# Eulerian discrete kinetic framework in co-moving reference frame for hypersonic flows: Supplementary materials 

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## 1. 2-D Riemann problems

In this part, we illustrate the simulation results to the configurations presented by Lax \& Liu (1998) one by one. Each initial conditions are listed in the table 1 and are easily compared with the reference solutions. From Figs. 1 to 4 , the complex phenomenology in the 2-D Riemann problems are observed and main features are quite close to the reference solutions (Lax \& Liu 1998, Kurganov \& Tadmor 2002).

## REFERENCES

Kurganov, Alexander \& Tadmor, Eitan 2002 Solution of two-dimensional riemann problems for gas dynamics without riemann problem solvers. Numerical Methods for Partial Differential Equations: An International Journal 18 (5), 584-608.
Lax, Peter D \& Liu, Xu-Dong 1998 Solution of two-dimensional riemann problems of gas dynamics by positive schemes. SIAM Journal on Scientific Computing 19 (2), 319-340.

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| configuration | quadrant 1 | quadrant 2 | quadrant 3 | quadrant 4 |
| 5 | $(1,1,0,0)$ | $(0.5197,0.4,-0.7259,0)$ | $(0.1072,0.0439,-0.7259,-1.4045)$ | $(0.2579,0.15,0,-1.4045)$ |
| 6 | $(1,1,0,0)$ | $(0.5197,0.4,-0.7259,0)$ | $(1,1,-0.7259,-0.7259)$ | $(0.5197,0.4,0,-0.7259)$ |
| 7 | $(1.1,1.1,0,0)$ | $(0.5065,0.35,0.8939,0)$ | $(1.1,1.1,0.8939,0.8939)$ | $(0.5065,0.35,0,0.8939)$ |
| 8 | $(1,1,-0.75,-0.5)$ | $(2,1,-0.75,0.5)$ | $(1,1,0.75,0.5)$ | $(3,1,0.75,-0.5)$ |
| 9 | $(1,1,0.1,0.1)$ | $(0.5197,0.4,-0.6259,0.1)$ | $(0.8,0.4,0.1,0.1)$ | $(0.5197,0.4,0.1,-0.6259)$ |
| 10 | $(0.5197,0.4,0.1,0.1)$ | $(1,1,-0.6259,0.1)$ | $(0.8,1,0.1,0.1)$ | $(1,1,0.1,-0.6259)$ |
| 11 | $(1,1,0,0.3)$ | $(2,1,0,-0.3)$ | $(1.039,0.4,0,-0.8133)$ | $(0.5197,0.4,0,-0.4259)$ |
| 12 | $(1,1,0,0.4297)$ | $(0.5,1,0,0.6076)$ | $(0.2281,0.3333,0,-0.6076)$ | $(0.4562,0.3333,0,-0.4297)$ |
| 13 | $(1,1,0.1,0)$ | $(0.5313,0.4,0.8276,0)$ | $(0.8,0.4,0.1,0)$ | $(0.5313,0.4,0.1,0.7276)$ |
| 14 | $(1,1,0,-0.3)$ | $(2,1,0,0.3)$ | $(1.0625,0.4,0,0.8145)$ | $(0.5313,0.4,0,0.4276)$ |
| 15 | $(2,8,0,-0.5606)$ | $(1,8,0,-1.2172)$ | $(0.4736,2.6667,0,1.2172)$ | $(0.9474,2.6667,0,1.1606)$ |
| 16 | $(1,1,0.1,-0.3)$ | $(0.5197,0.4,-0.6259,-0.3)$ | $(0.8,0.4,0.1,-0.3)$ | $(0.5313,0.4,0.1,0.4276)$ |
| 17 | $(0.5313,0.4,0.1,0.1)$ | $(1.0222,1,-0.6179,0.1)$ | $(0.8,1,0.1,0.1)$ | $(1,1,0.1,0.8276)$ |
| 18 | $(1,1,0,-0.4)$ | $(2,1,0,-0.3)$ | $(1.0625,0.4,0,0.2145)$ | $(0.5197,0.4,0,-1.1259)$ |
| 19 | $(1,1,0,1)$ | $(2,1,0,-0.3)$ | $(1.0625,0.4,0,0.2145)$ | $(0.5197,0.4,0,0.2741)$ |

Table 1: The initial conditions ( $\rho, p, u_{x}, u_{y}$ ) for the 2-D Riemann problems


Figure 1: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax \& Liu 1998)


Figure 2: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax \& Liu 1998)


Figure 3: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax \& Liu 1998)


Figure 4: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax \& Liu 1998)


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