

# Supplementary material: Parameters and core diagnostics of simulations

In this appendix, we present parameters and core diagnostics of all computed simulations. The spectral resolution ( $N, L, M$ ) refers to the maximum polynomial degree for the radial expansion in Jones-Worland and the horizontal expansion in spherical harmonics.

## $Pr = 1$ , fixed temperature and stress-free boundary conditions

$Ra$	Res. (N,L,M)	$Re$	$Nu$
3500	(17,34,34)	1.63	1.07
4000	(12,24,24)	2.47	1.14
4500	(15,30,30)	3.13	1.19
5000	(15,30,30)	3.69	1.24
5500	(17,34,34)	4.21	1.28
6000	(17,34,34)	4.69	1.32
6500	(17,34,34)	5.14	1.36
7000	(17,34,34)	5.57	1.39
7500	(17,34,34)	5.98	1.42
8000	(17,34,34)	6.38	1.45
8500	(17,34,34)	6.76	1.48
9000	(17,34,34)	7.14	1.50
9500	(18,36,36)	7.5	1.53
$10^4$	(18,36,36)	7.87	1.55
$1.05 \times 10^4$	(18,36,36)	8.23	1.58
$1.1 \times 10^4$	(18,36,36)	8.63	1.61

**Table 1:** Parameters and core diagnostics of simulations performed in the full sphere at  $Pr = 1$  with fixed temperature and stress-free boundary conditions: Rayleigh number, spectral resolution, Reynolds number and Nusselt number.

<i>Ra</i>	Res. (N,L,M)	<i>Re</i>	<i>Nu</i>
$1.15 \times 10^4$	(18,36,36)	9.05	1.66
$1.2 \times 10^4$	(18,36,36)	9.49	1.70
$1.25 \times 10^4$	(18,36,36)	9.92	1.75
$1.3 \times 10^4$	(18,36,36)	10.34	1.80
$1.35 \times 10^4$	(18,36,36)	10.76	1.85
$1.4 \times 10^4$	(18,36,36)	11.16	1.89
$1.45 \times 10^4$	(18,36,36)	11.55	1.94
$1.5 \times 10^4$	(20,40,40)	11.93	1.99
$2 \times 10^4$	(20,40,40)	15.32	2.42
$2.5 \times 10^4$	(25,50,50)	18.15	2.80
$2.8 \times 10^4$	(25,50,50)	19.68	3.0
$3 \times 10^4$	(25,50,50)	19.3	2.66
$3.3 \times 10^4$	(30,60,60)	20.01	2.58
$4 \times 10^4$	(30,60,60)	21.93	2.58
$5 \times 10^4$	(40,80,80)	24.44	2.67
$6 \times 10^4$	(40,80,80)	26.69	2.77
$7 \times 10^4$	(40,80,80)	28.75	2.89
$8 \times 10^4$	(50,100,100)	30.66	3.0
$10^5$	(50,100,100)	8.29	34.23
$1.5 \times 10^5$	(60,120,120)	40.99	3.69
$2 \times 10^5$	(60,120,120)	46.95	4.44
$3 \times 10^5$	(69,139,139)	55.32	5.01
$5 \times 10^5$	(69,139,139)	68.53	5.64
$10^6$	(79,159,159)	90.11	6.83
$1.5 \times 10^6$	(79,159,159)	104.8	7.49
$2 \times 10^6$	(89,179,179)	118.2	8.25
$3 \times 10^6$	(89,179,179)	136.8	9.04
$5 \times 10^6$	(39,79,79)	162.4	10.01
$10^7$	(63,127,127)	203.8	12.04
$1.5 \times 10^7$	(63,127,127)	234.5	13.32
$2 \times 10^7$	(63,127,127)	260.2	14.22
$3 \times 10^7$	(63,127,127)	295.3	15.55
$5 \times 10^7$	(81,161,161)	361.7	17.11
$10^8$	(89,179,179)	449.5	20.31
$2 \times 10^8$	(89,179,179)	571.1	23.98
$3 \times 10^8$	(89,179,179)	657.9	26.44
$5 \times 10^8$	(89,179,179)	789.2	29.69
$10^9$	(89,179,179)	1009.8	34.91
$1.5 \times 10^9$	(89,179,179)	1159.1	38.60
$2 \times 10^9$	(89,179,179)	1280.0	41.30
$3 \times 10^9$	(99,199,199)	1477.3	45.67
$5 \times 10^9$	(99,199,199)	1767.3	52.02

**Table 2:** Parameters and core diagnostics of simulations performed in the full sphere at  $Pr = 1$  with fixed temperature and stress-free boundary conditions: Rayleigh number, spectral resolution, Reynolds and Nusselt number.

