Internet Appendix for

"Mutual Funds' Conditional Performance Free of Data Snooping Bias"

I. Comparison of Procedures' Estimation

In this section, we recall the procedure of BSW in estimating FDR for selecting out-performing mutual funds and illustrate the differences with our $fFDR^+$ procedure.

The starting point for both procedures is to controlling for the type I error as in Benjamini and Hochberg (1995):

$$\text{FDR} = \mathbb{E}\left(\frac{V}{\max\{R,1\}}\right) = \mathbb{E}\left(\frac{V}{R}\middle| R>0\right)\mathbb{P}(R>0) = \text{pFDR}\cdot\mathbb{P}(R>0), \tag{1}$$

where the last equality follows from (4). This implies that controlling for pFDR at a given target τ, also controls for FDR at the same target. Furthermore, for a large number of tests, controlling for pFDR and FDR is equivalent (see Storey (2002) and Storey (2003)).

Consider the m tests (2) in the absence of the covariate Z and let t_i be the test statistic of test i. Storey (2002) assumes that $t_1, \ldots, t_m$ are independent and the statuses of the null hypotheses $h_1, \ldots, h_m$ are independent Bernoulli random variables with $\mathbb{P}(h_i = 0) = \pi_0$. Additionally, across i, $(t_i|h_i = 0)$ and $(t_i|h_i = 1)$ are identically distributed. When we reject based on the p-values, for some $\lambda \in [0, 1)$, π_0 can be estimated as

$$\hat{\pi}_0(\lambda) = \frac{\#\{p_i|p_i > \lambda, i = 1, \ldots, m\}}{(1-\lambda)m} \tag{2}$$

where $\#$ returns the number of elements in the set; this estimate is conservative biased.[35] BSW

[35]Under independence, there are $m\pi_0$ funds with truly zero alpha and their p-values have a uniform distribution in $[0, 1]$. Hence, we expect $m\pi_0(1-\lambda)$ p-values in the set to fall in $[\lambda, 1]$. This number can be conservatively

choose $\lambda = \lambda^*$ on the grid $\{0.3, 0.35, \ldots, 0.7\}$ such that the mean square error (MSE) of $\hat{\pi}_0(\lambda)$ is minimal.[36] We set $\hat{\pi}_0 = \hat{\pi}_0(\lambda^*)$.

To select out-performing funds with control for the FDR, BSW define FDR$^+$ as a measure of FDR in a group of funds selected as having significant and positive estimated alphas as

$$\text{FDR}^+ = \mathbb{E}\left(\frac{V^+}{\max\{R^+, 1\}}\right). \tag{3}$$

With a significant threshold γ and a procedure which selects a fund with a positive estimated alpha whenever its p-value $\leq \gamma$, BSW estimate FDR^+ as

$$\widehat{\text{FDR}}^+_\gamma = \frac{\hat{\pi}_0 \gamma/2}{\hat{R}^+/m}, \tag{4}$$

where $\hat{R}^+$ is the empirical number of funds selected as out-performers, i.e., $\hat{R}^+ = \#\{i | p_i \leq \gamma, \hat{\alpha}_i > 0\}$. When using this approach to determine out-performing funds with controlling for FDR$^+$ at a given target τ, we estimate the FDR$^+$ based on a grid of the threshold γ and use as the rejection threshold the one that producing $\widehat{\text{FDR}}^+$ closest to the target τ. We refer to this procedure as the FDR^+.

Next, we conduct an illustration to show the differences in estimation between the $fFDR^+$ and FDR^+. For this, we opt for a sub-period of five years from 2001 to 2004 and implement the FDR^+ and $fFDR^+$ to detect positive alpha funds, with the alphas determined by the four-factor model of Carhart (1997). In this case, the R-square of the model is used as the

approximated by $\#\{p_i | p_i > \lambda\}$, from which we get (2). For a larger λ, the estimate $\hat{\pi}_0$ becomes less conservative, as there are fewer p-values under the alternative belonging to $[\lambda, 1]$, but its variance gets higher.

[36]In MSE $= \mathbb{E}(\hat{\pi}_0(\lambda) - \pi_0)^2$, the unknown π_0 is replaced by $\min_\lambda \hat{\pi}_0(\lambda)$ over the λ grid.

covariate for $fFDR^+$.[37] In Figure IA1, we demonstrate how the two procedures work. Based on the p-values of all considered funds, the FDR^+ estimates the proportion of zero-alpha funds in the whole sample, as a first step, giving $\hat{\pi}_0 \approx 0.84$. It then selects the positive estimated alpha funds with smallest p-values until the estimated $\widehat{\text{FDR}}_\gamma^+$ reaches a given FDR target. For the sake of exemplification, we choose the FDR target $\tau = 45\%$, so that both methods select a substantial number of funds.[38] Here, all funds with p-values less than or equal to $\gamma = 0.008$ are selected by the FDR^+. The threshold γ is depicted by the green dashed line in Panel C and all funds corresponding to the points on the left of the vertical line are selected. By contrast, the $fFDR^+$ considers only the set of positive estimated alpha funds and estimates the proportion of zero-alpha funds in this set as a step function of z (the quantiles of R-square).

[Insert Figure IA1 approximately here]

In this experiment, we split the sample into five bins based on the ranking of the covariate z; thus, $\hat{\pi}_0(z)$ is a function with five "steps". In this particular case, $\hat{\pi}_0(z)$ is a non-decreasing function of z. The procedure continues with the estimation of the density function $f(p, z)$ and of the functional q-value $q(p, z)$. The $fFDR^+$ selects all funds with estimated q-value less than or equal to 0.45: those funds correspond to the points below the red dashed line (the q-value $= 0.45$ line) in Panel C. This clearly shows that, for the same target, the $fFDR^+$ selects significantly more funds than FDR^+ (185 versus 16). More importantly, the funds selected by the FDR^+ are not merely a subset of those selected by $fFDR^+$. Panel D displays the distribution of the selected funds with respect to the p-value and z.

We observe that the $fFDR^+$ assigns more weight to some funds with smaller z (thus,

[37]The details of the funds and the calculation of the p-values are deferred to Section VI.

[38]If we choose any target $\tau \leq 40\%$, the FDR^+ selects no funds.

smaller R-square), but the weight is not equally distributed across the funds with the same level of z. To explain this, we investigate further the second component of the posterior $r(p, z)$, the density function $f(p, z)$. We produce in Figure IA2 the heatmap of the density function and see that the value of $f(p, z)$ is higher where z is small. This combines with the low value of $\pi_0(z)$ for small z, which is presented in Panel B of Figure IA1, implying a low value of the posterior probability of being null $r(p, z)$ – which is used to determine the threshold for rejecting the null. Consequently, more funds with small z are selected by the $fFDR^+$ as profitable, regardless of the fact that the $p-$value may get some relatively high values (0.2–0.6) as shown in Panel D of Figure IA1.

[Insert Figure IA2 approximately here]

II. Simulation Execution

We summarize the simulation procedure as follows.

As a first step, we generate the covariate and alpha for each of the m funds. We generate the covariate vector $(z_1, z_2, \ldots, z_m)$ with each element drawn from the uniform distribution $[0, 1]$ and assign them to the funds. For the cases (11) or (12), we determine c in (14) such that $\int_0^1 \pi_0(z)dz = \pi_0$ for a given $\pi_0 > 0$. For each fund i, we draw h_i from the Bernoulli distribution with success probability $1 - \pi_0(z_i)$ and assign a zero alpha to fund i with $h_i = 0$. Finally, for the remaining funds, we draw true non-zero alphas from the given distribution (11) or (12) and assign them such that a fund with a smaller z has a smaller alpha. For the case (13), we draw alphas from the distribution and then assign them to the funds; again, a fund with a smaller z has a smaller alpha.

In the second step, we simulate the return factors from a normal distribution with parameters equal to their sample counterparts. The loadings of these factors are also drawn from a normal distribution with parameters equal to their sample counterparts obtained from the fund level estimation of equation (10). We consider balanced panel data for $2,000$ funds with 284 time-series observations; the number of $2,000$ is chosen to be close to our real sample of 2,224 funds, whereas the number of 284 periods is the median of our sample funds' observations. In unbalanced panel data, the number of observations for each fund is drawn randomly with replacement from the set of the number of observations of the funds in the real-data counterpart. Under cross-sectional independence, the noise term $\varepsilon_{i,t}$ is drawn from a normal distribution $\mathcal{N}(0, \sigma_\varepsilon^2)$, where, as in Barras et al. (2020), σ_ε is set equal to the median of its real-data counterpart, that is, approximately 0.0183 for our sample. Under cross-sectional dependence, we follow Barras et al. (2010) (BSW henceforth) and assume that all fund residuals load on a common latent factor G_t, whereas the out-performing and under-performing funds load on the specific factors G_t^+ and G_t^-, respectively. Thus,

$$\varepsilon_{i,t} = \gamma G_t + \gamma G_t^+ \mathbb{1}_{\alpha_i > 0} + \gamma G_t^- \mathbb{1}_{\alpha_i < 0} + \eta_{i,t}, \tag{5}$$

where $\mathbb{1}_{\alpha_i > 0}$ and $\mathbb{1}_{\alpha_i < 0}$ are, respectively, out-performing and under-performing indicators taking the value 1 if the fund i is out-performing or under-performing, and 0 otherwise. The three latent factors G_t, G_t^+ and G_t^- are assumed to be mutually orthogonal and to the four risk factors and have a normal distribution $\mathcal{N}(0, \sigma_G^2)$, where, from BSW, σ_G is set equal to the average of the monthly standard deviations of the three risk factors (size, book-to-market and momentum). The coefficient γ is set equal to the average of the loading of the three risk factors of the $2{,}224$ funds in our sample. Finally, $\{\eta_{i,t}\}_i$ are uncorrelated and drawn from the normal distribution $\mathcal{N}(0, \sigma_\eta^2)$, where σ_η is chosen such that σ_ε equates to the median of its real-data counterpart, as in the independent case.

In the last step, we implement the $fFDR^+$ and FDR^+ and compute their performance metrics. More specifically, based on the simulated data from the previous step, we calculate the Carhart four-factor model alpha and the corresponding p-value for each fund. We use the resulting p-value, the estimated alpha and the covariate as inputs to the $fFDR^+$ and FDR^+ procedures. At a given target of FDR, we calculate for each method a ratio of falsely classified funds $\widetilde{FDR}^+$ and a detected ratio $\widetilde{Power}^+$:

$$\widetilde{FDR}^+ = \frac{\widetilde{V}^+}{\max\left\{\widetilde{R}^+, 1\right\}} \quad \text{and} \quad \widetilde{Power}^+ = \frac{\widetilde{C}^+}{\widetilde{T}^+}, \tag{6}$$

where $\widetilde{R}^+$ is the number of classified out-performing funds and, among them, $\widetilde{V}^+$ funds are truly zero-alpha or under-performing funds. $\widetilde{T}^+$ is the number of truly out-performing funds in the population and, among them, $\widetilde{C}^+$ funds are classified correctly.

