## Eulerian discrete kinetic framework in co-moving reference frame for hypersonic flows: Supplementary materials

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## 1. 2-D Riemann problems

In this part, we illustrate the simulation results to the configurations presented by Lax & Liu (1998) one by one. Each initial conditions are listed in the table 1 and are easily compared with the reference solutions. From Figs. 1 to 4, the complex phenomenology in the 2-D Riemann problems are observed and main features are quite close to the reference solutions (Lax & Liu 1998; Kurganov & Tadmor 2002).

## REFERENCES

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configuration	quadrant 1	quadrant 2	quadrant 3	quadrant 4
5	(1,1,0,0)	(0.5197,0.4,-0.7259,0)	(0.1072, 0.0439, -0.7259, -1.4045)	(0.2579, 0.15, 0, -1.4045)
6	(1,1,0,0)	(0.5197,0.4,-0.7259,0)	(1,1,-0.7259,-0.7259)	(0.5197, 0.4, 0, -0.7259)
7	(1.1, 1.1, 0, 0)	(0.5065, 0.35, 0.8939, 0)	(1.1,1.1,0.8939,0.8939)	(0.5065,0.35,0,0.8939)
8	(1,1,-0.75,-0.5)	(2,1,-0.75,0.5)	(1,1,0.75,0.5)	(3,1,0.75,-0.5)
9	(1,1,0.1,0.1)	(0.5197,0.4,-0.6259,0.1)	(0.8,0.4,0.1,0.1)	(0.5197, 0.4, 0.1, -0.6259)
10	(0.5197, 0.4, 0.1, 0.1)	(1,1,-0.6259,0.1)	(0.8,1,0.1,0.1)	(1,1,0.1,-0.6259)
11	(1,1,0,0.3)	(2,1,0,-0.3)	(1.039,0.4,0,-0.8133)	(0.5197, 0.4, 0, -0.4259)
12	(1,1,0,0.4297)	(0.5,1,0,0.6076)	(0.2281, 0.3333, 0, -0.6076)	(0.4562, 0.3333, 0, -0.4297)
13	(1,1,0.1,0)	(0.5313, 0.4, 0.8276, 0)	(0.8,0.4,0.1,0)	(0.5313,0.4,0.1,0.7276)
14	(1,1,0,-0.3)	(2,1,0,0.3)	(1.0625, 0.4, 0, 0.8145)	(0.5313,0.4,0,0.4276)
15	(2,8,0,-0.5606)	(1,8,0,-1.2172)	(0.4736,2.6667,0,1.2172)	(0.9474,2.6667,0,1.1606)
16	(1,1,0.1,-0.3)	(0.5197,0.4,-0.6259,-0.3)	(0.8,0.4,0.1,-0.3)	(0.5313,0.4,0.1,0.4276)
17	(0.5313,0.4,0.1,0.1)	(1.0222,1,-0.6179,0.1)	(0.8, 1, 0.1, 0.1)	(1,1,0.1,0.8276)
18	(1,1,0,-0.4)	(2,1,0,-0.3)	(1.0625, 0.4, 0, 0.2145)	(0.5197, 0.4, 0, -1.1259)
19	(1,1,0,1)	(2,1,0,-0.3)	(1.0625, 0.4, 0, 0.2145)	(0.5197,0.4,0,0.2741)

Table 1: The initial conditions  $(\rho, p, u_x, u_y)$  for the 2-D Riemann problems



Figure 1: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax & Liu 1998)



Figure 2: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax & Liu 1998)



Figure 3: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax & Liu 1998)



Figure 4: The density contour of 2-D Riemann problem with different initial configurations. Left column: present solution; Right column: reference solution (Lax & Liu 1998)