Supplementary Material for

"*Wall-damped Faraday waves in horizontally oscillating two-layer fluid flows*".

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This Supplemental Information provides further discussion of the meniscus at rest for different fluid pairs investigated and the nature of subharmonic waves in the main text.

Meniscus at the endwall for superposed fluid layers at rest

We measured the typical length of the meniscus formed on the endwall of the test container at rest for different fluid pairs investigated, as shown in **Fig. S1**. At rest, **Fig. S1** indicates that the interface between the silicone oil and the perfluorinated fluid is almost flat and horizontal with a minimal meniscus at the endwall. Our measurements conducted for different fluid pairs have consistently shown that the typical length of the meniscus at rest lies within a range of 0.76 mm to 1.01 mm (0.89 ± 0.07 mm). For the five fluid pairs investigated, the calculated capillary lengths, $l_{ca} = \sqrt{\sigma/[(\rho_l - \rho_u)g]}$ (σ: interfacial tension, ρ_l: density of the lower layer, ρ_u: density of the upper layer, g: gravity), are as follows: 0.92 mm for HT135-SO10, 0.93 mm for HT135-SO20, 0.94 mm for HT135-SO50, 0.98 mm for HT270-SO50, and 0.94 mm for HT135-SO100. This result demonstrates that the capillary length remains consistent across the different fluid pairs, with no significant variation observed [*Talib et al. 2007 JFM*; *Jalikop, Juel, 2009 JFM*]. Thus, the typical meniscus length (l_m) is on the order of one capillary length, $l_m = 0.95 \pm 0.07 l_{ca}$.

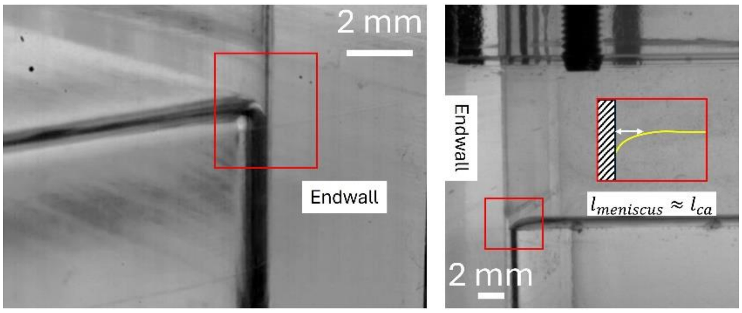

Fig. S1: Representative snapshots showing the menisci at the endwall for the fluid pair of HT135 and SO50 with the container at rest.

Subharmonic standing waves

Figure S2 shows large-amplitude subharmonic waves (SWs) which retain fixed **y** positions for over 1000 periods of oscillation. The vibrational parameter is set to be sufficiently large that droplets pinch off the large-amplitude SWs, and these droplets can be used to track the history of the wave. The droplets are observed to propagate along straight trajectories toward the centre of the container without exhibiting lateral tilting behaviours, suggesting that SWs do not exhibit lateral motion along the walls in the y-direction.

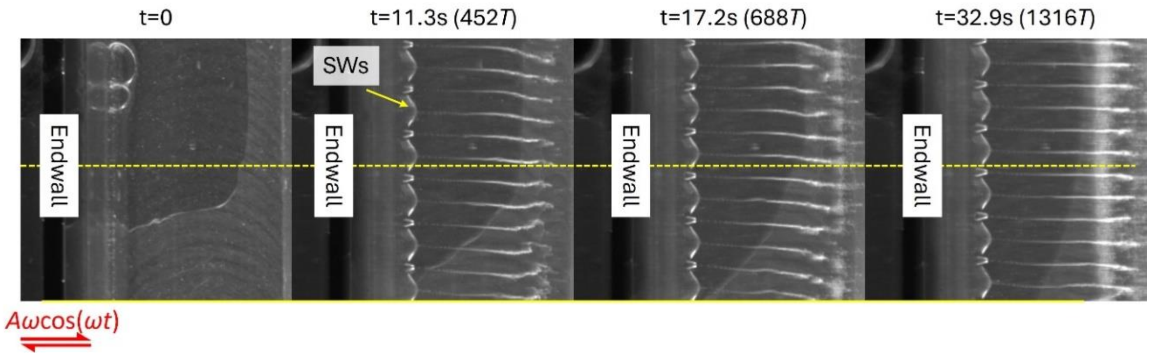

Fig. S2: Sequential images captured at f=40 Hz and A=1.5 mm for the fluid pair HT135-SO50. The yellow dashed line serves as the guide to check the alignment of the waves in time, and the yellow arrow indicates the SWs. The period of oscillation $T = 1/f = 0.25$ s.

The experimental evidence presented in Figure S2 shows that the subharmonic waves are standing waves, in spite of unpinned meniscus/contact line.

References

Talib, E., Jalikop, S. V., & Juel, A. (2007). The influence of viscosity on the frozen wave instability: theory and experiment. Journal of Fluid Mechanics, 584, 45-68.

Jalikop, S. V., & Juel, A. (2009). Steep capillary-gravity waves in oscillatory shear-driven flows. Journal of fluid mechanics, 640, 131-150.