The previous three steps are repeated $1{,}000$ times and we use the average $\widetilde{FDR}^+$ and $\widetilde{Power}^+$ as estimates for the actual FDR and power.

III. Variance Comparison of FDR Estimation

In this section, we investigate the performance of the two methods in terms of FDR estimation variance. As described in Section II, the actual FDR of the two methods is estimated by the average of the ratio of falsely classified funds $\widetilde{FDR}^+$. As the iterations are independent, the variance of the estimated actual FDR is proportionate to the variance of the $\widetilde{FDR}^+$. In

Figures IA3, IA4 and IA5, we report the gap in variance of the $\widetilde{FDR}^{+}$ of the FDR^{+} over the $fFDR^{+}$. We observe that the gap curves are either varying close to zero or positive for most of the cases of the distributions. This implies that the variance of the estimated actual FDR of the $fFDR^{+}$ is less than that of the FDR^{+}.

[Insert Figure IA3 approximately here]

[Insert Figure IA4 approximately here]

[Insert Figure IA5 approximately here]

IV. Additional Simulation Results

In supplementing Section V of the main manuscript, we present here the performance of the $fFDR^{+}$ in terms of FDR control and power under several settings. We first present the performance of $fFDR^{+}$, where $\pi_0(z)$ can take three different forms. We then show the results corresponding to balanced panel data under cross-sectional dependence, before proceeding to the results for unbalanced panel data under both cross-sectional independence and dependence. Finally, we exhibit simulation results for the case where alphas are drawn from a single normal distribution.

A. Results for balanced panel data under cross-sectional dependence

We present in Figures IA6–IA8 the cases where the data are generated as balanced panels under cross-sectional dependent errors. The comparisons in terms of power between $fFDR^{+}$ and FDR^{+} are reported in Tables IA2–IA6.

[Insert Figure IA6 approximately here]

[Insert Figure IA7 approximately here]

[Insert Figure IA8 approximately here]

[Insert Table IA2 approximately here]

[Insert Table IA3 approximately here]

[Insert Table IA4 approximately here]

[Insert Table IA6 approximately here]

B. Results for unbalanced panel data

We present the performance of the $fFDR^{+}$ under both cross-sectional independence and dependence. Figures IA9–IA11 depict the FDR control of the $fFDR^{+}$, while the power comparisons are given in Tables IA7–IA9.

[Insert Figure IA9 approximately here]

[Insert Figure IA10 approximately here]

[Insert Figure IA11 approximately here]

[Insert Table IA7 approximately here]

[Insert Table IA8 approximately here]

[Insert Table IA9 approximately here]

C. Simulation results for single normal distribution

We present the simulation results for a special case of continuous distribution where the mixture (13) has only one component. Specifically, we consider the case $\pi_2 = 0$, $\alpha \sim \mathcal{N}(\mu, \sigma^2)$ and, based on Jones and Shanken (2005) and Fama and French (2010), we use $\mu \in \{-0.8, -0.5, 0\}$ and $\sigma \in \{1, 1.5, 2, 2.5, 3\}$ (both parameters' values are annualized and in % terms).[39]

Figures IA12 and IA13 present the performance of the $fFDR^+$ procedure when the alphas are drawn from balanced and unbalanced panel data, respectively. It is shown that the FDR is controlled at any given target.

[Insert Figure IA12 approximately here]

[Insert Figure IA13 approximately here]

In Table IA10, we compare the performance of $fFDR^+$ and FDR^+ in terms of power. As π^+ depends on both the mean μ and variance σ^2 of the distribution, we need to distinguish the value of π^+ from the pairs (μ, σ). We provide in Panel A additional information about π^+, which helps us assess the impact of the magnitude of positive alphas on the power. For instance, for $\pi^+ \approx 40\%$, the power of the two procedures for $(\mu, \sigma) = (-0.8, 3)$ is significantly higher than for $(\mu, \sigma) = (-0.5, 2)$. We observe a boost in power for both methods with increasing σ (for given non-positive μ), resulting in larger proportion and magnitude of positive alphas. In all the cases

[39] Jones and Shanken (2005) assume that the fund alphas are drawn from a normal distribution and their estimates for the mean and standard deviation are based on prior beliefs. They find that the mean is 1.3%-1.4% per annum before expenses (about 2%) and the standard deviation is 1.5%-1.8%. In addition, Fama and French (2010) assume that the fund (gross) alpha population has a normal distribution centered at 0.

under consideration, the $fFDR^+$ dominates FDR^+ in terms of power and this gap soon becomes omnipresent for $\sigma \geq 1.5$ reaching up to 18%.

[Insert Table IA10 approximately here]

D. Results for alternative forms of $\pi_0(z)$

We consider three forms of $\pi_0(z)$, including decreasing, increasing and constant with respect to z. For the first two cases, we specify $\pi_0(z)$ based on $f(z) = -1.5(z - 0.5)^3 + c$ or $f(z) = 1.5(z - 0.5)^3 + c$. In the interest of space, we present in Tables IA11–IA13 results in terms of power for the mass distribution of alphas with balanced panel data which is generated under cross-sectional independence. For all forms of $\pi_0(z)$, even when this is constant, we conclude similarly to the case of $\pi_0(z)$ with an up-and-down shape presented in the main manuscript. Results for other distributions as well as under cross-sectional dependence convey the same message and are available upon request.

[Insert Table IA11 approximately here]

[Insert Table IA12 approximately here]

[Insert Table IA13 approximately here]

V. Performance of $fFDR^+$ when Using a Non-informative Covariate

In what follows, we present the simulation results when a non-informative covariate is used instead of the informative as in the simulations in the main paper. The simulated data is the same as in the main paper, except that for each iteration a covariate is drawn randomly from the uniform distribution on $[0, 1]$ and is used as covariate input of the $fFDR^{+}$. This covariate is non-informative in that it has no connection to the true alpha of funds and, thus, no information for detecting truly positive alpha funds. We see that the $fFDR^{+}$ controls well FDR under all alpha distributions, similarly to Figures 1, 2 and 3 main paper. In the interest of space, these results are not reported but can be made available upon request. In terms of power, the $fFDR^{+}$ with use of the non-informative covariate performs very similarly to the FDR^{+} as exhibited in Table IA14.

[Insert Table IA14 approximately here]

VI. Varying Number of Observations and Funds

In the simulations in the main paper, we have assumed a sample of $m = 2{,}000$ funds, which reflects our actual dataset for the whole period from 1975 to 2022. When constructing a portfolio, we usually use sub-periods of five years and the number of alive funds in these sub-periods naturally varies. In this section, we investigate the impact of varying number of observations T per fund and the number of funds m on the power.

In Table IA15, we present the outcomes for different underlying distributions of fund alphas, when we control FDR at a 10% target and use balanced panel data with cross-sectional independence. We vary m from 500 to $3{,}000$ and T from 120 months (i.e., 10 years) to 420 months (i.e., 35 years). It is evident from the reports that the power of the $fFDR^{+}$ increases at a much faster pace with increasing T. The power of the $fFDR^{+}$ slightly decreases with rising m, whereas such is observed for the FDR^{+} mainly in Panel C. This is not a substantial concern, though, as in reality we do not have a very large number of alive funds in a given sub-period. Overall, the power difference between the $fFDR^{+}$ and the FDR^{+} can reach 34%.

[Insert Table IA15 approximately here]

For $T = 120$, both procedures have low power. Empirically, when constructing a portfolio of mutual funds, we usually use in-sample sub-periods of 5 years. In these cases, the investors may have to raise the FDR target to a higher level as explained in the previous section.[40] In Table IA16, we focus the spotlight on (small) $m = 500$ and $T = 60$ (i.e., 5 years). It is shown there that both methods yield even lower power at the FDR target of 10%. By increasing the target, the power of the $fFDR^{+}$ in detecting out-performing funds rises faster than that of the FDR^{+}, especially for the discrete and mixed normal distributions.

[Insert Table IA16 approximately here]

VII. Estimation Errors in Covariates

In the simulations in the main paper, we consider a simple covariate where in the set of *non-zero* alpha funds, the ranking of the funds' alpha is the same as that of the funds' covariate. This does not hold in the whole population. Put differently, one cannot simply rank the funds based on a covariate to distinguish the out-performing funds from the zero-alpha and the under-performing ones.

Here, we further study the behaviour of our $fFDR^{+}$ approach by adding a noise to the original covariate that reflects potential estimation biases, as all covariates in the real data are

[40] In fact, in order to construct non-empty FDR-based portfolios with use of five-year in-samples, BSW introduce a procedure where they allow the estimate of FDR to be above 70% for several years.

calculated based on a certain sample period. More specifically, instead of using the covariate Z as in our previous simulations, we use $Z' = (z'_1, \ldots, z'_m)$ given by

$$z'_i = z_i + \eta_i, \tag{7}$$

where η_i denotes the noise and is generated independently from a normal distribution $N(0, \sigma_\eta^2)$. Alternatively Z' can be viewed as a realization of some fund characteristic which aims to capture Z. Depending on the scale of the estimation error, the realized covariate can have different levels of information. In reality, we do not know the actual estimation errors in the covariates. Thus, we simulate low to high noise in our covariates. In particular, we consider two different values of σ_η including $\sigma_1 = 0.5/\sqrt{12}$ and $\sigma_2 = 1/\sqrt{12}$. These values are based on the fact that the covariate $Z \sim U[0, 1]$, which has a standard deviation of $1/\sqrt{12}$. We confirm that the $fFDR^+$ controls well for the FDR in this setting and the figures are virtually the same as those presented in Section V.A in the main paper. This is the most important characteristic of the $fFDR^+$ we would expect, that is, the ability to control well for the risk even when the new information contains noise.

In Table IA17, we provide further information by presenting the power (at FDR target of 10%) of the $fFDR^+$. Comparing with Table 1, the power is lower but still remarkably higher than that of the FDR^+ with a varying gap across cases of the alpha distribution and the choice of σ_η. As will be shown in our empirical analysis, the $fFDR^+$ with use of each covariate gains significant power over the FDR^+. Therefore, we can assume that the covariates in our application have relatively less noise than the ones in this simulation.

[Insert Table IA17 approximately here]

VIII. Fund Characteristics as Informative Covariates

As part of our empirical investigation of the $fFDR^+$ approach, we consider six covariates that may convey information about the performance of mutual funds. They are shown to be persistent and, therefore, can predict the performance of mutual funds. We also propose four new covariates based on asset pricing models.

First, we study the R-square of Amihud and Goyenko (2013), which is estimated from the Carhart four-factor model and measures the activeness of a fund. When a fund replicates the market, the R-square is close to one; if, instead, it is more active, it has a small R-square and, in this case, according to the authors, funds tend to perform better.

