Supplementary Material for Acoustothermal effect: mechanism and quantification of the heat source

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S1 Determination of the actuation frequency

We used a 128° YX lithium-niobate-based surface acoustic wave (SAW) device to generate standing SAWs with a wavelength of $120 \,\mu$ m. Frequency sweep was performed to determine the actuation frequency corresponding to the minimum insertion loss (see Figure S1). Our device has an insertion loss of only $-6.16 \,d$ B at $32.016 \,M$ Hz (actuation frequency).



Figure S1: Insertion loss as a function of frequency for the 128° YX lithium niobate-based SAW device used for the present study.

S2 Contact angle measurements

We have carried out contact angle measurements for a water droplet on LiNbO₃ substrates. The average static contact angle was estimated to be $\phi_c \sim 100.6^{\circ}$, based on three independent measurements, as shown in Figure S2a-c.



Figure S2: Contact angle measurements for sessile water droplet on LiNbO₃ substrate.

S3 Materials parameters

In Table S1, we present the material parameters utilized in our numerical studies to compute acoustothermal heating in sessile water droplets actuated by standing surface acoustic waves. The thermophysical properties of water are sourced from the COMSOL Material Library. For all calculations, we consider $T_0 = 294.15$ K.

Table S1: Thermophysical properties and other parameters used in the numerical studies of the acoustothermal effects in sessile water droplets (T is in K).

Fluid, water				
Density, ρ	$\begin{array}{l}(0.0000103350533197T^3-0.0133950656344527T^2+4.9692888326551607T+\\ 432.257114008512){\rm Kg.m^{-3}}\end{array}$			
Shear viscosity, μ	$\begin{array}{l}(1.3799566804-0.0212240191517T+1.3604562827\times10^{-4}T^2-4.6454090319\times10^{-7}T^3+8.9042735735\times10^{-10}T^4-9.0790692686\times10^{-13}T^5+3.8457331488\times10^{-16}T^6)\mathrm{Pa.s}\end{array}$			
Bulk viscosity, μ_b	$2.79\mu(T)$			
Thermal conductivity, k_{tb}	$(-0.869083936 + 0.00894880345T - 1.58366345 \times 10^{-5}T^{2} + 7.97543259 \times 10^{-9}T^{3})$ W.m ⁻¹ .K ⁻¹			
Specific heat, c_p	$(12010.1471 - 80.4072879T + 0.309866854T^2 - 5.38186884 \times 10^{-4}T^3 + 3.62536437 \times 10^{-7}T^4)$ J.Kg ⁻¹ .K ⁻¹			
Sonic speed, c_0	$\begin{array}{rrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrrr$			
Isobaric thermal expansion coefficient, α_p	$\begin{array}{l} (-6.2142868017\times10^{-1}+9.6088951712\times10^{-3}T-5.9809671543\times10^{-5}T^2+\\ 1.8708110371\times10^{-7}T^3\ -\ 2.9366903765\times10^{-10}T^4\ +\ 1.8495258230\times10^{-13}T^5)\mathrm{K}^{-1} \end{array}$			
Ratio of specific heats, γ	1.0106			
Lithium niobate				
Density, ρ_{LN} Sonic speed, c_{LN} Ratio of longitudinal and transverse wave ampli- tude, ξ	$4700 \mathrm{Kg.m^{-3}}$ $3841.92 \mathrm{m.s^{-1}}$ 0.86			
Heat transfer parameters				
Convective heat transfer coefficient, h_{amb}	$5 \mathrm{W.m^{-2}.K^{-1}}$			

In order to investigate the influence of viscosity on acoustothermal phenomena, we considered droplets composed of a glycerol–water mixture and numerically computed the associated acoustic fields and acoustothermal profiles. The droplet volume was fixed at 5 µL, with a prescribed contact angle of 100.63°. The relevant thermophysical properties, including density, viscosity, thermal expansion coefficient, thermal conductivity, and specific heat capacity, were adopted from well-established literature sources (Delgado et al., 2012; Cheng, 2008; Perry, 1950). The material parameters utilized in the simulation studies are detailed in Table S2. The ratio of specific heats, γ , is taken as 1.01. To facilitate a fundamental understanding of viscous dissipation effects, we assumed these thermophysical properties to remain invariant across the considered temperature range. Furthermore, all simulations were conducted using the finite element-based commercial solver COMSOL Multiphysics. It is well recognized that the displacement amplitude of the surface acoustic wave (SAW) exhibits dependence on the input poweras $\frac{u_0}{\lambda} \propto P_{\rm SAW}^{1/2}$, where the proportionality constant was taken as $1.8 \times 10^{-5} \, {\rm W}^{-1/2}$, determined through fitting to the acoustothermal temperature rise

and consistent with previously reported displacement field results (Sano et al., 1998).

Table S2: Thermophysical properties of glycerol-water used in the numerical studies of the acoustothermal effects in section 5.4.

