Supplement 3: Character Tables

This supplement provides character tables for some of the small finite groups associated with symmetry-breaking bifurcations. The character tables were calculated using the GAP software with an algorithm for calculating isotropy subgroups described by Matthews [2004].

The first table on each page is a standard character table but with an extra column on the right, following a similar format as those in Matthews [2004]. The header of the table '1a', '2a', etc. lists the conjugacy classes where the number indicates the order of the elements. The next row (labelled 'size') gives the number of elements in each conjugacy class. For some of the tables, the next rows give the power map, with rows labelled '2P', '3P', etc. These give the conjugacy class when a given element is raised to a certain prime-order power. The remaining rows give the characters of the irreps, labelled R_1, R_2, etc. A letter F is added to denote that the representation is faithful.

The column on the far right gives the axial isotropy subgroups associated with each representation. Where numbers in brackets are given, the quotient group $N_\Gamma(\Sigma)/\Sigma$ is non-trivial and the number gives the order of the quotient group. Here $N_\Gamma(\Sigma)$ is the normalizer, defined by

$$N_\Gamma(\Sigma) = \left\{\gamma \in \Gamma : \gamma^{-1}\Sigma\gamma = \Sigma\right\}, \tag{1}$$

Γ is the parent group and Σ is the isotropy subgroup. Those isotopy subgroups with (2) denoted will be associated with pitchfork bifurcations. Those without a number may be transcritical, but having $N_\Gamma(\Sigma)/\Sigma = 1$ is not a sufficient condition for being transcritical. D_5 provides an example where the bifurcation associated with the faithful irrep is not transcritical, but $N_\Gamma(\Sigma)/\Sigma = 1$.

The second table gives the dimensions of the spaces of polynomial invariants $I(k)$ of degree k, and the third table gives the dimensions of the spaces of polynomial equivariants $E(k)$ of degree k. These tables help further classify the types of bifurcation and can be used to constrain the number of free parameters needed in the amplitude equations.

References

P. C. Matthews. Automating Symmetry-Breaking Calculations. *LMS Journal of Computation and Mathematics*, 7:101–119, feb 2004. ISSN 1461-1570. doi: 10.1112/S1461157000001066.

C_2

	1a	2a	axial subgroups
size	1	1	
R_1	1	1	C_2
R_2F	1	-1	$C_1(2)$

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2F	1	0	1	0	1

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2F	1	0	1	0	1

The faithful irrep R_2F is associated with a pitchfork bifurcation.

D_3

	1a	2a	3a	axial subgroups
size	1	3	2	
2P	1a	1a	3a	
3P	1a	2a	1a	
R_1	1	1	1	D_3
R_2	1	-1	1	$C_3(2)$
R_3F	2	0	-1	C_2

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3F	1	1	1	1	2

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3F	1	1	1	2	2

The faithful irrep R_3F is associated with a transcritical bifurcation.

D_4

	1a	2a	2b	2c	4a	axial subgroups
size	1	1	2	2	2	
R_1	1	1	1	1	1	D_4
R_2	1	1	-1	-1	1	$C_4(2)$
R_3	1	1	1	-1	-1	$C_2^2(2)$
R_4	1	1	-1	1	-1	$C_2^2(2)$
R_5F	2	-2	0	0	0	$C_2(2)$, $C_2(2)$

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	1	0	1	0	1
R_4	1	0	1	0	1
R_5F	1	0	2	0	2

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	1	0	1	0	1
R_4	1	0	1	0	1
R_5F	1	0	2	0	3

The faithful irrep R_5F is associated with a pitchfork bifurcation.

D_5

	1a	2a	5a	5b	axial subgroups
size	1	5	2	2	
2P	1a	1a	5b	5a	
3P	1a	2a	5b	5a	
5P	1a	2a	1a	1a	
R_1	1	1	1	1	D_5
R_2	1	-1	1	1	$C_5(2)$
R_3F	2	0	$\frac{-1-\sqrt{5}}{2}$	$\frac{-1+\sqrt{5}}{2}$	C_2
R_4F	2	0	$\frac{-1+\sqrt{5}}{2}$	$\frac{-1-\sqrt{5}}{2}$	C_2

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3F	1	0	1	1	1
R_4F	1	0	1	1	1

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3F	1	0	1	1	1
R_4F	1	0	1	1	1

The faithful irreps R_3F and R_4F are associated with pitchfork bifurcations.

S_4

	1a	2a	2b	3a	4a	axial subgroups
size	1	3	6	8	6	
2P	1a	1a	1a	3a	2a	
3P	1a	2a	2b	1a	4a	
R_1	1	1	1	1	1	S_4
R_2	1	1	-1	1	-1	$A_4(2)$
R_3	2	2	0	-1	0	D_4
R_4F	3	-1	-1	0	1	$C_2(2)$, $C_3(2)$, $C_4(2)$
R_5F	3	-1	1	0	-1	$C_2^2(2)$, D_3

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	1	1	1	1	2
R_4F	1	0	2	0	3
R_5F	1	1	2	1	3

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	1	1	1	2	2
R_4F	1	0	2	1	4
R_5F	1	1	2	2	4

The faithful irreps R_4F is associated with a pitchfork bifurcation, the faithful irrep R_5 with a transcritical bifurcation.

