Shear-induced depinning of thin droplets on rough substrates (Supplementary Material)

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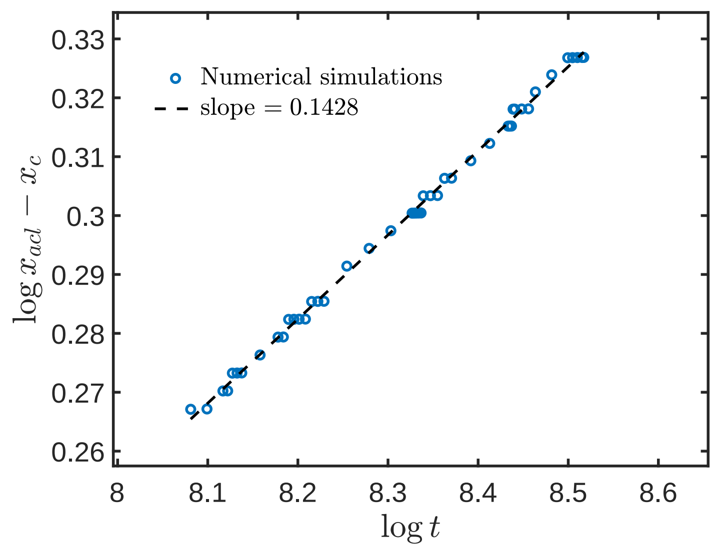

Figure 1: $\log x_{acl} - x_c$ vs. $\log t$. The other parameters are $L = 20$, $\mu_r = 0$, $A = 0$ ($\theta_{eq} = 0°$), $b = 0.001$, $\Delta P = 0$, $v_0 = 0.2$, $h_d = 0$, and $w_d = 0$.

Figure 1 shows $\log(x_{acl} - x_c)$ vs. $\log t$ for a droplet spreading on a perfectly wetting substrate without a surrounding fluid and an external pressure gradient, where x_{acl} is the advancing contact line position and x_c is the droplet center. Here, the open blue circles show numerical calculations and the dashed black line shows a slope of $0.1428 \approx 1/7$. The close agreement between the two indicates that our model recovers Tanner's spreading law ($x_{acl} - x_c \sim t^{1/7}$).

Figure 2 shows the force acting on a pinned droplet vs. ΔP. The open red circles show the total drag force acting on the droplet ($\int_s \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{e_x}\, ds$), the open blue circles show the skin drag component ($\int_s \mathbf{n} \cdot \boldsymbol{\tau} \cdot \mathbf{e_x}\, ds$), and the open yellow circles show the shear force acting on the droplet ($\int_s \mathbf{n} \cdot \mathbf{T} \cdot \mathbf{t}\, ds$). Here, $\mathbf{n}$ and $\mathbf{t}$ are the unit normal and tangent vectors at the interface, $\mathbf{T}$ is the droplet stress tensor, $\mathbf{e_x}$ is the unit vector in the x-direction, and $\boldsymbol{\tau}$ is the deviatoric stress tensor. It can be seen that skin drag is the primary component of the total drag force since they are approximately equal, and the contribution of pressure drag (total drag - skin drag) is negligible. Also, the shear force is nearly equal to the total drag force due to the droplet being thin.

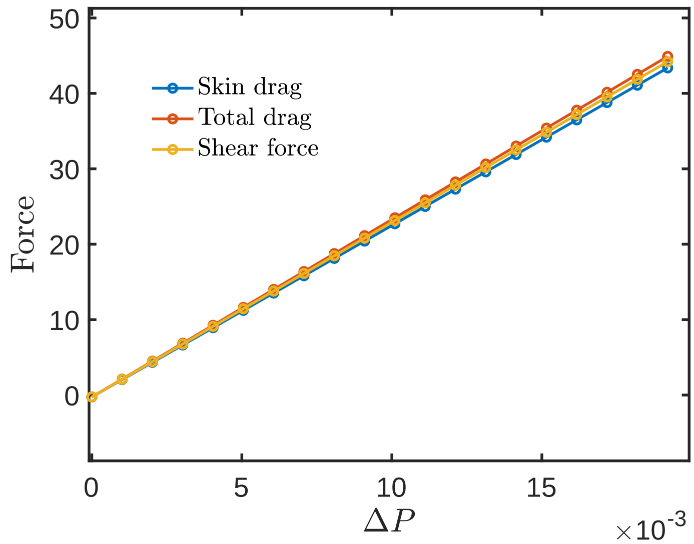

Figure 2: Drag force vs. ΔP. The other parameters are $L = 9$, $\mu_r = 0.01$, $A = 10^5$ ($\theta_{eq} = 10^o$), $b = 0.001$, $v_0 = 0.2$, $h_d = 0.02h_{max}$, and $w_d = 2h_d$.

Figure 3 shows the total interfacial pressure (capillary and disjoining) vs. x for a pinned droplet. It can be seen that the total pressure gradient within the droplet is negative near the receding contact line located at $x = 4.63$, and positive near the advancing contact line located at $x = 5.56$. These opposing pressure gradients keep the droplet pinned.

Figure 4 shows $\log \Delta P_{crit}$ vs. $\log v_0$ for $\mu_r = 100$, where the open blue circles show numerical calculations and the dashed line shows a slope of -0.5. The close agreement between the two indicates that $\Delta P_{crit} \sim v_0^{-0.5}$, which is consistent with the scaling relation derived in §5.2.