The second covariate is the Fund Size of Harvey and Liu (2017). This takes into account both the fund size, which is the total net assets under management (TNA) of a fund, and the industry size, which is the total assets under management of all active mutual funds in the sample (sum of TNA). More specifically, for fund i at time t, it is defined as

$$\text{Fund Size}_{i,t} = \ln \frac{\text{TNA}_{i,t}}{\text{IndustrySize}_t} - \ln \frac{\text{TNA}_{i,0^*}}{\text{IndustrySize}_{0^*}}, \tag{8}$$

where $t = 0^*$ corresponds to the time of the first TNA observation in our sample. The Fund Size reflects the growth in scale of a fund relative to the whole active mutual fund market. Harvey and Liu (2017) show a significant negative relationship between Fund Size and funds' performance.[41]

[41] Pastor, Stambaugh, and Taylor (2015) and Chen, Hong, Huang, and Kubik (2004) as well as Zhu (2018), respectively, argue that the industry size and the fund size (approximated by the logarithm of the fund's TNA) have a negative impact on the funds' performance. We use the Fund Size of Harvey and Liu (2017) as it incorporates information of both covariates. Other studies on the relationship between fund size and performance and funds'

The third covariate is the Return Gap of Kacperczyk et al. (2008), which aims to reflect the unobserved actions of the funds. Mutual funds usually disclose their portfolio holdings and return periodically, e.g., quarterly or semi-annually. The investors are unaware of the funds' trading activities in the period of consecutive reports. The Return Gap of a fund is defined as the difference between the return that is disclosed by the fund and the return that the fund would have based on disclosure of its last portfolio holdings. Kacperczyk et al. (2008) show that the funds' performance can be predicted by their past return gaps; mutual funds with higher past return gap tend to perform better in the future.

Our fourth covariate is the Active Weight of Doshi et al. (2015), which aims to gauge the fund's activeness level and is given by the sum of the absolute differences of the stock value weights and the actual weights that the fund assigns to the stocks in its portfolio holdings. They show that funds with higher active weight tend to perform better. We note that the active weight is also related to the fund's turnover, which plays a role in explaining performance as pointed out in Pastor, Stambaugh, and Taylor (2017) and BSW. To obtain meaningful values for the active weight and the return gap, as in Kacperczyk et al. (2008) and Doshi et al. (2015), we require each mutual fund to hold at least 10 stocks in its portfolio at any time.

The fifth covariate is the Fund Flow. The interaction of fund flow and funds' performance has been studied quite extensively such as in Sirri and Tufano (1998), Berk and Green (2004), Harvey and Liu (2017), Capponi, Glasserman, and Weber (2020) and Bessembinder, Chen, Cooper, Xue, and Zhang (2023), among others. Zheng (1999), in particular, discovers that funds receiving money perform better than those that lose money. The author also shows that investors

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holding liquidity (e.g., Yan (2008)) or funds' merger (i.e., McLemore (2019)) document the same conclusion. Fund size is also strongly related to skills as great investment idea is difficult to scaled up (see Barras et al. (2022)).

can earn abnormal returns using small funds' flow information. Here, we follow Bris, Gulen, Kadiyala, and Rau (2007) and define Fund Flow at time t as

$$\text{Fund Flow}_t = \frac{\text{TNA}_t - (1 + r_t)\text{TNA}_{t-1}}{(1 + r_t)\text{TNA}_{t-1}}, \tag{9}$$

where r_t is the return of the fund in the period $t - 1$ to t.

Finally, we study the information carried by expenses and fees, which is reflected in expense ratio. The impact and informativeness of this funds' characteristic on active mutual fund performance has been discussed by Berk and Green (2004), BSW, and Berk and van Binsbergen (2015).

IX. Data-based Simulations

In this simulation experiment, we design a setting close to our later empirical exercises, in which the simulation data retains the dependencies among alphas and covariates as in the real data.[42] Since we construct portfolios based on in-sample of five-year data (as will be presented in Section VI in the main paper), we opt data on returns and covariates of all funds in a five-year period from January 1999 to December 2003. This is the period that offers us the largest number of funds (1,567) having all ten covariates.

[42]We additionally conduct a simulation set which is similar to BSW and Andrikogiannopoulou and Papakonstantinou (2019). Our conclusions on the advantages of the covariate-augmented method remain under this setting regardless of different alpha distributions, balanced and unbalanced data structures, and cross-sectional dependent or independent error terms. For the sake of space, the results are available upon request.

A. Data generating process

The data generating process is as follows.

- First, we calculate each fund's alpha, beta coefficients (b, s, h, m) and residuals of the Carhart four-factor model. We then calculate the correlation coefficients between alpha and all betas (b, s, h, m) across all 1,567 funds. For each fund, we also calculate the covariates mentioned in Section VIII where R-square, Beta, Sigma, Sharpe and Treynor ratios are from the asset pricing models, whereas Active Weight, Return Gap, FundSize, Expense Ratio and Fund Flow are obtained from averaging the available values realised over the five years.[43] Similarly, we calculate the correlation coefficient matrix of 11 vectors for alpha and ten covariates across 1,567 funds. To determine the probability of being truly zero-alpha of each fund, we estimate $\pi_0(z)$ corresponding to each of the 10 covariates, then average across the covariates to have an empirically representative $\hat{\pi}_0(z)$.

- Second, in each iteration of the simulation,

  – we generate simulated alphas for funds such that the correlation coefficients between the alpha and the b, s, h, m are the same as in the real counterpart. We then assign zero for zero-alpha funds based on the representative $\hat{\pi}_0(z)$. We denote the simulated alpha as α^s.

  – we generate 10 simulated covariates such that the matrix of correlation coefficients

[43]The results are similar if we instead use the average over the final year. Averaging available values of each covariate over the five years gives us a higher number of funds as there are a number of funds missing covariate values in the final year.

among the 11 vectors including covariates and α^s is the same as the real counterpart calculated above.[44]

– we generate the simulated return of each fund via the following formula

$$R_{i,t} = \alpha_i^s + b_i.R_{mkt,t} + s_i.SMB_t + h_i.HML_t + m_i.Mom_t + \epsilon_{it}, \tag{10}$$

where the noise $\epsilon_{i,t}$ is randomly drawn from the collected residuals via the stationary bootstrap procedure of Politis and Romano (1994) with an average block length of 10 following literature.

– we regress the simulated returns of each fund on the four factors to obtain $\hat{\alpha}$ and calculate the related $p-$value (based on two-sided $t-$tests).

– then, generated covariates are transformed to a unit interval $[0, 1]$ as the formula described in Section II.A, and we implement the FDR^+ and $fFDR^+$ procedures, control for FDR at predetermined targets, to detect truly positive Carhart alpha funds with use of the $\hat{\alpha}$s, calculated $p-$values and simulated covariates.

- In each iteration, by comparing the simulated α^s and the selected out-performing funds, we compute the rate of falsely selected funds among those classified as out-performers and the rate of truly out-performing funds detected. The two metrics are averaged across 1,000 replications to obtain estimates for the actual FDR and the power of each procedure.

We present our analyses of the performance of the $fFDR^+$ with use of each covariate. Figure IA14 depicts the performance of the $fFDR^+$ in terms of FDR control in its left panel and

[44] We present details in Section B the procedure generating the correlated vectors.

power in its right panel (presented by thin dotted or dashed lines) and compare with that of the FDR^+ (presented by a thick red solid line). First, it is clear that all procedures asymptotically control for FDR at any given target (from 0.05 to 0.95), and the $fFDR^+$ with use of any one of the covariates, gains higher power than the FDR^+. Second, the covariates differ in informativeness level, for example, the Sigma gains higher power with a gap of around 10% compared to the FDR^+, while the Expense Ratio and Fund Flow are the least powerful among the $fFDR^+$ ones.

[Insert Figure IA14 approximately here]

B. Dependent data generating process

In this section, we formally present our procedure to generate dependent data described in Section IX. Given a set of linearly independent vectors $\{X_1, \ldots, X_k\}$ in vector space $\mathbb{R}^n$, $n > k$ and (column) vector of correlation coefficients $\rho = (\rho_1, \ldots, \rho_k)'$, we generate a vector Y in $\mathbb{R}^n$ such that correlation coefficient of Y and X_i is ρ_i, $i = 1, \ldots, k$. The mechanism to generate Y is designed as follows.[45]

First, we scale $X_1, \ldots, X_k$ so that each of them has mean zero and standard deviation of one and denote by X the matrix with columns the X_is. To ease notation, we keep using these notations after scaling. Note that the correlation coefficient of Y and X_i is not effected by the aforementioned scaling.

Next, we generate a vector U of length n from the Gaussian distribution. We denote the residuals of the multivariate regression of U on X by e, which is in $\mathbb{R}^n$. We find

[45] Readers can easily implement the mechanism using the freely available `R` package `faux` of DeBruine (2021).

$X^* = \{X_1^*, \ldots, X_k^*\}$ such that the scalar product $X_i \cdot X_j^* = 1$ if $i = j$ and 0 if otherwise. This can be done via singular value decomposition $X = TDV'$, where T and V are orthogonal matrices of sizes $n \times n$ and $k \times k$, respectively, and D a $n \times k$ matrix with zeros everywhere except the main diagonal elements which are positive. The columns of $W = T\tilde{D}V'$ form the X^*, where $\tilde{D}$ satisfies $D'\tilde{D} = I_k$. Indeed, $X'W = VD'T'T\tilde{D}V' = I_k$ which is identity matrix of size k.

Finally, set $Y = W\rho + \sigma e$ where $\sigma^2 = \frac{1-\rho' Cov(W)\rho}{Var(e)}$ and Cov and Var are covariance and variance, respectively. We have that $Cov(Y, X_i) = Cov(W\rho + \sigma e, X_i) = Cov(W\rho, X_i) = \sum_{j=1}^k \rho_j Cov(X_j^*, X_i) = \sum_{j=1}^k \rho_j [\mathbb{E}(X_j^* X_i) - \mathbb{E}X_j^* \mathbb{E}X_i] = \rho_i$ since $\mathbb{E}X_i = 0$ and $\mathbb{E}(X_j^* X_i) = 0$ for $i \neq j$ and 1 otherwise. From the scaling step $Var(X_i) = 1$, and it is easy to see that $Var(Y) = Var(W\rho) + \sigma^2 Var(e) = \rho' Cov(W)\rho + \sigma^2 Var(e) = 1$. Thus, the correlation coefficient of Y and X_i is $\frac{Cov(Y,X_i)}{\sqrt{Var(Y)Var(X_i)}} = \rho_i$.

When generating the simulated α^s, the X consists of b, s, h, m, and ρ consists of correlation coefficients of α and each of the b, s, h, m.