A_4

	1a	2a	3a	3b	axial subgroups
size	1	3	4	4	
2P	1a	1a	3b	3a	
3P	1a	2a	1a	1a	
R_1	1	1	1	1	A_4
R_2	1	1	$e^{4\pi i/3}$	$e^{2\pi i/3}$	$C_2^2(3)$
R_3	1	1	$e^{2\pi i/3}$	$e^{4\pi i/3}$	$C_2^2(3)$
R_4F	3	-1	0	0	$C_2(2)$, C_3

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	0	1	0	0	1
R_3	0	1	0	0	1
R_4F	1	1	2	1	4

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	0	1	0	0	1
R_3	0	1	0	0	1
R_4F	1	1	3	3	6

The faithful irrep R_4F is associated with a transcritical bifurcation.

$C_5 \rtimes C_4$

	1a	2a	4a	4b	5a	axial subgroups
size	1	5	5	5	4	
2P	1a	1a	2a	2a	5a	
3P	1a	2a	4b	4a	5a	
5P	1a	2a	4a	4b	1a	
R_1	1	1	1	1	1	$C_5 \rtimes C_4$
R_2	1	1	-1	-1	1	$D_5(2)$
R_3	1	-1	$-i$	i	1	$C_5(4)$
R_4	1	-1	i	$-i$	1	$C_5(4)$
R_5F	4	0	0	0	-1	C_4

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	0	0	1	0	0
R_4	0	0	1	0	0
R_5F	1	1	3	3	5

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	0	0	1	0	0
R_4	0	0	1	0	0
R_5F	1	2	4	7	11

The faithful irrep R_5F is associated with a transcritical bifurcation.

$C_3^2 \rtimes C_4$

	1a	2a	3a	3b	4a	4b	axial subgroups
size	1	9	4	4	9	9	
2P	1a	1a	3a	3b	2a	2a	
3P	1a	2a	1a	1a	4b	4a	
R_1	1	1	1	1	1	1	$C_3^2 \rtimes C_4$
R_2	1	1	1	1	-1	-1	$C_3^2 \rtimes C_2(2)$
R_3	1	-1	1	1	$-i$	i	$C_3 \times C_3(4)$
R_4	1	-1	1	1	i	$-i$	$C_3 \times C_3(4)$
R_5F	4	0	1	-2	0	0	$C_4,\ D_3$
R_6F	4	0	-2	1	0	0	$C_4,\ D_3$

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	0	0	1	0	0
R_4	0	0	1	0	0
R_5F	1	1	2	2	4
R_6F	1	1	2	2	4

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	0	0	1	0	0
R_4	0	0	1	0	0
R_5F	1	1	2	5	6
R_6F	1	1	2	5	6

The faithful irreps R_5F and R_6F are both associated with transcritical bifurcations.

$C_3^2 \rtimes D_4$

	1a	2a	2b	2c	3a	3b	4a	6a	6b	axial subgroups
size	1	6	6	9	4	4	18	12	12	
2P	1a	1a	1a	1a	3a	3b	2c	3a	3b	
3P	1a	2a	2b	2c	1a	1a	4a	2a	2b	
5P	1a	2a	2b	2c	3a	3b	4a	6a	6b	
R_1	1	1	1	1	1	1	1	1	1	$C_3^2 \rtimes D_4$
R_2	1	-1	-1	1	1	1	1	-1	-1	$C_3^2 \rtimes C_4(2)$
R_3	1	-1	1	1	1	1	-1	-1	1	$D_3^2(2)$
R_4	1	1	-1	1	1	1	-1	1	-1	$D_3^2(2)$
R_5	2	0	0	-2	2	2	0	0	0	$C_3 \times D_3(2)$, $C_3 \times D_3(2)$
R_6F	4	-2	0	0	1	-2	0	1	0	$C_2^2(2)$, $C_4(2)$, $D_3(2)$, $C_6(2)$
R_7F	4	0	-2	0	-2	1	0	0	1	$C_2^2(2)$, $C_4(2)$, $D_3(2)$, $C_6(2)$
R_8F	4	0	2	0	-2	1	0	0	-1	D_4, D_6
R_9F	4	2	0	0	1	-2	0	-1	0	D_4, D_6

	$I(2)$	$I(3)$	$I(4)$	$I(5)$	$I(6)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	1	0	1	0	1
R_4	1	0	1	0	1
R_5	1	0	2	0	2
R_6F	1	0	2	0	4
R_7F	1	0	2	0	4
R_8F	1	1	2	2	4
R_9F	1	1	2	2	4

	$E(1)$	$E(2)$	$E(3)$	$E(4)$	$E(5)$
R_1	1	1	1	1	1
R_2	1	0	1	0	1
R_3	1	0	1	0	1
R_4	1	0	1	0	1
R_5	1	0	2	0	3
R_6F	1	0	2	1	5
R_7F	1	0	2	1	5
R_8F	1	1	2	4	5
R_9F	1	1	2	4	5

The faithful irreps R_6F and R_7F are associated with pitchfork bifurcations, the irreps R_7F and R_8F with transcritical bifurcations.