Supplementary Material for "Correlation times of velocity and kinetic helicity fluctuations in nonhelical hydrodynamic turbulence"

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1 Evolution of the relative helicity

In Figure 1 we plot the evolution of the volume-average helicity normalized by $u_{\rm rms}^2 k_{\rm f}$, $\langle \boldsymbol{u} \cdot \boldsymbol{\omega} \rangle / u_{\rm rms}^2 k_{\rm f}$, where $\langle \boldsymbol{u} \cdot \boldsymbol{\omega} \rangle$ is the mean helicity density in the simulation domain. Note that for the rotation (R) runs, $u_{\rm rms}$ does not include the large-scale components as explained in Section 4 in the main text. All the curves represent data from the last $tu_{\rm rms}k_{\rm f} = 200$ time intervals at the end of the simulations.

2 Correlation times for axisymmetric modes, $k_y = 0$

Here, we depict the correlation times obtained from the auto-correlations of $k_y = 0$ modes, as given by Equation (A10) in the main text, for shearing turbulence (Figure 2), shearing burgulence (Figure 3), and Keplerian turbulence (Figure 4). Unlike the k_x -integrated auto-correlations, these on the $k_y = 0$ plane (axisymmetric modes) reveal significant scale separations across broader ranges of modes.

3 Velocity and helicity energy densities for the shearing flow

Here we showcase the energy densities of velocity and helicity (quadratic in \tilde{u} and \tilde{g}) in the context of shearing flows. The representations include slices in the *x*-integrated plane (Figure 5) and slices on the $k_y = 0$ plane (Figure 6).



Figure 1: Evolution of the normalized volume-averaged helicity in the steady state. For all panels except the first, the color gradient from blue to red represents an increasing rotation or shearing rate, following the order of the visible spectrum. See Table 1 in the manuscript for notation and strength of rotation or shear.



Figure 2: For shearing turbulence, correlation times measured from velocity and helicity correlations in the $(k_x, k_y = 0, k_z)$ plane and normalized by the eddy turnover time (upper and middle rows, \log_{10} scale), and their ratios (bottom row, linear scale).



Figure 3: For shearing burgulence, correlation times measured from velocity and helicity correlations in the $(k_x, k_y = 0, k_z)$ plane and normalized by the eddy turnover time (upper and middle rows, \log_{10} scale), and their ratios (bottom row, linear scale).



Figure 4: For Keplerian turbulence, correlation times measured from velocity and helicity correlations in the $(k_x, k_y = 0, k_z)$ plane and normalized by the eddy turnover time (upper and middle rows, \log_{10} scale), and their ratios (bottom row, linear scale).



Figure 5: For shearing turbulence, velocity $(|\tilde{u}|^2, \text{top})$ and helicity $(|\tilde{g}|^2, \text{bottom})$ energy densities on a \log_{10} scale in the *x*-integrated (k_y, k_z) plane.



Figure 6: For shearing turbulence, velocity $(|\tilde{u}|^2, \text{top})$ and helicity $(|\tilde{g}|^2, \text{bottom})$ energy densities on a \log_{10} scale in the $k_y = 0$ plane.