When generating the 10 simulated covariates, given α^s and the real correlation coefficient matrix of 10 vectors, the first covariate Z_1 is generated such that its correlation coefficient with α^s equals the real one. The second covariates Z_2 is generated such that its correlation coefficients with Z_1 and α^s are the same as the real ones, and so on. The final covariate Z_9 is generated such that its correlation coefficients with $Z_1, \ldots, Z_9$ and α^s are the same those in the real correlation coefficient matrix. It is also noted that, in our simulation, $n =$1,567 which is a very large number comparing to 11. Thus, the k' generated vectors (α^s and $k' - 1$ covariates) are likely to be linear independent in $\mathbb{R}^n$, thus the $(k'+1)^{th}$ vector can be generated, $k' = 1, \ldots, 10$.

X. Results for Alternative Target of FDR

In this section, we repeat the exercise with the FDR target of 20%. Figure IA17 presents the alpha evolution of the individual covariates. Table IA18 shows the average n-year alpha of those portfolios. Finally, Table IA19 presents the statistic metrics for all mentioned portfolios.

[Insert Figure IA17 approximately here]

[Insert Table IA18 approximately here]

[Insert Table IA19 approximately here]

XI. Results from Using an Alternative Proxy of Covariates

Here, we present in Figure IA18 the alpha evolution of $fFDR10\%$ portfolios where the proxy for each covariate is based on whole data in the in-sample period instead of the data in the final year as in the main manuscript. We see that the performance of the portfolios does not vary significantly.

[Insert Figure IA18 approximately here]

XII. Comparison of Portfolios' Trading Metrics

Here, we evaluate our portfolios in regard to a set of trading metrics, including the annualized estimated alpha $\hat{\alpha}$ of the Carhart four-factor model, its bootstrap p-value and t-statistic (with use of heteroskedasticity and autocorrelation-consistent standard error), the annual standard deviation of the four-factor model residuals ($\hat{\sigma}_{\varepsilon}$), the geometric mean return in excess of the one-month T-bill rate, the annual Sharpe ratio and the annual Information Ratio $\hat{\alpha}/\hat{\sigma}_{\varepsilon}$. All metrics are presented in Table IA1.

[Insert Table IA1 approximately here]

XIII. Wealth Evolution

In Figure 4 in the main paper, we study the alpha evolution of the portfolios over time. However, an investor may be interested in the gain in value. Figure IA15 shows the growth of 1 dollar that the investor invests in each portfolio at the beginning of 1982. Ultimately, at the end of 2022, this amount grows to about 68 dollars if she chooses the $fFDR10\%$ portfolio with Active Weight as the covariate, as opposed to just 41, 51 and 48 dollars with the $FDR10\%$, the equal weight plus and equally weighted portfolios, respectively. This exercise reveals the potential profitability of an investor who had a perfect oracle in 1982 about the methods and the covariate that would play out over the next 41 years.

[Insert Figure IA15 approximately here]

Similarly to Figure IA15, in Figure IA16 we depict the wealth evolution of one dollar invested in the $fFDR10\%$ portfolios based on the combined covariates. At the end of 2022, 1 dollar grows to about 64 to 76 dollars if the investor invests in one of the $fFDR10\%$ portfolios with the covariates obtained from LASSO, elastic net, Ridge, PC1 and OLS regressions.

[Insert Figure IA16 approximately here]

XIV. Sub-period Performance

By construction, Figure 4 in the main paper contains returns which start from January 1982 and are not representative of the recent mutual fund performance. In order to investigate the contribution of the returns over different periods to the performance of the portfolios, we split the whole period into four non-overlapping sub-periods including the first three decades 1982–1991 (P1), 1992–2001 (P2), 2002–2011 (P3), and the remainder. For the final sub-period, we consider separately the part that does not cover the Covid-19 pandemic, that is, 2012–2019 (P4a), and the part that does, that is, 2012–2022 (P4b). We present in Table IA20 the alpha, its $t-$statistic and Sharpe ratio of portfolios (with a FDR target $\tau = 10\%$) in the sub-periods.

[Insert Table IA20 approximately here]

In terms of alphas, it is clear that all portfolios performing well in the first two sub-periods suffer a decline in the third sub-period. In P3, we observe negative alphas for the $FDR10\%$ portfolio and the $fFDR10\%$ portfolios with FundSize, Active Weight, Return Gap, Expense Ratio covariates and combined covariates based on shrinkage methods. Note that this sub-period suffers from the global financial crisis 2007–2008. In the sub-period P4a, this decrease continues for 6 out of 15 $fFDR10\%$ portfolios (including those with combined covariates) while the others rebounce. Moving from P3 to P4b, we find that alphas of all portfolios are decreasing and this reflects the severe impact of the pandemic. The t-statistic columns show that most portfolios have significantly positive alphas in the first sub-period. Interestingly, for the Sharpe ratio, we witness the highest reports in the sub-period P4a and even in P4b. This also occurs in the Equal Weight portfolio, that is, the portfolio that selects all the eligible funds in the in-sample windows and invests them equally in the following year; thus, the high Sharpe ratio in the final sub-period partially comes from the whole mutual fund market. The Equal Weight Plus portfolio, which invests in all funds with positive estimated alphas in the previous five years, is always better than the Equal Weight one. The alphas of the $fFDR10\%$ portfolios, by contrast, are nuanced depending on the covariate used; except in the sub-period P4b, most of them beat the equally weighted one in all the other sub-periods in terms of alpha (with notable exceptions of the FundSize, Active Weight and Return Gap covariates in the third sub-period).

XV. Comparison to Sorting Portfolios

First, we describe the single- and double-sorting portfolios which are traditionally constructed in the literature. Specifically, the single-sorting portfolios based on a covariate are as in Kacperczyk et al. (2008) and Doshi et al. (2015), and the double-sorting based on a covariate and the past alpha are as in Amihud and Goyenko (2013).

To construct the single-sorting portfolio for each covariate, at the end of each year from 1981, all the mutual funds are sorted into deciles (quintiles) according to the given covariate. For the covariate that has a negative/positive relationship with the performance of the funds, the funds in the bottom/top deciles (quintiles) are selected. These form a portfolio to be invested in the following year. To form the double-sorting portfolio, the funds selected in the single-sorting portfolio are again sorted into decile (quintile) according to the past alpha. The funds in the top decile (quintile) form the portfolio to be invested in the following year. This process is rolled forward until the end of the sample period. For consistency with the $fFDR$ portfolios, we use the same type of 5-year rolling window, i.e., each time we use the aforementioned observed covariates and the alpha and covariates calculated based on the last five years.

The performance in terms of alpha of those portfolios from 1982 to 2022 is presented in Table IA21. Our results suggest that most of the sorting portfolios, except the Active Weight and Sharpe ratio, have negative or negligible positive alphas at the end of 2022, which contrasts with the assumption of a linear relationship between the covariate and the funds' performance.

Obviously, sorted portfolios perform better if they are based on the correct sign of the correlation between the underlying covariate and our funds' performance. We see that the portfolios based on $fFDR$ gain significant positive alphas and beat the corresponding sorted portfolios in most cases.

[Insert Table IA21 approximately here]

XVI. $FDR\tau$ Portfolios Conditional on Covariates

In what follows, we combine fund characteristics and FDR^{+} to construct $FDR\tau$ portfolios conditional on covariates. For each covariate, we first partition the sample into quintiles based on the value of each mutual fund's covariate (in ascending order). We then implement the BSW approach to select funds in each quintile, generating five $FDR10\%$ portfolios. The portfolios are constructed in the same fashion as the $FDR10\%$ portfolios in the main paper. This way we are able to examine the performance of the portfolio of selected funds in each quintile.

Table IA22 presents the portfolios' alphas based on monthly fund returns from 1982 to 2022. For convenience, suppose we look at the R-square covariate. Starting from the end of December 1981, we use past five years historical data to calculate inputs including the $p-$value, estimated alpha and R-square. We partition the funds into quintiles based on the covariate. For each quintile $q = 1, \ldots, 5$, we implement the FDR^{+} procedure of BSW at FDR target of 10% to select a group of funds invested in the year 1982. At the end of December 1982, we repeat the process: we calculate the $p-$value, the alpha estimate and the R-square based on the past five years' data; we partition funds into quintiles, and for each of them we implement the FDR^{+} to select funds to invest in 1983. We repeat this process until the end of December 2021 to select funds for each quintile q portfolio. For ten covariates, we obtain $10 \times 5 = 50$ portfolios which we present in Table IA22.

[Insert Table IA22 approximately here]

We observe that among the five well-known fund characteristics (R-square, Active Weight, FundSize, Return Gap and Fund Flow), only Active Weight shows a clear pattern in which the portfolio of funds from the group of higher active weight performs better. Our experiment suggests that, except from Fund Flow case, the two-step procedure (i.e., first ranking funds based on a covariate and then implementing the $FDR10\%$) does not exploit the informativeness of the covariate or offer a clear strategy. This implies that our $fFDR$ method cannot be substituted by or be considered as overlapping with the two-step procedure.

XVII. Restricted Data

As supplementary to our empirical study of Section VI, we repeat here our experiments for a data subset where a mutual fund enters the sample when its TNA reaches $\$15$ million (adjusted for inflation as of January 2022). This choice of threshold is consistent with Pastor et al. (2015) and Zhu (2018). Figure IA19 exhibits the alpha evolution while Table IA23 shows the average n-year alpha for the $fFDR10\%$ portfolios based on each individual covariate. Similarly, we present in Figure IA20 and Table IA24 the $fFDR10\%$ results of the portfolios based on combinations of the covariates.

[Insert Figure IA19 approximately here]

[Insert Table IA23 approximately here]

[Insert Figure IA20 approximately here]

[Insert Table IA24 approximately here]

XVIII. Selecting Unprofitable Funds with fFDR

In this part, we obtain, by analogy to the $fFDR\tau$ portfolio, a selection of unprofitable funds. First, consider a selection of R^- under-performing funds including V^- wrongly selected zero-alpha or out-performing funds. We define

$$\text{FDR}^- = \mathbb{E}\left(\frac{V^-}{\max\{R^-, 1\}}\right) \tag{11}$$

and

$$\text{pFDR}^- = \mathbb{E}\left(\frac{V^-}{R^-}\middle| R^- > 0\right). \tag{12}$$

If a fund i with p-value p_i and negative estimated alpha ($\hat{\alpha}_i < 0$) is selected as under-performing fund whenever $p_i < \gamma$, then FDR^- is estimated as

$$\widehat{\text{FDR}}^-_\gamma = \frac{\hat{\pi}_0 \gamma/2}{\hat{R}^-/m} \tag{13}$$

where $\hat{R}^- = \#\{i|p_i < \gamma, \hat{\alpha}_i < 0\}$ and $\hat{\pi}_0$ is calculated as in equation (2) in the main manuscript.