Figure 5 shows ΔP_{crit} vs. w_d/h_{max}, where w_d is the maximum defect width. It can be seen that the droplet depins at a higher ΔP_{crit} for a narrower defect. Thus, making the defect taller and narrower have the same qualitative effect on droplet depinning.

We have also included two videos showing droplet pinning at the defects for $\Delta P < \Delta P_{crit}$ ($\Delta P = 0.05$), and droplet depinning and sliding on the substrate for $\Delta P > \Delta P_{crit}$ ($\Delta P = 0.07$). The parameter values are $L = 9$, $\mu_r = 0.01$, $A = 10^5$ ($\theta_{eq} = 10°$), $b = 0.001$, $v_0 = 0.2$, $h_d = 0.02h_{max}$, and $w_d = 2h_d$.

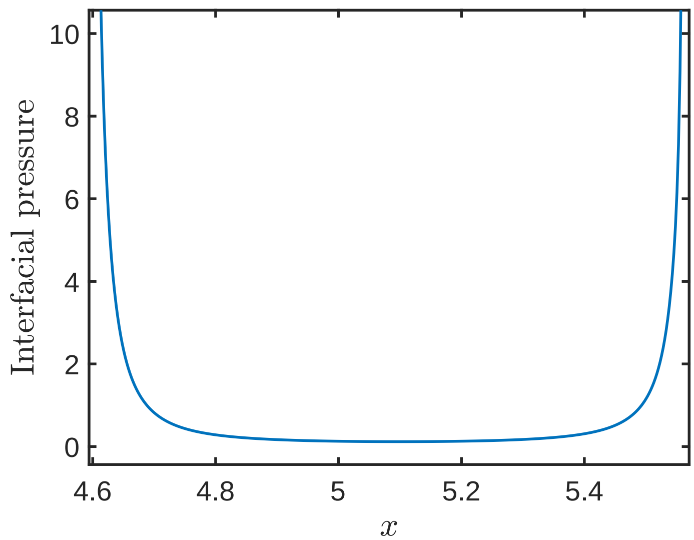

Figure 3: Interfacial pressure vs. x for a pinned droplet ($\Delta P = 0.05$). The other parameters are $L = 9$, $\mu_r = 0.01$, $A = 10^5$ ($\theta_{eq} = 10^o$), $b = 0.001$, $v_0 = 0.2$, $h_d = 0.02h_{max}$, and $w_d = 2h_d$.

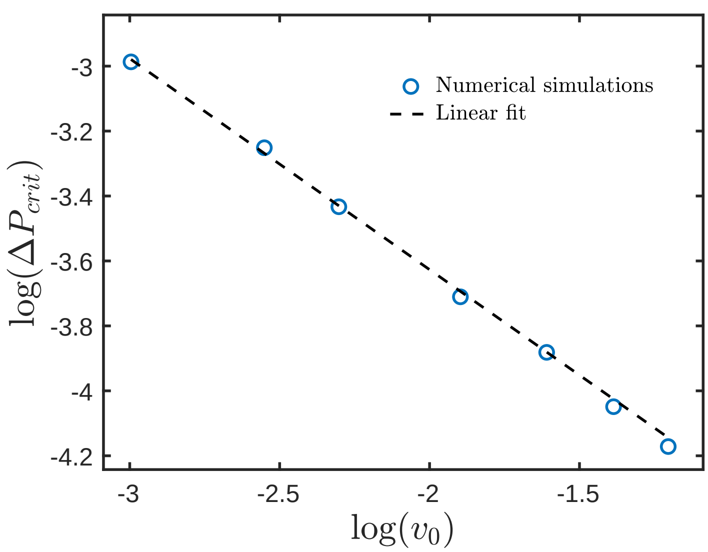

Figure 4: $\log \Delta P_{crit}$ vs. $\log v_0$. The other parameters are $L = 9$, $\mu_r = 100$, $A = 10^5$ ($\theta_{eq} = 10^o$), $b = 0.001$, $h_d = 0.02h_{max}$, and $w_d = 2h_d$.

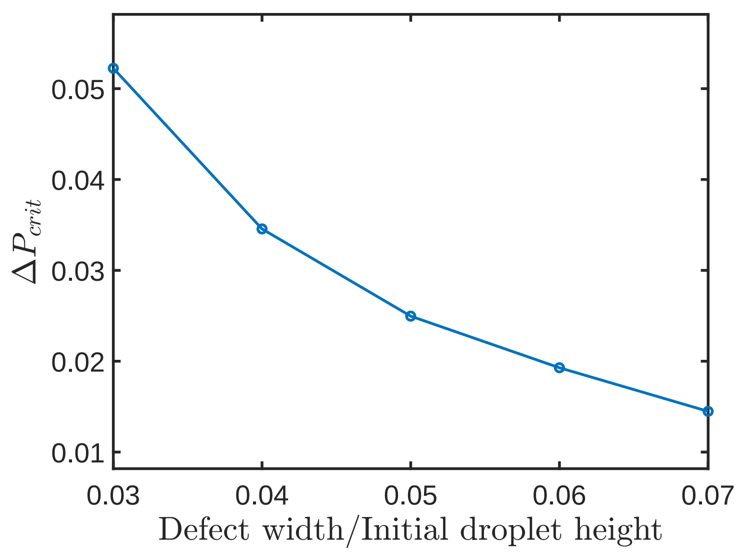

Figure 5: ΔP_{crit} vs. w_d/h_{max}. The other parameters are $L = 9$, $\mu_r = 0.01$, $A = 10^5$ ($\theta_{eq} = 10^o$), $b = 0.001$, $v_0 = 0.2$, and $h_d = 0.02h_{max}$.