**Supplemental Material for**

**A simple prediction of time-mean and wave orbital velocity in submerged canopy**

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The MATLAB code solving Eq. 23 and Eq. 25 in the main manuscript are described in part A and B, respectively.

# MATLAB code solving Eq. 23

Eq. 23 described the total in-canopy velocity $U\_{1}$ as a function of free stream velocity $U\_{2}$ and the canopy properties. To make it easier to read, Eq. 23 was repeated below,

 $\frac{∂U\_{2}}{∂t}-\frac{∂U\_{1}}{∂t}=\frac{1}{ρ}\frac{F\_{d}N\_{s}}{h\_{p}(1-λ\_{p})}-\frac{h}{h\_{p}(h-h\_{p})}\left.τ\_{xz}\right|\_{h\_{p}}$

The MATLAB code solving Eq. 23 is given below.

%The following input parameters was given as an example for rigid cylinders.

%Note they need to be set according to the relevant conditions

Ns=100;

D=0.01;

hp=0.3;

h=0.6;

Uc2=0.1;

Uw2=0.1;

Tw=1;

CDs=1;

CMs=2;

rho=1000; %density of water

Nt=100; %number of phases in one wave cycle

Nw=100; %number of wave periods to process the code to obtain a steady velocity state

dt=Tw/Nt; %length of the time step

t=((1:1:Nt\*Nw).\*dt)'; % length of the total modeled time

f=1/Tw;

omega=2\*pi/Tw;

U2=Uc2+Uw2.\*cos(omega.\*t);

dU2=-omega\*Uw2.\*sin(omega.\*t);

Umax=max(abs(U2));

Astem=pi\*D^2/4;

a=Ns\*D;

lamda\_p=Ns\*Astem;

CDstem=0.5\*rho\*CDs\*D\*hp;

CMstem=rho\*CMs\*Astem\*hp;

CDleaf=0;

CMleaf=0;

deltE=min([0.3/(CDs\*Ns\*D),h-hp,hp]);

C=0.07\*(deltE/h)^(1/3);

Cp=Ns/rho/hp/(1-lamda\_p);

B=C\*h/(h-hp)/hp;

U1(1,1)=U2(1,1);

for i=2:1:Nw\*Nt

 A=1+Cp\*(CMleaf+CMstem);

 D=Cp\*(CDleaf+CDstem);

 T1=dt/A\*(dU2(i-1,1)+dU2(i,1))/2;

 T2=dt/A\*B\*abs(U2(i-1,1)-U1(i-1,1));

 T3=dt/A\*D\*abs(U1(i-1,1));

 T4=U1(i-1,1);

 Tleft=1+T3+T2;

 Tright=T4+T1+T2\*U2(i,1);

 U1(i,1)=Tright/Tleft;

end

plot(t,U2,t,U1);

legend('U2','U1');

Uc1=mean(U1(end-Nt+1:end,1));

Uw1=sqrt(2\*sum((U1(end-Nt+1:end,1)- Uc1).\* (U1(end-Nt+1:end,1)- Uc1))/Nt);

alphas\_Uc=Uc1./Uc2;

alphas\_Uw=Uw1./Uw2;

# MATLAB code solving Eq. 25

Eq. 25 described the total in-canopy velocity $U\_{1}$ as a function of imposed velocity $U$ and the canopy properties. Eq. 25 is repeated below,

 $R\_{1}\frac{∂U}{∂t}-(R\_{2}+1)\frac{∂U\_{1}}{∂t}=\frac{1}{ρ}\frac{F\_{d}N\_{s}}{h\_{p}(1-λ\_{p})}-\frac{R\_{1}}{h\_{p}}C\left|R\_{1}U-(R\_{2}+1)U\_{1}\right|(R\_{1}U-(R\_{2}+1)U\_{1})$

In which $U\_{2}=RU+R\_{1}U\_{1}$ with $R=h/(h-h\_{p})$, $R\_{1}=h\_{p}(1-λ\_{p})/(h-h\_{p})$. The MATLAB code solving Eq. 25 is given below.

%The following input parameters was given as an example for rigid cylinders.

%Note they need to be set according to the relevant conditions

Ns=100;

D=0.01;

hp=0.3;

h=0.6;

Uc=0.1;

Uw=0.1;

Tw=1;

CDs=1;

CMs=2;

rho=1000; %density of water

Nt=100; %number of phases in one wave cycle

Nw=100; %number of wave periods to process the code to obtain a steady velocity state

dt=Tw/Nt;%length of the time step

t=((1:1:Nt\*Nw).\*dt)'; % length of the total modeled time

omega=2\*pi/Tw;

U=Uc+Uw.\*cos(omega.\*t);

dU=-omega\*Uw.\*sin(omega.\*t);

Astem=pi\*D^2/4;

a=Ns\*D;

lamda\_p=Ns\*Astem;

CDstem=0.5\*rho\*CDs\*D\*hp;

CMstem=rho\*CMs\*Astem\*hp;

CDleaf=0;

CMleaf=0;

Cp=Ns/rho/hp/(1-lamda\_p);

A=(h-hp\*lamda\_p)/(h-hp);

B=h/(h-hp);

deltE=min([0.3/(CDs\*Ns\*D),h-hp,hp]);

C=0.07\*(deltE/h)^(1/3);

D=C\*h/hp/((h-hp)^3);

U1(1,1)=U(1,1);

for i=2:1:Nw\*Nt

 T1=dt\*B\*(dU(i-1,1)+dU(i,1))/2;

 T2=dt\*D\*abs(U(i-1,1)\*h-U1(i-1,1)\*(h-hp\*lamda\_p));

 T3=dt\*Cp\*(CDleaf+CDstem)\*abs(U1(i-1,1));

 T4=A+Cp\*(CMleaf+CMstem);

 Tleft=T4\*U1(i-1,1)+T1+T2\*h\*U(i,1);

 Tright=T4+T2\*(h-hp\*lamda\_p)+T3;

 U1(i,1)=Tleft/Tright;

end

plot(t,U,t,U1);

legend('U','U1');

Uc1=mean(U1(end-Nt+1:end,1));

Uw1=sqrt(2\*sum((U1(end-Nt+1:end,1)- Uc1).\* (U1(end-Nt+1:end,1)- Uc1))/Nt);

alphas\_Uc=Uc1./Uc;

alphas\_Uw=Uw1./Uw;