General model for segregation forces in flowing granular mixtures

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Supplementary Material

1. Force and momentum balance

For size-bidisperse mixtures the momentum balance for each species i takes the form

$$\frac{\partial}{\partial t}(\rho_i \boldsymbol{u}_i) + \boldsymbol{\nabla} \boldsymbol{\cdot} (\rho_i \boldsymbol{u}_i \otimes \boldsymbol{u}_i) = -\nabla p_i + \rho_i \boldsymbol{g} + \boldsymbol{\beta}_i, \quad i = l, s, \tag{1.1}$$

where t is time, $\rho_i = \rho\phi c_i$ is the partial density, $\boldsymbol{u}_i$ is the vector velocity, and $\boldsymbol{\beta}_i$ is the interspecies drag vector. We assume that flow is incompressible and that vertical acceleration terms are negligible, which is reasonable for the relatively slow segregation that occurs in many common granular flows. By converting the mixture segregation force into the partial pressure gradient and substituting it into the momentum equation, we get the simplified momentum equation in the z direction:

$$-\frac{\partial p_i}{\partial z} - \rho_i g + \beta_i = 0, \tag{1.2}$$

or force balance at the particle level,

$$F_i - m_i g + \beta_i / n_i = 0, \tag{1.3}$$

noting that the partial pressure gradient can be written as segregation force, i.e. $\frac{\partial p_i}{\partial z} = -n_i F_i$ (Duan *et al.* 2022).

The key idea of the force measurement approach is to employ runtime adjustable restoring forces that function like drag to balance the segregation force or the partial pressure gradient, ensuring the segregation velocity between the two species $\Delta w_i = w_i - w_j$ is close to zero. Following this approach, we choose flows with uniform concentration profiles to isolate the segregation force from the remixing or diffusive forces. As a result, the interspecies drag β_i ($\beta_i \propto \Delta w_i$) becomes negligible, and the updated momentum balance reflecting the added restoring force is:

$$-\frac{\partial p_i}{\partial z} - \rho_i g + n_i F_{res,i} = 0, \tag{1.4}$$

where n_i is the number density, and $F_{res,i}$ is the restoring force. Similarly, the force balance at the particle level is

$$F_i - m_i g + F_{res,i} = 0. \tag{1.5}$$

In this way, the segregation force F_i or the partial pressure gradient $\frac{\partial p_i}{\partial z}$ can be calculated using the time-average restoring force $F_{res,i}$ determined from the simulations.

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However, for cases with varying concentration profiles like those in figure 7, additional remixing/diffusive forces can potentially impact the above balance equation, and the measured segregation force based on the combined restoring force and the particle weight could be inaccurate. Nevertheless, the segregation force model does not depend on segregation velocity or concentration gradient. The fact that the segregation force model (3.3) still predicts the measured segregation force reasonably well for the varying concentration field cases in figure 7 indicates that the concentration-gradient-induced diffusive force in these cases is negligible compared to the segregation force.

2. Results for $R = 3$

We additionally evaluate the concentration-dependent segregation force model for the $R = 3$ case ($d_l = 4\,\mathrm{mm}$, $d_s = 4/3\,\mathrm{mm}$). Flow field details of the four controlled shear flows are shown in figure 1. Results for the segregation force are shown in figure 2. Under these conditions, the pressures due to the constant domain volume constraint when $g = 0$ are $P_0 \approx 0.24\rho\phi g_0 H$ for the exponential profile and $P_0 \approx 1.06\rho\phi g_0 H$ for the parabolic profile.

The quality of the match between the segregation force model predictions and the DEM measurements of the segregation force profiles is similar for $R = 2$ (see main text) and $R = 3$. The model, whether using imposed profiles for $\dot{\gamma}$, $\partial\dot{\gamma}/\partial z$, p, and $\partial p/\partial z$ or measured profiles for these quantities, generally coincide. Perhaps more importantly, the measured segregation force obtained directly from the DEM results matches both models reasonably well for $R = 3$. Of course, the standard deviation of the DEM data is large compared to the values of $\hat{F}_i$, and the agreement is not as good near the walls and around the singularity at $z/H = 0.5$ where $\dot{\gamma} = 0$ for the parabolic case. However, it is clear that the model predictions for concentration dependence of the segregation force, (3.3), using the gravity-induced and kinematics-induced terms in the single intruder segregation force model, (1.2), agree reasonably well with the DEM measurements over a range of inertial numbers that are consistent with dense granular flow.