Properties	$W_{\rm glyc}$ (%)				
	0	5	10	15	20
Density, ρ (Kg.m ⁻³)	998	1009.91	1021.82	1034.20	1046.57
Shear viscosity, μ (Pa.s)	0.892	1.0856	1.2792	1.4921	1.7049
Bulk viscosity, μ_b (Pa.s)	2.485	2.948	3.411	3.870	4.329
Thermal conductivity, k_{th} (W.m ⁻¹ .K ⁻¹)	0.606	0.583	0.560	0.539	0.519
Specific heat, c_p (J.Kg ⁻¹ .K ⁻¹)	4180	4101.0	4022.0	3952.1	3882.1
Sonic speed, $c_0 \ (m.s^{-1})$	1495	1518	1540	1560	1580
Isobaric thermal expansion coefficient, $\alpha_p (10^{-4} \text{ K}^{-1})$	2.57	2.70	2.83	2.99	3.15

S4 Acoustic Poynting vector and the power flow

We plot the dimensionless Poynting vector magnitude $|\langle \hat{p}^* \hat{\mathbf{v}}^* \rangle|$ for a $2 \mu \ell$ sessile water droplet and observe that the maximum acoustic power flow is concentrated at two distinct regions, called "eyes", as illustrated in Figure S3. These eyes are found to be symmetrically positioned along the meridian plane y = 0.



Figure S3: Acoustic energy flow inside a $2 \mu \ell$ sessile water droplet actuated by 32.016 MHz standing SAW. The magnitude of time-averaged acoustic energy flux reveals the presence of two eyes, indicative of regions with substantial acoustic power flow, corresponding to $|\langle \hat{p}^* \hat{\mathbf{v}}^* \rangle| = 200$.

S5 Equivalent acoustothermal heat sources

Since directly computing $\nabla \cdot \langle \tilde{\mathbf{v}} \cdot \tilde{\boldsymbol{\sigma}} \rangle$ for our droplet system is highly challenging, we simulate a simplified 2D system, analogous to our previous work (Das et al., 2019), to illustrate the heat source components governed by Eqs. (4.9)–(4.11) and Eq. (5.4) in the main text and thus reinforce our theory developed in this study. We consider a PDMS microchannel with a width of 2λ , and a height of λ , where $\lambda = 120 \,\mu$ m. The microchannel is actuated from the bottom by a standing SAW with a displacement amplitude of $u_0 = 1$ nm. The thermophysical properties of the fluid are identical to those of the droplet system. The boundary conditions on the PDMS walls are treated using acoustic impedance, following the approach outlined in our earlier work (Das et al., 2019). Figure S4 presents the acoustothermal heat sources: Eq. (4.9) in Figure S4a, Eq. (4.10) in Figure S4b, Eq. (4.11) in Figure S4c, and the time-averaged viscous dissipation associated with Eq. (5.4) in Figure S4d. Our analysis reveals that the contribution of $\nabla \cdot \langle \tilde{\mathbf{v}} \cdot \tilde{\boldsymbol{\sigma}} \rangle$ to the temperature rise is negligibly small (< 0.6%), as evident from the temperature distributions in Figures S4a-b. Additionally, computing the viscous dissipation term remains highly challenging for relatively large systems, such as our current droplet configuration.

In this context, the closed-form expression i.e., Eq. (4.11) of the main text, developed in this study, is extremely useful, as its estimation deviates by no more than 0.5% from the prediction based on viscous dissipation (Figures S4c–d).



Figure S4: Acoustothermal temperature rise distribution for a standing SAW-actuated PDMS microchannel with different heat source considerations. The heat source is calculated as $\langle q_{ac} \rangle = \nabla \cdot [\langle \tilde{\mathbf{v}} \cdot \tilde{\boldsymbol{\sigma}} \rangle - \langle \mathbf{\Pi}_{ac} \rangle]$ (a), $\langle q_{ac} \rangle = -\nabla \cdot \langle \mathbf{\Pi}_{ac} \rangle$ (b), $\langle q_{ac} \rangle = \Gamma \bar{\rho} \omega \langle \tilde{\mathbf{v}} \cdot \tilde{\mathbf{v}} \rangle$ (c), and $\langle q_{ac} \rangle = \langle \nabla \tilde{\mathbf{v}} : \tilde{\boldsymbol{\sigma}} \rangle$ (d).

S6 Correction of the acoustothermal dynamics data

The main text already underscores that surface acoustic waves (SAWs) induce an increase in substrate temperature, even in the absence of a sessile water droplet. This thermal elevation primarily



Figure S5: Correction of the acoustothermal dynamics dataset: a representative case for a 2µL sessile water droplet at a total SAW input power of $P_{\text{SAW},t} = 232 \text{ mW}$. (a) Temporal evolution of the temperature rise for both the droplet and the lithium niobate substrate, as measured via thermal imaging, alongside the corrected temperature rise that excludes extraneous heat sources, thereby isolating the acoustothermal heating contribution within the droplet. (b) Numerical simulation results depicting the steady-state temperature distribution in the droplet for $(\overline{T}-T_0)$ with the droplet base temperature set to $T_b = T_0 + \Delta T_b$, where $\Delta T_b = 1.08 \text{ °C}$. (c) Numerical simulation results for $(\overline{T} - T_b)$, under the same base temperature conditions as (b). (d) Numerical simulation results for $(\overline{T} - T_0)$, with the droplet base temperature set to $T_b = T_0$, effectively isolating the acoustothermal heating effect from extraneous heat sources.