At a given target τ of FDR^-, we form the $FDR^-\tau$ ($fFDR^-\tau$) portfolio of under-performing funds similarly to the $FDR\tau$ ($fFDR\tau$) portfolio of out-performing funds. Specifically, we establish the $FDR^-\tau$ portfolio using the same γ grid as for the $FDR\tau$ and form the $fFDR^-\tau$ portfolio by implementing the $fFDR$ procedure (with a specific covariate) on the set of non-positive estimated alpha funds to control pFDR$^-$ at the same level as the portfolio $FDR^-\tau$. The following tables present the average n-year alpha of the portfolios at target

$\tau = 10\%$ (Table IA25) and their trading metrics (Table IA26). We also construct the Equal Weight Minus portfolio, which includes all funds with negative estimated in-sample alpha invested in the following year.

[Insert Table IA25 approximately here]

[Insert Table IA26 approximately here]

TABLE IA1

Comparison of performance statistics of all considered portfolios with $\tau = 10\%$

The table compares the portfolios with regard to metrics including the annual Carhart four-factor alpha ($\hat{\alpha}$, in $\%$) with its bootstrap p-value and t-statistic (with use of Newey–West heteroskedasticity and autocorrelation-consistent standard error), the annual standard deviation of the four-factor model residuals ($\hat{\sigma}_{\varepsilon}$, in $\%$), the mean return in excess of the one-month T-bill rate (in %), the annual Sharpe ratio and the annual Information Ratio ($IR = \hat{\alpha}/\hat{\sigma}_{\varepsilon}$). Panel A reports the metrics calculated based on the portfolios' return from January 1982 to December 2022 while Panel B reports those based on return from January 1982 to December 2019.

Covariate	$\hat{\alpha}$ (p-value)	t-statistic	$\hat{\sigma}_{\varepsilon}$	Mean Return	Sharpe Ratio	IR
Panel A: Whole sample						
R-square	0.84 (0.41)	0.83	4.97	6.71	0.51	0.17
Fund Size	0.20 (0.89)	0.19	4.98	6.13	0.46	0.04
Active Weight	0.73 (0.41)	0.84	4.16	7.09	0.51	0.18
Return Gap	0.31 (0.75)	0.34	4.46	6.70	0.49	0.07
Fund Flow	0.33 (0.75)	0.33	4.57	6.54	0.49	0.07
Expense Ratio	0.94 (0.31)	0.97	4.24	6.95	0.54	0.22
Sharpe	0.17 (0.91)	0.17	4.34	6.53	0.51	0.04
Treynor	0.24 (0.84)	0.25	4.42	6.48	0.50	0.05
Beta	1.11 (0.30)	1.02	5.60	6.57	0.48	0.20
Sigma	0.34 (0.78)	0.31	5.48	6.25	0.46	0.06
OLS	0.55 (0.52)	0.66	3.96	7.34	0.53	0.14
Ridge	0.89 (0.33)	0.97	4.50	7.18	0.51	0.20
LASSO	0.61 (0.49)	0.70	4.28	6.90	0.50	0.14
Elastic Net	0.86 90.30)	1.01	4.29	7.11	0.51	0.20
PC 1	0.69 (0.38)	0.86	3.65	7.33	0.55	0.19
$FDR10\%$	-0.05 (0.94)	-0.05	5.27	5.77	0.45	-0.01
Equal Weight	-0.93 (0.02)	-2.36	1.92	6.12	0.48	-0.49
Equal Weight Plus	-0.51 (0.22)	-1.14	2.22	6.30	0.49	-0.23
Panel B: Sample period until December 2019						
R-square	1.29 (0.27)	1.19	5.06	7.16	0.55	0.25
Fund Size	0.57 (0.62)	0.51	5.08	6.53	0.50	0.11
Active Weight	1.05 (0.26)	1.14	4.24	7.41	0.54	0.25
Return Gap	0.62 (0.55)	0.64	4.55	7.01	0.52	0.14
Fund Flow	0.70 (0.54)	0.67	4.65	6.98	0.53	0.15
Expense Ratio	1.39 (0.15)	1.35	4.29	7.41	0.59	0.32
Sharpe	0.53 (0.66)	0.52	4.40	6.96	0.55	0.12
Treynor	0.60 (0.60)	0.59	4.49	6.91	0.54	0.13
Beta	1.58 (0.16)	1.37	5.72	7.00	0.52	0.28
Sigma	0.72 (0.56)	0.61	5.60	6.66	0.50	0.13
OLS	0.81 (0.37)	0.91	4.03	7.69	0.57	0.20
Ridge	1.20 (0.22)	1.21	4.59	7.51	0.54	0.26
LASSO	0.89 (0.36)	0.96	4.36	7.22	0.53	0.20
Elastic Net	1.17 (0.18)	1.29	4.37	7.44	0.54	0.27
PC 1	0.99 (0.22)	1.16	3.70	7.68	0.58	0.27
$FDR10\%$	0.32 (0.81)	0.28	5.39	6.14	0.48	0.06
Equal Weight	-0.80 (0.03)	-2.00	1.85	6.26	0.50	-0.43
Equal Weight Plus	-0.29 (0.44)	-0.61	2.18	6.62	0.52	-0.13

TABLE IA2

Power comparison (in %) for discrete distribution

The table compares the power of the $fFDR^{+}$ and FDR^{+} at FDR target of 10% when the alphas of 2,000 funds are drawn from a discrete distribution: $\alpha \sim \pi^{+}\delta_{\alpha=\alpha^*} + \pi_0\delta_{\alpha=0} + \pi^{-}\delta_{\alpha=-\alpha^*}$ with varying α^* (annualized, in %) and proportions $(\pi^{+}, \pi_0, \pi^{-})$. The simulated data are a balanced panel with 284 observations per fund and generated with cross-sectional dependence.

$(\pi^{+}, \pi_0, \pi^{-})$	Procedure	$\alpha^* = 1.5$	$\alpha^* = 2$	$\alpha^* = 2.5$	$\alpha^* = 3$	$\alpha^* = 3.5$
$(10, 75, 15)\%$	$fFDR^{+}$	1	6.8	23.9	46.6	68.7
	FDR^{+}	0.6	2.9	13.9	33.6	55.3
$(10, 60, 30)\%$	$fFDR^{+}$	2	12.6	35.5	59.6	77.8
	FDR^{+}	0.5	3.4	16.2	37.7	58.5
$(10, 30, 60)\%$	$fFDR^{+}$	5.5	26	54	77.6	90.2
	FDR^{+}	0.6	5.3	23.3	49.9	71.3
$(13, 67.5, 19.5)\%$	$fFDR^{+}$	1.8	11.5	32.8	56.7	76.7
	FDR^{+}	0.7	5	19.9	41.7	62.8
$(13, 48, 39)\%$	$fFDR^{+}$	3.8	19.3	44.6	70	85.1
	FDR^{+}	0.7	5.5	23.5	48.5	68.3
$(13, 9, 78)\%$	$fFDR^{+}$	9.7	37.6	70.7	91.5	97.8
	FDR^{+}	0.9	10	41	73.4	89.8

TABLE IA3

Power comparison (in %) for discrete-normal distribution mixture

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when alphas of 2,000 funds are drawn from a discrete-normal distribution mixture:
$\alpha \sim \pi_0 \delta_{\alpha=0} + (1-\pi_0)\mathcal{N}(0, \sigma^2)$ with varying σ (annualized, in %) and null proportion π_0. The simulated data are a balanced panel with 284 observations per fund and generated with cross-sectional dependence.

π_0	Procedure	$\sigma = 1$	$\sigma = 2$	$\sigma = 3$	$\sigma = 4$	$\sigma = 5$
75%	$fFDR^+$	0.6	16.8	37.3	51.8	61.3
	FDR^+	0.3	9.2	27.7	42.4	52.9
60%	$fFDR^+$	1.8	22.6	44.2	58.1	67.2
	FDR^+	0.4	12.3	32.8	47.5	57.8
30%	$fFDR^+$	5.1	32.9	54.9	68.1	75.5
	FDR^+	0.6	18.7	41.3	56.5	66.1
67.5%	$fFDR^+$	1.1	20.1	40.9	55.3	64.2
	FDR^+	0.3	11	30.4	45.3	55.7
48%	$fFDR^+$	3.2	27.9	49.1	62.8	71.6
	FDR^+	0.4	15.4	36.4	51.5	61.4
9%	$fFDR^+$	7.5	39.8	62.2	74.6	81.4
	FDR^+	0.9	23.5	48.7	63.9	73.1

TABLE IA4

Power comparison (in %) for mixture of two normal distributions

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when alphas of 2,000 funds are drawn from a mixture of two normal distributions: $\alpha \sim 0.3\mathcal{N}(\mu_1, \sigma_1^2) + 0.7\mathcal{N}(\mu_2, \sigma_2^2)$ with varying standard deviation pairs (σ_1, σ_2) and mean pairs (μ_1, μ_2) (both parameters' pairs are annualized and in %). The simulated data are a balanced panel with 284 observations per fund and generated with cross-sectional dependence.

		(σ_1, σ_2)				
(μ_1, μ_2)	Procedure	$(1, 0.5)$	$(1.5, 0.6)$	$(2, 1)$	$(2.5, 1.25)$	$(3, 1.5)$
		$\pi^+ = 6\%$	$\pi^+ = 10.4\%$	$\pi^+ = 20.7\%$	$\pi^+ = 25.5\%$	$\pi^+ = 29.1\%$
$(-2.3, -0.7)$	$fFDR^+$	0.1	0.5	5.8	14.4	24.5
	FDR^+	0	0	0.4	2.4	8.1
		$\pi^+ = 11.8\%$	$\pi^+ = 16.9\%$	$\pi^+ = 26.4\%$	$\pi^+ = 30.5\%$	$\pi^+ = 33.4\%$
$(-2, -0.5)$	$fFDR^+$	0.1	0.7	7	16.5	26.5
	FDR^+	0	0	0.6	3.6	10.1
		$\pi^+ = 35.2\%$	$\pi^+ = 36.4\%$	$\pi^+ = 38.2\%$	$\pi^+ = 39.8\%$	$\pi^+ = 41.1\%$
$(-2.5, 0)$	$fFDR^+$	0.5	1.1	9.9	19.3	29.4
	FDR^+	0	0.1	1.1	5.1	12.7

TABLE IA5

Power comparison (in %) for varying sample size and observation length

The table compares the power of the $fFDR^{+}$ and FDR^{+} in a balanced panel data with varying number of observations per fund (T) and number of funds (m). We present three cases where alphas of m funds are drawn from i) discrete distribution: $\alpha \sim 0.1\delta_{\alpha=2} + 0.3\delta_{\alpha=0} + 0.6\delta_{\alpha=-2}$ (Panel A); ii) discrete-normal mixture: $\alpha \sim 0.3\delta_{\alpha=0} + 0.7\mathcal{N}(0, 2^2)$ (Panel B); and mixture of two normal distributions: $\alpha \sim 0.3\mathcal{N}(-2, 2^2) + 0.7\mathcal{N}(-0.5, 1)$ (Panel C). For each alpha population, we generate data with cross-sectional dependence.