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$Ra$	Res. (N,L,M)	$Re$	$Nu$
$10^{10}$	(127,255,255)	2249.2	60.51
$1.5 \times 10^{10}$	(139,279,279)	2567.2	66.89
$2 \times 10^{10}$	(159,299,299)	2819.4	71.94
$3 \times 10^{10}$	(159,319,319)	3329.2	80.01

**Table 3:** Parameters and core diagnostics of simulations performed in the full sphere at  $Pr = 1$  with fixed temperature and stress-free boundary conditions: Rayleigh number, spectral resolution, Reynolds number and Nusselt number.

## $Pr = 1$ , fixed temperature and no-slip boundary conditions

$Ra$	Res. (N,L,M)	$Re$	$Nu$
$10^4$	(19,39,39)	1.63	1.07
$1.5 \times 10^4$	(19,39,39)	3.58	1.23
$2 \times 10^4$	(19,39,39)	5.35	1.45
$3 \times 10^4$	(19,39,39)	7.98	1.76
$5 \times 10^4$	(19,39,39)	12.08	1.93
$10^5$	(19,39,39)	20.21	2.99
$1.5 \times 10^5$	(19,39,39)	25.3	3.38
$2 \times 10^5$	(19,39,39)	29.51	3.56
$3 \times 10^5$	(19,39,39)	36.6	3.98
$5 \times 10^5$	(29,59,59)	46.21	4.52
$10^6$	(29,59,59)	63.61	5.29
$2 \times 10^6$	(29,59,59)	84.73	6.15
$3 \times 10^6$	(29,59,59)	100.09	6.53
$5 \times 10^6$	(39,79,79)	121.69	7.29
$10^7$	(39,79,79)	157.85	8.52
$1.5 \times 10^7$	(49,99,99)	183.03	9.28
$2 \times 10^7$	(49,99,99)	202.12	9.92
$3 \times 10^7$	(49,99,99)	232.78	10.87
$5 \times 10^7$	(59,119,119)	277.75	12.18
$10^8$	(59,119,119)	361.51	14.17
$1.5 \times 10^8$	(59,119,119)	420.92	15.48
$2 \times 10^8$	(69,139,139)	467.9	16.5
$3 \times 10^8$	(79,159,159)	541.96	18.12
$5 \times 10^8$	(79,159,159)	651.88	20.4
$10^9$	(89,179,179)	862.09	24.26
$1.5 \times 10^9$	(89,179,179)	1001.84	26.79

**Table 4:** Parameters and core diagnostics of simulations performed in the full sphere at  $Pr = 1$  with fixed temperature and no-slip boundary conditions: Rayleigh number, spectral resolution, Reynolds number and Nusselt number.

$Ra$	Res. (N,L,M)	$Re$	$Nu$
$2 \times 10^9$	(89,179,179)	1118.02	28.90
$3 \times 10^9$	(89,179,179)	1297.61	32.06
$5 \times 10^9$	(89,179,179)	1560.35	36.83
$10^{10}$	(99,199,199)	2001.52	43.75
$1.5 \times 10^{10}$	(99,199,199)	2308.93	49.31
$2 \times 10^{10}$	(99,199,199)	2555.32	53.69
$3 \times 10^{10}$	(99,199,199)	2956.13	60.22
$5 \times 10^{10}$	(99,199,199)	3542.7	69.52
$10^{11}$	(99,199,199)	4515.86	83.94
$1.5 \times 10^{11}$	(99,199,199)	5213.19	93.1
$2 \times 10^{11}$	(99,199,199)	5759.03	99.88
$3 \times 10^{11}$	(99,199,199)	6638.36	110.7
$5 \times 10^{11}$	(99,199,199)	7899.42	125.4
$10^{12}$	(99,199,199)	$1.01 \times 10^4$	146.9
$1.5 \times 10^{12}$	(99,199,199)	$1.17 \times 10^4$	161.1
$2 \times 10^{12}$	(99,199,199)	$1.29 \times 10^4$	172.6

**Table 5:** Parameters and core diagnostics of simulations performed in the full sphere at  $Pr = 1$  with fixed temperature and no-slip boundary conditions: Rayleigh number, spectral resolution, Reynolds and Nusselt number.

## Fixed temperature and stress-free boundary conditions, varying $Pr$

$Pr$	$Ra$	Res. (N,L,M)	$Re$	$Nu$
0.1	3122	(19,39,39)	1.23	1.00
0.1	$3.5 \times 10^3$	(19,39,39)	5.06	1.01
0.1	$5 \times 10^3$	(19,39,39)	23.61	1.10
0.1	$6 \times 10^3$	(19,39,39)	36.87	1.20
0.1	$7.5 \times 10^3$	(19,39,39)	38.64	1.19
0.1	$10^4$	(19,39,39)	42.43	1.19
0.1	$1.5 \times 10^4$	(19,39,39)	63.65	1.36
0.1	$2 \times 10^4$	(19,39,39)	76.88	1.46
0.1	$3 \times 10^4$	(19,39,39)	96.79	1.61
0.1	$5 \times 10^4$	(19,39,39)	126.99	1.83
0.1	$10^5$	(29,59,59)	176.32	2.19
0.1	$1.5 \times 10^5$	(49,99,99)	206.89	2.39
0.1	$2 \times 10^5$	(49,99,99)	237.81	2.62
0.1	$3 \times 10^5$	(49,99,99)	282.11	2.94

**Table 6:** Parameters and core diagnostics of simulations performed in the full sphere at different  $Pr$  with fixed temperature and stress-free boundary conditions: Prandtl number, Rayleigh number, spectral resolution, Reynolds number and Nusselt number.