originates from extraneous sources, including Joule heating generated by the interdigital transducers (IDTs) and additional contributions from dielectric losses. Consequently, in our experimental dataset, substrate heating due to these extraneous factors constitutes an inherent and unavoidable element, rendering the precise isolation of acoustothermal effects highly challenging Shilton et al. (2015). To address this, we employed a two-step correction method:

- 1. Quantification of extraneous heating: We conducted experiments at designated SAW power levels without sessile droplets, allowing the acoustic wave to heat the substrate exclusively through extraneous heat sources, thereby raising its temperature to $T_b > T_0$.
- 2. Decoupling acoustothermal heating: Numerical simulations were performed to assess the effect of elevated substrate temperature (i.e., $T_b > T_0$) on droplet heating and to predict equivalent droplet temperature rise in the absence of extraneous heat sources (i.e. by setting $T_b = T_0$).

To illustrate the dataset correction, we present a representative case for a 2 µL sessile water droplet at a total SAW input power of $P_{\text{SAW,t}} = 232 \text{ mW}$, where the maximum substrate temperature rise in the absence of a droplet is ~ 1.08 °C. Figure S5a depicts the temperature rise, $\Delta T = (T_{\text{max}} - T_0)$, for both the droplet ([ΔT]_{experimental}) and the lithium niobate substrate (ΔT_b), as obtained



Figure S6: Experimentally observed temperature rise and the corresponding corrected acoustothermal dynamics for 2 µL (a–c) and 5 µL (d–f) sessile water droplets at total SAW input power of $P_{\text{SAW,t}} = 151.3 \text{ mW}$ (a,d), $P_{\text{SAW,t}} = 183.7 \text{ mW}$ (b,e), and $P_{\text{SAW,t}} = 232.0 \text{ mW}$ (c,f). The blue and red lines denote the experimentally measured and corrected droplet temperature rise, respectively, while the black lines indicate the substrate temperature rise in the absence of sessile water droplets.

from thermal camera measurements, in the presence and the absence of the sessile water droplet, as a function of time, τ . Notably, ΔT at the end of the SAW actuation period (i.e. at $\tau = 90$ s), the observed steady temperature rise are $[\Delta T]_{\text{experimental},\text{ss}} = 15.01 \,^{\circ}\text{C}$ and $\Delta T_{b,\text{ss}} = 1.08 \,^{\circ}\text{C}$.

A numerical simulation was conducted with a 2 µL water droplet, setting its base temperature to $T_{b,ss} = T_0 + \Delta T_{b,ss}$, to match the steady-state droplet temperature rise to $[\Delta T]_{\text{experimental,ss}}$, and determine the corresponding heat generation rate. A second simulation, performed by setting base temperature $T_{b,ss} = T_0$, while maintaining the same heat generation rate, yielded steady-state droplet temperature rise, indicating the temperature rise only due to the acoustothermal effect. This temperature rise is denoted as the corrected temperature rise, $[\Delta T]_{\text{corrected,ss}}$ (see Figure S5a).

Figure S5b illustrates the steady-state temperature distribution $(\overline{T} - T_0)$ for $T_b = T_0 + \Delta T_b$, while Figure S5c presents $(\overline{T} - T_b)$ under identical conditions. Figure S5d displays the corrected steady-state temperature distribution $(\overline{T} - T_0)$ for $T_b = T_0$, isolating acoustothermal heating from extraneous sources. Notably, for our representative case, the experimentally observed maximum value of $(\overline{T} - T_b)$ is 13.94 °C, closely aligning with the numerically corrected value of 13.98 °C. This agreement arises because the specific heat of water C_p remains nearly unchanged between $T_0 = 294.15$ K and $T_b = T_0 + 1.08$ °C, resulting in an almost identical temperature difference. Thus, the substrate temperature, T_b , can be directly subtracted from the experimentally observed droplet temperature to obtain the corrected temperature rise.

By employing the aforementioned techniques, the acoustothermal temperature rise dataset has been systematically corrected. Figure S6 illustrates a comprehensive depiction of the temporal evolution of both the experimentally recorded and the associated corrected temperature rise profiles, along with the substrate temperature rise. These datasets are delineated for droplet volumes of 2 µL and 5 µL under total surface acoustic wave (SAW) input power conditions $P_{\text{SAW,t}} = 151.3, 183.7$, and 232 mW. The corrected temperature rise data not only mitigate extraneous heat-induced discrepancies but also refine the accuracy of the acoustothermal characterization of the sessile droplet system, thereby enabling a more rigorous and nuanced interpretation of the underlying acoustic-fluid interactions.

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