		Number of observations per fund					
m	Procedure	$T = 120$	$T = 180$	$T = 240$	$T = 300$	$T = 360$	$T = 420$
		Panel A: Discrete distribution					
500	$fFDR^{+}$	3.9	10.3	20.8	33.3	45.1	54.3
	FDR^{+}	0.7	1.7	3.4	6.7	12.3	18.8
1000	$fFDR^{+}$	2.4	7.8	18.0	30.8	41.6	52.7
	FDR^{+}	0.4	1.1	2.7	6.6	12.1	19.8
2000	$fFDR^{+}$	2.2	7.4	17.7	28.9	41.2	50.6
	FDR^{+}	0.3	0.9	2.7	6.6	12.9	19.7
3000	$fFDR^{+}$	2.2	6.8	16.2	28.1	39.2	50.4
	FDR^{+}	0.2	0.7	2.3	6.0	12.5	20.7
		Panel B: Mixture of Discrete and Normal distributions					
500	$fFDR^{+}$	12.7	22.3	30.4	36.4	40.7	46.2
	FDR^{+}	2.9	8.5	14.8	20.4	25.3	30.6
1000	$fFDR^{+}$	12.7	21.7	29.1	35.6	40.7	44.8
	FDR^{+}	2.9	8.5	14.6	20.6	25.6	30.1
2000	$fFDR^{+}$	12.1	21.4	28.7	35.3	39.9	44.4
	FDR^{+}	2.8	8.5	14.5	20.6	25.2	30.0
3000	$fFDR^{+}$	12.2	21.0	28.2	34.7	39.6	43.8
	FDR^{+}	2.9	8.4	14.4	20.4	25.4	29.8
		Panel C: Mixture of Normal distributions					
500	$fFDR^{+}$	1.8	3.8	6.0	9.3	11.8	15.1
	FDR^{+}	0.2	0.4	0.6	1.0	1.4	2.3
1000	$fFDR^{+}$	1.4	3.1	5.2	8.3	11.1	13.6
	FDR^{+}	0.1	0.2	0.4	0.8	1.3	1.9
2000	$fFDR^{+}$	1.1	2.7	5.2	7.7	10.6	13.2
	FDR^{+}	0.1	0.1	0.3	0.6	1.2	2.0
3000	$fFDR^{+}$	1.2	2.8	5.0	7.9	10.6	12.8
	FDR^{+}	0.0	0.1	0.3	0.7	1.2	2.0

TABLE IA6

Power comparison (in %): small size and small number of observations

In this table, we consider three distributions as in Table IA5 for samples consisting of $m = 500$ funds (balanced panels) with $T = 60$ observations per fund (5 years) under cross-sectional dependence.

		FDR target								
Distribution	Procedure	10	20	30	40	50	60	70	80	90
Discrete	$fFDR^+$	0.7	3.4	8	14.7	22.9	32.4	42.3	53.2	65.2
	FDR^+	0.2	0.4	0.7	1	1.4	2.1	3	4.5	6.5
Mixture of discrete and normal	$fFDR^+$	3.3	8.9	16	24.2	33.1	42.2	51.6	61.4	67.3
	FDR^+	0.5	1.3	2.7	5.3	9.1	14.9	22.6	32.8	43.3
Mixture of normals	$fFDR^+$	0.6	1.9	4.3	8.3	13.4	20	28.2	38.4	50.9
	FDR^+	0.1	0.2	0.3	0.5	0.9	1.2	1.8	3.1	5.3

TABLE IA7

Power comparison (in $\%$) for discrete distribution

The table compares the power of the $fFDR^{+}$ and FDR^{+} at FDR target of 10% when the alphas of 2,000 funds are drawn from a discrete distribution: $\alpha \sim \pi^{+}\delta_{\alpha=\alpha^{*}} + \pi_{0}\delta_{\alpha=0} + \pi^{-}\delta_{\alpha=-\alpha^{*}}$ with varying α^{*} (annualized, in %) and proportions $(\pi^{+}, \pi_{0}, \pi^{-})$. The simulated data are an unbalanced panel with the number of observations of each fund drawn randomly with replacement from the real-data counterpart. We study the simulated data with both cross-sectional independence (left-hand side) and cross-sectional dependence (right-hand side).

		Cross-sectional Independence					Cross-sectional Dependence				
$(\pi^{+}, \pi_{0}, \pi^{-})$	Procedure	$\alpha^{*} = 1.5$	$\alpha^{*} = 2$	$\alpha^{*} = 2.5$	$\alpha^{*} = 3$	$\alpha^{*} = 3.5$	$\alpha^{*} = 1.5$	$\alpha^{*} = 2$	$\alpha^{*} = 2.5$	$\alpha^{*} = 3$	$\alpha^{*} = 3.5$
$(10, 75, 15)\%$	$fFDR^{+}$	0.5	7	24.2	44.6	61.2	0.8	7.1	23.4	43.1	60.6
	FDR^{+}	0.5	3.2	15.6	33	49.5	0.6	3.6	14.9	31.8	48.6
$(10, 60, 30)\%$	$fFDR^{+}$	1.4	12	33.5	54.2	69.3	1.8	12	32.4	53.2	68.9
	FDR^{+}	0.5	3.5	16.9	35.3	52.1	0.6	4	16.3	34.5	51.4
$(10, 30, 60)\%$	$fFDR^{+}$	4.2	22.4	49.3	68.3	80.8	4.6	21.8	48.4	67.8	80.5
	FDR^{+}	0.6	4.4	22	43.1	60.3	0.7	4.9	21.4	42.1	59.8
$(13, 67.5, 19.5)\%$	$fFDR^{+}$	1.1	10.7	30.6	51.1	67.2	1.5	11.2	29.8	50.1	66.2
	FDR^{+}	0.6	4.8	20.1	38.4	55	0.7	5.4	19.4	37.4	53.8
$(13, 48, 39)\%$	$fFDR^{+}$	2.9	17.7	40.9	61	75.1	3.5	17.9	40.3	60.2	74.3
	FDR^{+}	0.7	5.5	23.2	42.6	59	0.8	6.2	22.5	42	58.2
$(13, 9, 78)\%$	$fFDR^{+}$	7.6	31.2	63.2	79.6	89.7	8.2	31.4	62.3	79.8	89.4
	FDR^{+}	0.8	8.8	32.9	56.9	74.4	0.9	9.3	32.9	57.2	74.3

TABLE IA8

Power comparison (in $\%$) for discrete-normal distribution mixture

The table compares the power of the $fFDR^{+}$ and FDR^{+} at FDR target of 10% when alphas of 2,000 funds are drawn from a discrete-normal distribution mixture: $\alpha \sim \pi_0 \delta_{\alpha=0} + (1-\pi_0)\mathcal{N}(0, \sigma^2)$ with varying σ (annualized, in %) and null proportion π_0. The simulated data are an unbalanced panel with the number of observations of each fund drawn randomly with replacement from the real-data counterpart. We study the simulated data with both cross-sectional independence (left-hand side) and cross-sectional dependence (right-hand side).

		Cross-sectional Independence					Cross-sectional Dependence				
π_0	Procedure	$\sigma=1$	$\sigma=2$	$\sigma=3$	$\sigma=4$	$\sigma=5$	$\sigma=1$	$\sigma=2$	$\sigma=3$	$\sigma=4$	$\sigma=5$
75%	$fFDR^{+}$	0.5	15	33.1	47	56.5	0.5	14.8	32.5	46.6	56.1
	FDR^{+}	0.3	8.7	24.6	38.3	48.4	0.3	8.5	24.1	37.9	48.1
60%	$fFDR^{+}$	1.5	20.4	39.3	52.7	61.7	1.6	20.2	39	52.5	61.5
	FDR^{+}	0.4	11.5	29.1	43	53.1	0.4	11.3	28.7	42.6	52.8
30%	$fFDR^{+}$	4.1	28.7	49.4	62.7	71	4.3	28.9	49.7	62.7	71.1
	FDR^{+}	0.5	16.6	37.1	51.8	61.5	0.6	16.5	37	51.6	61.6
67.5%	$fFDR^{+}$	0.9	17.9	36.2	50	58.9	0.9	17.9	36	49.5	59
	FDR^{+}	0.3	10.1	26.8	40.7	50.7	0.3	10.2	26.6	40.3	50.6
48%	$fFDR^{+}$	2.4	23.9	43.3	56.9	65.9	2.7	23.8	43.6	56.9	65.6
	FDR^{+}	0.4	13.6	32.3	46.6	56.4	0.4	13.4	32.3	46.3	56.2
9%	$fFDR^{+}$	6.1	34.6	55.8	69.2	77.4	6.1	34.7	56	69	77.2
	FDR^{+}	0.7	20.3	42.7	58.3	68.1	0.9	20.5	42.8	58.1	68

TABLE IA9

Power comparison (in %) for mixture of two normal distributions

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when alphas of 2,000 funds are drawn from a mixture of two normal distributions: $\alpha \sim 0.3\mathcal{N}(\mu_1, \sigma_1^2) + 0.7\mathcal{N}(\mu_2, \sigma_2^2)$ with varying standard deviation pairs (σ_1, σ_2) and mean pairs (μ_1, μ_2) (both parameters' pairs are annualized and in %). The simulated data are an unbalanced panel with the number of observations of each fund drawn randomly with replacement from the real-data counterpart. We study the simulated data with both cross-sectional independence (left-hand side) and cross-sectional dependence (right-hand side).

		Cross-sectional Independence					Cross-sectional Dependence				
(μ_1, μ_2)	Procedure	σ^1	σ^2	σ^3	σ^4	σ^5	σ^1	σ^2	σ^3	σ^4	σ^5
$(-2.3, -0.7)$	$fFDR^+$	0	0.3	4.2	12	20.7	0	0.4	4.6	12.3	20.8
	FDR^+	0	0	0.4	2.2	6.9	0	0.1	0.4	2.3	7
$(-2, -0.5)$	$fFDR^+$	0	0.5	5.7	14	22.7	0.1	0.6	5.9	14.1	23.2
	FDR^+	0	0.1	0.5	3.2	8.6	0	0.1	0.6	3.2	8.9
$(-2.5, 0)$	$fFDR^+$	0.2	0.6	8	16.2	25.4	0.3	0.9	8.5	16.8	25.6
	FDR^+	0	0.1	0.9	4.6	10.8	0	0.1	1	4.9	11.2

where $\sigma^1 = (1, 0.5), \sigma^2 = (1.5, 0.6), \sigma^3 = (2, 1), \sigma^4 = (2.5, 1.25), \sigma^5 = (3, 1.5)$.

