## Self-similarity of the dipole–multipole transition in rapidly rotating dynamos (Supplementary Material)

Debarshi Majumder, Binod Sreenivasan and Gaurav Maurya

Centre for Earth Sciences, Indian Institute of Science, Bangalore 560012, India



Figure S1: The figure shows the evolution in time (measured in units of the magnetic diffusion time) of the root mean square value of the magnetic field intensity,  $\bar{B}$  (solid blue line), the axial dipole part of the field,  $\bar{B}_{10}$  (dashed blue line), and the relative dipole field strength at the outer boundary,  $f_{dip}$  (red line). The run starts from a dipole seed magnetic field. The vertical dotted line marks the emergence of the slow MAC waves, which happens when  $|\omega_M| \approx |\omega_A|$  (Varma and Sreenivasan, 2022); the vertical dashed line denotes  $f_{dip} = 0.35$ , which is suggested as the lower bound for the existence of dipole-dominated numerical dynamos (Christensen and Aubert, 2006). Point 'A' ( $f_{dip} = 0.095$ ) is a multipolar state before the appearance of the slow MAC waves and point 'B' ( $f_{dip} = 0.212$ ) is a multipolar state where slow waves are present. The dynamo parameters are  $E = 6 \times 10^{-5}$ , Pm = Pr = 5 and Ra = 18000.



Figure S2: Horizontal section plots at a distance z = 0.4 below the equator of the square of the total magnetic field,  $B^2$  (left panels) and the kinetic helicity  $h = \mathbf{u} \cdot \boldsymbol{\zeta}$  (right panels) for the energy-containing range of spherical harmonic degrees  $l \leq 25$ . Figures (a) and (b) represent point 'A' in figure S1 while figures (c) and (d) represent point 'B' in figure S1. The minimum and maximum values of the contours in (a), (b), (c) and (d) are [0, 17],  $[0, 2.6 \times 10^6]$ , [0, 46] and  $[0, 5.8 \times 10^6]$  respectively. The dynamo parameters are  $E = 6 \times 10^{-5}$ , Pm = Pr = 5 and Ra = 18000.

## References

- Christensen, U. R. and Aubert, J. (2006). Scaling properties of convection-driven dynamos in rotating spherical shells and application to planetary magnetic fields. *Geophys. J. Int.*, 166(1):97–114.
- Varma, A. and Sreenivasan, B. (2022). The role of slow magnetostrophic waves in the formation of the axial dipole in planetary dynamos. *Phys. Earth Planet. Inter.*, 333:106944.