i}{\partial t} = \frac{\rho_i \dot{A}}{d} \tag{S.5}$$

This treatment avoids the complexity of having to deform the upper surface of the geometry by both the velocity and the ablation rate.

To implement the third treatment, pressure $p$ was specified as a boundary condition at the surface and base of the ice geometry, $z_{\text{surface}}(r,t)$ and $z_{\text{base}}(r,t)$, respectively. At $z = z_{\text{surface}}(r,t)$ where there was melting with meltwater runoff, the runoff was assumed to be instantaneous. This was assumed to be the case for the sloping ramp transition of the geometry between the basin and the surrounding ice shelf and for the surrounding ice shelf. For these two areas, the pressure was specified as zero, and the effect of melting and runoff was handled by altering the density of the ice in the upper layer as described by Eqn. S.5. For the basin, local melting was assumed not to runoff, but to remain in place. Also for the basin, it was assumed that meltwater runoff from the ramp part of the geometry would distribute itself in a layer of constant thickness across the basin. The pressure boundary condition on the surface for the basin was thus specified according to the growing influx of meltwater from the ramp,

$$p(t)\Big|_{z=z_{\mathrm{surface}}(r,t)} = \frac{\rho_i g \dot{A}\left(R_t^2 - R_b^2\right)}{R_b^2} \tag{S.6}$$

where $R_t$ and $R_b$ are the radii of the basin and outer edge of the doline ramp (where the annulus with variable ice thickness meets the surrounding ice shelf), taken to be 130 and 330 m, respectively, where $\dot{A}$ is the melt rate of the ramp (assumed to be constant), and where $\rho_i$ is taken to be the density of solid ice (900 kg m$^{-3}$).

At the base of the geometry, $z = z_{\mathrm{base}}(r,t)$, where ice is in contact with seawater, the pressure is specified to be that of a hydrostatic seawater pressure field,

$$p(t)\Big|_{z=z_{\mathrm{base}}(r,t)} = -\rho_{sw} g z_{\mathrm{base}}(r,t) \tag{S.7}$$

where the minus sign accounts for the expectation that $z_{\mathrm{base}}(r,t) < 0$. No additional pressures at the base of the geometry such as may arise from deviations of ocean pressure due to currents and tides were accounted for.

The fourth treatment allowing for time dependence of the model was the specification of a boundary condition at the outer radius of the model domain. This boundary was taken to be at $r = 2$ km so as to isolate the doline region from the arbitrary specification of flow at the artificial outer boundary. This boundary was taken to be purely vertical where a radial component of velocity was specified (assumed to be independent of $z$) to account for divergence in the surrounding ice shelf needed to eliminate the doline closing in upon itself. For the vertical velocity at the boundary a free-slip condition was specified so as to allow the ice shelf to remain in hydrostatic equilibrium over the ice geometry as a whole. To specify the radial component of the velocity specified at the outer boundary, a series of trial and error model runs were made to select a velocity value that would eliminate convergence of ice into the doline. This arbitrary velocity specification was chosen to avoid having the model-predicted velocity field be dominated by converging inflow associated with the radical ice-thickness gradient between the basin and the surrounding ice shelf.

One final boundary condition was specified at $r = 0$, the origin of the axisymmetric ice geometry. This boundary condition is that radial derivatives of all variables are zero and that the radial component of ice velocity is zero along the vertical extent of this boundary. This boundary condition arbitrarily eliminates the possibility that holes or voids develop in the doline basin centred at $r = 0$ if, for example, there were strong ice divergence that induced fracture there.

## Numerical Implementation in COMSOL

To model the idealised ice geometry through an idealised representation of a melt season, the above-described equations were set up in a COMSOL (version 6.0) model following the user interface associated with this commercially available software. The documentation for this software is available on a open-access basis, and can be referred to for questions regarding numerical implementation. The specific model set-up details used in this study are recorded in a .pdf document generated automatically by the COMSOL software (provided as a separate Supplementary Materials document). The details in this report are for Experiment 3 (reports for Experiments 1 and 2 are sufficiently similar that they are not included in the Supplementary Material), and a comprehensive description of all equations, numerical solvers, parameter and boundary condition settings, and model results in graphical form. The organisation of this document is described in the open-access documentation for COMSOL software.