TABLE IA10

Power comparison (in %) for single normal distribution

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when alphas of 2,000 funds are drawn from a normal distribution: $\alpha \sim \mathcal{N}(\mu, \sigma^2)$ with varying standard deviation σ and mean μ (both parameters are annualized and in $\%$). In Panel A the simulated data are a balanced panel with 284 observations per fund, whereas in Panel B an unbalanced panel with the number of observations of each fund drawn randomly with replacement from the real-data counterpart. For each type of panel data, we generate data cross-sectional independence (left-hand side) and with cross-sectional dependence (right-hand side).

		Cross-sectional Independence					Cross-sectional Dependence				
		σ					σ				
μ	Procedure	1	1.5	2	2.5	3	1	1.5	2	2.5	3
Panel A: Balanced Data											
	π^+	21.2	29.7	34.5	37.4	39.5	21.2	29.7	34.5	37.4	39.5
-0.8	$fFDR^+$	1.8	15.1	31.6	45.5	56.2	2.4	15.5	32.6	46.1	56
	FDR^+	0.1	2.2	13.4	28.5	42	0.1	2.4	14.4	29.1	41.5
	π^+	30.9	36.9	40.1	42.1	43.4	30.9	36.9	40.1	42.1	43.4
-0.5	$fFDR^+$	3.3	18.5	35.5	48.7	58.9	4.2	19.4	35.7	49.3	58.9
	FDR^+	0.1	4	17.7	32.8	45.5	0.2	4.5	17.9	33.1	45.5
	π^+	50	50	50	50	50	50	50	50	50	50
0	$fFDR^+$	8.3	26	41.9	54	63.5	9.3	27	42.3	54.4	63.9
	FDR^+	0.7	9.9	25.6	40.2	51.7	1.2	10.5	25.7	40.3	51.8
Panel B: Unbalanced Data											
-0.8	$fFDR^+$	1.7	13.3	27.6	40.8	50.7	1.9	13.5	27.9	40.8	50.7
	FDR^+	0.1	2.3	11.9	24.5	36.4	0.1	2.4	11.9	24.7	36.1
-0.5	$fFDR^+$	3	16.2	30.6	43.5	53.4	3.5	16.8	31.4	43.8	53.6
	FDR^+	0.2	4	15.2	28.3	39.8	0.2	4.3	15.5	28.4	39.9
0	$fFDR^+$	7.5	22.5	37.2	48.9	58.2	7.7	22.9	37.2	48.3	58.2
	FDR^+	0.8	9	22.1	35.1	45.8	1	9.1	22.1	34.4	45.9

TABLE IA11

Power comparison (in %) when $\pi_0(z)$ is an increasing function

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when the alphas of 2,000 funds are drawn from a discrete distribution: $\alpha \sim \pi^+\delta_{\alpha=\alpha^*} + \pi_0\delta_{\alpha=0} + \pi^-\delta_{\alpha=-\alpha^*}$ with varying α^* (annualized, in %) and proportions (π^+, π_0, π^-). The simulated data are a balanced panel with 284 observations per fund and are generated with cross-sectional independence.

(π^+, π_0, π^-)	Procedure	$\alpha^* = 1.5$	$\alpha^* = 2$	$\alpha^* = 2.5$	$\alpha^* = 3$	$\alpha^* = 3.5$
$(10, 75, 15)\%$	$fFDR^+$	0.3	4.7	20.3	43.8	65.8
	FDR^+	0.4	2.6	14.1	35.2	56.5
$(10, 60, 30)\%$	$fFDR^+$	0.9	9.1	30.6	55.5	75.1
	FDR^+	0.4	2.7	16.2	39.3	60.6
$(10, 30, 60)\%$	$fFDR^+$	3.7	22.7	51.9	76.5	89.7
	FDR^+	0.5	4.1	24.3	51.3	72.6
$(13, 67.5, 19.5)\%$	$fFDR^+$	0.7	8.3	28.8	53.8	73.4
	FDR^+	0.5	3.8	19.9	43.5	63.5
$(13, 48, 39)\%$	$fFDR^+$	2.1	15.9	42.7	68.1	84.6
	FDR^+	0.5	4.7	24.7	49.5	69.8
$(13, 9, 78)\%$	$fFDR^+$	7.6	37.4	68.8	91	97.7
	FDR^+	0.6	8.9	41.7	72.7	90.1

TABLE IA12

Power comparison (in $\%$) when $\pi_0(z)$ is a decreasing function

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when the alphas of 2,000 funds are drawn from a discrete distribution: $\alpha \sim \pi^+\delta_{\alpha=\alpha^*} + \pi_0\delta_{\alpha=0} + \pi^-\delta_{\alpha=-\alpha^*}$ with varying α^* (annualized, in %) and proportions (π^+, π_0, π^-). The simulated data are a balanced panel with 284 observations per fund and are generated with cross-sectional independence.

(π^+, π_0, π^-)	Procedure	$\alpha^* = 1.5$	$\alpha^* = 2$	$\alpha^* = 2.5$	$\alpha^* = 3$	$\alpha^* = 3.5$
$(10, 75, 15)\%$	$fFDR^+$	1.5	11.8	33.7	58.4	77.2
	FDR^+	0.4	2.4	13.9	35.4	56.8
$(10, 60, 30)\%$	$fFDR^+$	3.4	19	46.2	70.8	86.2
	FDR^+	0.4	2.7	16.1	39.4	60.8
$(10, 30, 60)\%$	$fFDR^+$	7.3	33.2	72.2	91	96.7
	FDR^+	0.5	3.9	24.9	51.2	72.4
$(13, 67.5, 19.5)\%$	$fFDR^+$	2.4	16	40.3	64.3	81
	FDR^+	0.5	3.7	20.2	43.1	63.7
$(13, 48, 39)\%$	$fFDR^+$	5.1	27.2	57.7	79.8	91.1
	FDR^+	0.6	4.5	24.5	49.5	69.9
$(13, 9, 78)\%$	$fFDR^+$	11.1	43.6	81.6	94.3	98.3
	FDR^+	0.6	9	41.4	72.8	89.9

TABLE IA13

Power comparison (in %) when $\pi_0(z)$ is a constant function

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when the alphas of 2,000 funds are drawn from a discrete distribution: $\alpha \sim \pi^+\delta_{\alpha=\alpha^*} + \pi_0\delta_{\alpha=0} + \pi^-\delta_{\alpha=-\alpha^*}$ with varying α^* (annualized, in %) and proportions (π^+, π_0, π^-). The simulated data are a balanced panel with 284 observations per fund and are generated with cross-sectional independence.

(π^+, π_0, π^-)	Procedure	$\alpha^* = 1.5$	$\alpha^* = 2$	$\alpha^* = 2.5$	$\alpha^* = 3$	$\alpha^* = 3.5$
$(10, 75, 15)\%$	$fFDR^+$	0.6	6.9	24.6	48.3	69.7
	FDR^+	0.4	2.4	13.9	35.1	56.4
$(10, 60, 30)\%$	$fFDR^+$	1.8	14.1	39	64.1	81.1
	FDR^+	0.5	2.7	16.7	39.3	60.6
$(10, 30, 60)\%$	$fFDR^+$	5.4	29.4	61.8	82.9	93
	FDR^+	0.5	4	24.3	51.2	72.4
$(13, 67.5, 19.5)\%$	$fFDR^+$	1.3	10.9	32.8	58	77.2
	FDR^+	0.5	3.7	19.8	43.1	64
$(13, 48, 39)\%$	$fFDR^+$	3.4	20.6	48.5	72.4	86.9
	FDR^+	0.6	4.6	24.4	49.2	69.8
$(13, 9, 78)\%$	$fFDR^+$	9.6	41.5	76	92.4	98.1
	FDR^+	0.6	9.4	41.8	72.8	89.8

TABLE IA14

Performance comparison in terms of power (%): non-informative covariate

The table compares the power of the $fFDR^+$ and FDR^+ at FDR target of 10% when the alphas of 2,000 funds are drawn from a discrete distribution, i.e. $\alpha \sim \pi^+\delta_{\alpha=\alpha^*} + \pi_0\delta_{\alpha=0} + \pi^-\delta_{\alpha=-\alpha^*}$ (Panel A), a discrete-normal distribution mixture, i.e. $\alpha \sim \pi_0\delta_{\alpha=0} + (1-\pi_0)\mathcal{N}(0,\sigma^2)$ (Panel B), and a mixture of two normal distributions, i.e. $\alpha \sim 0.3\mathcal{N}(\mu_1,\sigma_1^2) + 0.7\mathcal{N}(\mu_2,\sigma_2^2)$ (Panel C) under different combinations of parameter values. The simulated data are a balanced panel with 284 observations per fund and are generated with cross-sectional independence. The covariate input of the $fFDR^+$ is a random variable drawn randomly from the standard uniform distribution, Uniform$(0,1)$, without any connections to the alpha.

Panel A: discrete distribution

(π^+,π_0,π^-)	Procedure	$\alpha^*=1.5$	$\alpha^*=2$	$\alpha^*=2.5$	$\alpha^*=3$	$\alpha^*=3.5$
$(10,75,15)\%$	$fFDR^+$	0.2	2.7	14.1	34.5	57
	FDR^+	0.4	2.3	13.8	35.1	56.8
$(10,60,30)\%$	$fFDR^+$	0.2	3.5	16.9	39.1	61.4
	FDR^+	0.5	2.7	15.9	39.5	61.1
$(10,30,60)\%$	$fFDR^+$	0.4	6.3	26.2	52.3	73.4
	FDR^+	0.5	3.9	24.1	51.3	72.1
$(13,67.5,19.5)\%$	$fFDR^+$	0.3	4.7	19.6	42.8	63.9
	FDR^+	0.5	3.8	19.7	43.4	63.8
$(13,48,39)\%$	$fFDR^+$	0.5	6.4	24.9	49.1	70.3
	FDR^+	0.5	4.5	24.5	49.3	69.7
$(13,9,78)\%$	$fFDR^+$	1.1	14	46.9	77.2	92.1
	FDR^+	0.6	9.2	41.8	72.9	89.9

Panel B: discrete-normal distribution mixture

π_0	Procedure	$\sigma=1$	$\sigma=2$	$\sigma=3$	$\sigma=4$	$\sigma=5$
75%	$fFDR^+$	0	9.5	30.1	45.6	55.9
	FDR^+	0.2	8.9	27.9	43	53.3
60%	$fFDR^+$	0.1	13.2	34.6	49.9	59.7
	FDR^+	0.3	12.4	32.9	48	58.1
30%	$fFDR^+$	0.3	19.5	42.9	57.8	67.3
	FDR^+	0.4	18.7	42	56.9	66.4
67.5%	$fFDR^+$	0.1	11.4	32.5	47.8	57.8
	FDR^+	0.2	10.6	30.5	45.6	55.7
48%	$fFDR^+$	0.2	15.8	38	52.9	62.5
	FDR^+	0.3	14.9	36.5	51.5	61.4
9%	$fFDR^+$	0.6	24.2	49.3	64.5	73.5
	FDR^+	0.6	23.4	48.6	63.8	73