$Pr$	$Ra$	Res. (N,L,M)	$Re$	$Nu$
0.3	3122	(29,59,59)	0.95	1.00
0.3	$5 \times 10^3$	(29,59,59)	8.87	1.12
0.3	$6 \times 10^3$	(29,59,59)	12.06	1.18
0.3	$8 \times 10^3$	(29,59,59)	19.45	1.38
0.3	$10^4$	(29,59,59)	26.26	1.60
0.3	$1.2 \times 10^4$	(29,59,59)	32.03	1.80
0.3	$1.5 \times 10^4$	(29,59,59)	30.07	1.56
0.3	$1.7 \times 10^4$	(29,59,59)	31.69	1.56
0.3	$2 \times 10^4$	(29,59,59)	36.66	1.71
0.3	$3 \times 10^4$	(29,59,59)	47.75	2.05
0.3	$5 \times 10^4$	(29,59,59)	62.52	2.42
0.3	$10^5$	(39,79,79)	86.98	2.97
0.3	$1.5 \times 10^5$	(39,79,79)	104.19	3.27
0.3	$2 \times 10^5$	(39,79,79)	117.77	3.57
0.3	$3 \times 10^5$	(39,79,79)	136.29	3.90
3	$5 \times 10^3$	(19,39,39)	1.41	1.34
3	$10^4$	(19,39,39)	3.1	1.97
3	$2 \times 10^4$	(19,39,39)	5.33	3.09
3	$3 \times 10^4$	(19,39,39)	6.78	3.98
3	$4 \times 10^4$	(19,39,39)	7.79	4.68
3	$5 \times 10^4$	(19,39,39)	8.46	5.21
3	$6 \times 10^4$	(19,39,39)	8.83	5.57
3	$7 \times 10^4$	(29,59,59)	8.85	5.73
3	$8 \times 10^4$	(29,59,59)	9.39	5.92
3	$10^5$	(29,59,59)	10.84	6.28
3	$1.5 \times 10^5$	(29,59,59)	13.5	6.71
3	$2 \times 10^5$	(29,59,59)	15.7	7.51
3	$3 \times 10^5$	(29,59,59)	18.93	8.10
10	$5 \times 10^3$	(19,39,39)	0.44	1.38
10	$10^4$	(19,39,39)	0.95	2.15
10	$1.5 \times 10^4$	(19,39,39)	1.29	2.81
10	$2 \times 10^4$	(19,39,39)	1.55	3.39
10	$3 \times 10^4$	(19,39,39)	1.91	4.33
10	$5 \times 10^4$	(19,39,39)	2.36	5.56
10	$7 \times 10^4$	(19,39,39)	2.58	4.98
10	$8 \times 10^4$	(19,39,39)	2.92	6.17
10	$10^5$	(19,39,39)	3.19	6.37
10	$1.5 \times 10^5$	(19,39,39)	3.72	6.72
10	$2 \times 10^5$	(19,39,39)	4.12	6.64
10	$3 \times 10^5$	(19,39,39)	4.82	6.99

**Table 7:** Parameters and core diagnostics of simulations performed in the full sphere at different  $Pr$  with fixed temperature and stress-free boundary conditions: Prandtl number, Rayleigh number, spectral resolution, Reynolds number and Nusselt number.

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$Pr$	$Ra$	Res. (N,L,M)	$Re$	$Nu$
30	$5 \times 10^3$	(19,39,39)	0.15	1.39
30	$10^4$	(19,39,39)	0.32	2.19
30	$2 \times 10^4$	(19,39,39)	0.51	3.45
30	$3 \times 10^4$	(19,39,39)	0.63	4.38
30	$5 \times 10^4$	(19,39,39)	0.79	5.64
30	$10^5$	(19,39,39)	1.00	5.48
30	$1.5 \times 10^5$	(19,39,39)	1.25	6.36
30	$2 \times 10^5$	(19,39,39)	1.38	6.41
30	$3 \times 10^5$	(19,39,39)	1.62	6.69

**Table 8:** Parameters and core diagnostics of simulations performed in the full sphere at different  $Pr$  with fixed temperature and stress-free boundary conditions: Prandtl number, Rayleigh number, spectral resolution, Reynolds number and Nusselt number.