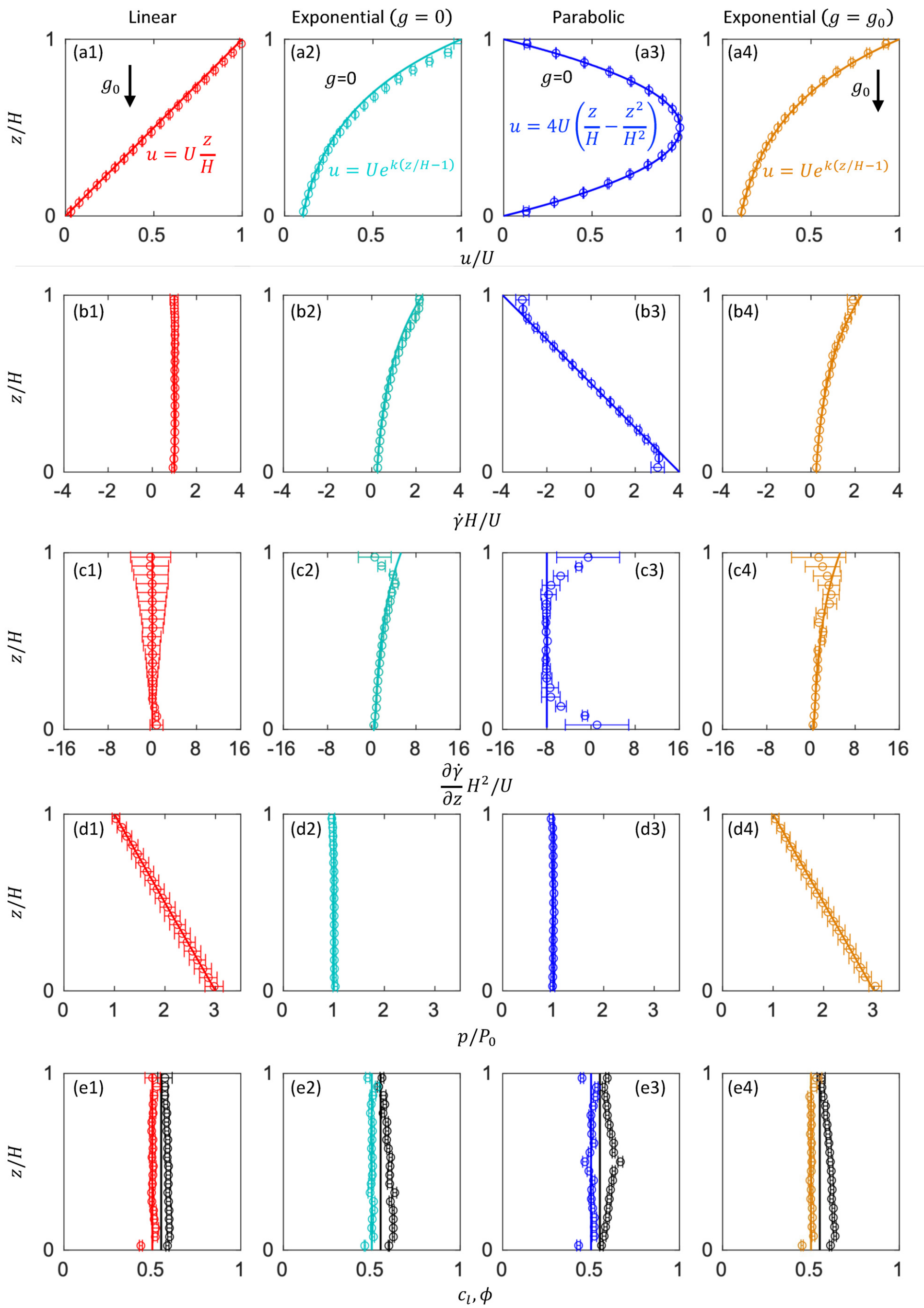

FIGURE 1. Scaled flow field profiles for controlled shear flows with different velocity profiles and $R = 3$. $d_l = 4\,\mathrm{mm}$, $d_s = 4/3\,\mathrm{mm}$, $\rho_l = \rho_s = 1\,\mathrm{g/cm^3}$, $H \approx 0.2\,\mathrm{m} = 50d_l$, and $U = 20\,\mathrm{m/s}$. $g = g_0 = 9.81\,\mathrm{m/s^2}$ in the negative z direction for columns 1 and 4, and $g = 0$ in columns 2 and 3. Values for P_0 are $P_0 = 0.5\rho\phi g_0 H$ for the applied overburden pressure (columns 1 and 4), $P_0 = 0.24\rho\phi g_0 H$ for the exponential profile (column 2), and $P_0 = 1.06\rho\phi g_0 H$ for the parabolic profile (column 3). In the bottom row (e), c_l is in color, and ϕ is black.

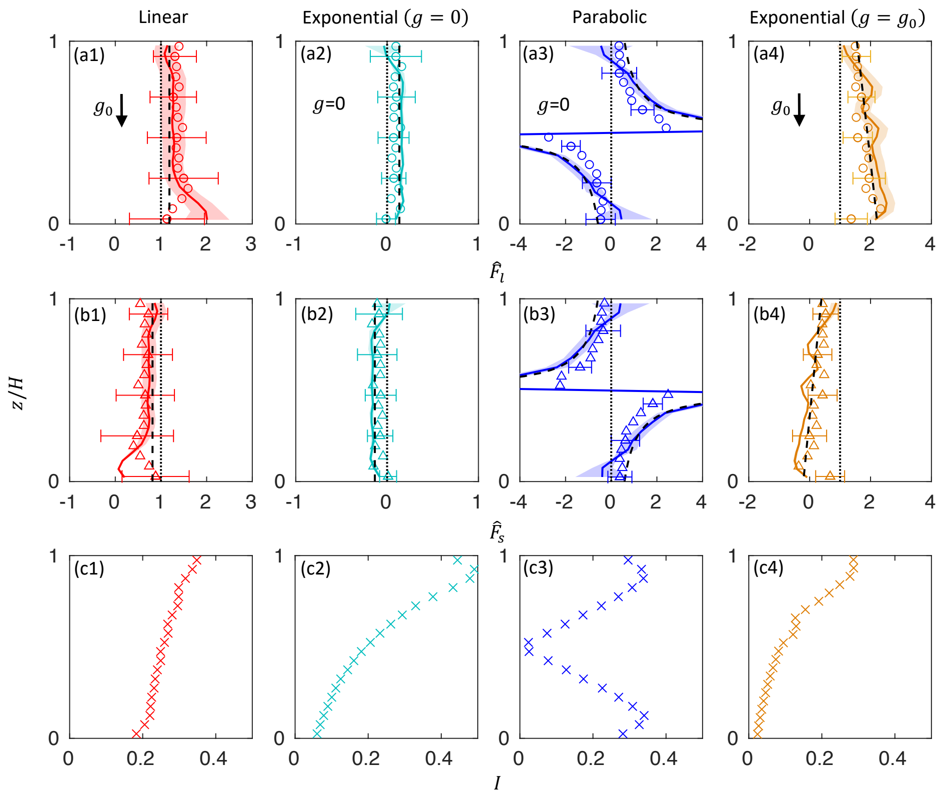

FIGURE 2. Segregation force profiles for large, $\hat{F}_l = F_l/m_l g_0$ (row a), and small, $\hat{F}_s = F_s/m_s g_0$ (row b), particles at $R = 3$ ($d_l = 4\,$mm, $d_s = 4/3\,$mm, $\rho_l = \rho_s = 1\,$g/cm^3, $H \approx 0.2\,$m$= 50d_l$, and $U = 20\,$m/s) from the model (3.3) using the imposed velocity profile (dashed black curves) and the measured profiles (solid color curves) in figure 1 as well as direct DEM measurements (symbols) from 2 s time averages in steady state. Note the different horizontal axes limits in rows a and b. Vertical dotted lines indicate the value about which $\hat{F}_l$ and $\hat{F}_s$ balances according to (3.4). Error bars indicate the DEM data standard deviation. Shaded bands represent the uncertainty of the segregation force calculated from the measured profiles. (row c) Inertial number profiles, I.