Panel C: mixture of two normal distributions

				(σ_1,σ_2)		
(μ_1,μ_2)	Procedure	$(1,0.5)$	$(1.5,0.6)$	$(2,1)$	$(2.5,1.25)$	$(3,1.5)$
		$\pi^+=6\%$	$\pi^+=10.4\%$	$\pi^+=20.7\%$	$\pi^+=25.5\%$	$\pi^+=29.1\%$
$(-2.3,-0.7)$	$fFDR^+$	0	0	0.2	2.8	8.8
	FDR^+	0	0.1	0.3	2.2	7.7
		$\pi^+=11.8\%$	$\pi^+=16.9\%$	$\pi^+=26.4\%$	$\pi^+=30.5\%$	$\pi^+=33.4\%$
$(-2,-0.5)$	$fFDR^+$	0	0	0.5	3.9	10.7
	FDR^+	0	0.1	0.4	3.2	9.7
		$\pi^+=35.2\%$	$\pi^+=36.4\%$	$\pi^+=38.2\%$	$\pi^+=39.8\%$	$\pi^+=41.1\%$
$(-2.5,0)$	$fFDR^+$	0	0	1	5.4	13.2
	FDR^+	0	0.1	0.7	4.8	12.4

TABLE IA15

Power comparison (in %) for varying sample size and observation length

The table compares the power of the $fFDR^+$ and FDR^+ in a balanced panel data with varying number of observations per fund (T) and number of funds (m). We present three cases where alphas of m funds are drawn from i) discrete distribution: $\alpha \sim 0.1\delta_{\alpha=2} + 0.3\delta_{\alpha=0} + 0.6\delta_{\alpha=-2}$ (Panel A); ii) discrete-normal mixture: $\alpha \sim 0.3\delta_{\alpha=0} + 0.7\mathcal{N}(0, 2^2)$ (Panel B); and mixture of two normal distributions: $\alpha \sim 0.3\mathcal{N}(-2, 2^2) + 0.7\mathcal{N}(-0.5, 1)$ (Panel C). The simulated data are balanced panels with cross-sectional independence.

		Number of observations per fund					
m	Procedure	$T = 120$	$T = 180$	$T = 240$	$T = 300$	$T = 360$	$T = 420$
		Panel A: Discrete distribution					
500	$fFDR^+$	2.7	8.5	20.6	33.3	46.2	55.2
	FDR^+	0.6	1.4	3.2	6.1	11.6	18.7
1000	$fFDR^+$	1.7	6.4	17.1	30.4	42.7	53.8
	FDR^+	0.4	0.9	2.2	5.2	11.3	19.8
2000	$fFDR^+$	1.2	6.2	15.8	29.3	41.5	52.0
	FDR^+	0.2	0.7	1.6	5.4	12.1	21.1
3000	$fFDR^+$	1.1	5.9	15.3	28.4	40.6	52.1
	FDR^+	0.2	0.5	1.6	5.3	12.7	21.7
		Panel B: Mixture of Discrete and Normal distributions					
500	$fFDR^+$	12.6	22.1	29.4	35.8	41.1	45.3
	FDR^+	2.6	7.9	14.1	20.3	25.6	30.2
1000	$fFDR^+$	12.0	21.1	28.8	35.1	40.7	44.4
	FDR^+	2.4	8.0	14.5	20.3	25.7	30.0
2000	$fFDR^+$	11.4	20.6	28.2	34.5	39.6	44.0
	FDR^+	2.3	8.2	14.4	20.4	25.4	29.8
3000	$fFDR^+$	11.4	20.5	28.0	34.2	39.2	43.6
	FDR^+	2.4	8.1	14.5	20.3	25.3	29.9
		Panel C: Mixture of Normal distributions					
500	$fFDR^+$	1.2	3.2	5.2	8.4	11.1	13.6
	FDR^+	0.2	0.3	0.5	0.9	1.3	1.8
1000	$fFDR^+$	0.9	2.4	4.9	7.6	10.2	13.3
	FDR^+	0.1	0.2	0.4	0.7	1.1	1.7
2000	$fFDR^+$	0.7	2.3	4.4	6.9	9.7	12.2
	FDR^+	0.1	0.1	0.3	0.5	0.9	1.6
3000	$fFDR^+$	0.7	2.1	4.4	6.8	9.6	12.1
	FDR^+	0.0	0.1	0.2	0.5	0.9	1.7

TABLE IA16

Power comparison (in %) for small size and small number of observations

In this table, we consider three distributions as in Table IA15 for samples consisting of $m = 500$ funds (balanced panels with cross-sectional independence) with $T = 60$ observations per fund (5 years).

Distribution	Procedure	FDR target								
		10	20	30	40	50	60	70	80	90
Discrete	$fFDR^{+}$	0.4	2	5.5	11.8	20.3	30.4	41.4	53.1	66.6
	FDR^{+}	0.2	0.4	0.6	0.9	1.1	1.5	1.9	2.5	3.4
Mixture of discrete and normal	$fFDR^{+}$	2.7	7.9	14.8	23.3	33	43.4	53.7	63.8	68.3
	FDR^{+}	0.4	1	1.9	3.4	6	11	19.4	32.7	48.9
Mixture of normals	$fFDR^{+}$	0.3	1.2	3.1	6.4	11.4	18.3	27	38.2	52.1
	FDR^{+}	0.1	0.2	0.3	0.3	0.5	0.6	0.8	1.1	1.5

TABLE IA17

Power (in %) of the $fFDR^+$ under covariate with noise

The data are generated as in Tables 1 except the use of a new covariate containing a noise: $Z' = Z + \eta$ instead of Z. The noise is drawn independently from normal distribution $\eta \sim N(0, \sigma_\eta^2)$ where $\sigma_\eta \in \{\sigma_1 = 0.5/\sqrt{12}, \sigma_2 = 1/\sqrt{12}\}$.

Panel A: Discrete distribution										
	$\alpha^* = 1.5$		$\alpha^* = 2$		$\alpha^* = 2.5$		$\alpha^* = 3$		$\alpha^* = 3.5$	
(π^+, π_0, π^-)	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
(10,75,15)%	0.5	0.4	5.7	4.7	21.9	19	45.4	41	67	62.7
(10,60,30)%	1.2	0.9	9.7	7.3	30.9	25.6	55.6	49.4	75	69.6
(10,30,60)%	3.2	1.9	18.9	14.1	47.6	39.5	72.8	65	87.4	82
(13,67.5,19.5)%	0.9	0.7	9.3	7.7	30.1	26	54.6	49.6	73.8	69.5
(13,48,39)%	2.3	1.7	16.1	12.5	41.7	35.2	66	59.3	82.7	77.7
(13,9,78)%	6	4	31.1	24.9	66.1	59.4	89.6	84.8	97.3	95.2
Panel B: Mixture of discrete and normal distributions										
	$\sigma = 1$		$\sigma = 2$		$\sigma = 3$		$\sigma = 4$		$\sigma = 5$	
π_0	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
75%	0.2	0.1	14.8	12.8	35.2	33	49.5	47.7	59.1	57.5
60%	0.8	0.4	20.2	17.4	40.7	37.9	54.8	52.3	63.4	61.3
30%	2.6	1.5	29.2	25	50.5	46.8	63.1	60.1	71	68.7
67.5%	0.5	0.3	17.7	15.2	38.2	35.7	52.2	50	61.3	59.5
48%	1.4	0.8	24	20.5	44.8	41.6	58	55.4	66.5	64.2
9%	4.1	2.4	35.4	30.3	57.1	52.9	69.3	66.3	76.6	74.6
Panel C: Mixture of two normal distributions										
	(σ_1, σ_2)									
	$(1, 0.5)$		$(1.5, 0.6)$		$(2, 1)$		$(2.5, 1.25)$		$(3, 1.5)$	
(μ_1, μ_2)	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2	σ_1	σ_2
(-2.3,-0.7)	0	0	0	0	1.3	0.6	7.2	4.7	16	12.1
(-2,-0.5)	0	0	0.1	0	2.5	1.3	9.6	6.6	19.1	14.8
(-2.5,0)	0	0	0.2	0.1	4.8	2.7	13.4	9.4	23.6	18.5

TABLE IA18

Comparison of portfolios' performances for varying time lengths of investing

In this table, we consider 10 portfolios including ten $fFDR20\%$ portfolios corresponding to the ten covariates and the $FDR20\%$ portfolio of BSW. We compare the average alphas (annualized and in %) of the portfolios that are kept in periods of exactly n consecutive years. For example, consider $n = 5$. For each portfolio, we calculate the alpha for the first 5 years based on the portfolios' returns from January 1982 to December 1986. Then, we roll forward by a month and calculate the second alpha. The process is repeated and the last alpha is estimated based on the portfolios' returns from January 2018 to December 2022. The average of these alphas is presented in the first row of Panel A of the table. Panel B reports similar metrics using the portfolios' return from January 1982 to December 2022.

	$fFDR20\%$										$FDR20\%$
n	R-square	Fund Size	Active Weight	Return Gap	Fund Flow	Expense	Sharpe	Treynor	Beta	Sigma	
Panel A: Whole sample											
5	0.90	0.01	0.55	0.41	0.31	1.42	-0.10	0.12	1.13	0.83	-0.03
10	0.84	-0.06	0.57	0.31	0.37	1.08	0.03	0.22	1.24	0.65	-0.23
15	0.95	-0.02	0.63	0.26	0.42	0.94	0.16	0.32	1.32	0.68	-0.25
20	1.18	0.21	0.83	0.44	0.57	0.96	0.34	0.48	1.45	0.94	-0.06
25	1.10	0.15	0.71	0.37	0.52	0.88	0.34	0.48	1.43	0.82	-0.08
30	0.85	0.04	0.50	0.27	0.43	0.75	0.23	0.40	1.29	0.58	-0.22
35	0.69	-0.00	0.50	0.30	0.39	0.94	0.15	0.31	1.13	0.48	-0.25
40	0.61	0.03	0.42	0.21	0.33	0.86	0.07	0.24	1.07	0.55	-0.14
41	0.70	0.12	0.50	0.29	0.41	0.90	0.08	0.26	1.16	0.62	-0.03
Panel B: Sample period until December 2019											
5	1.03	0.07	0.69	0.52	0.39	1.38	-0.04	0.19	1.28	0.94	0.02
10	1.09	0.15	0.74	0.45	0.56	1.13	0.24	0.44	1.53	0.90	-0.00
15	1.33	0.37	0.93	0.58	0.72	1.18	0.46	0.62	1.57	1.14	0.13
20	1.48	0.55	1.11	0.73	0.84	1.22	0.60	0.75	1.70	1.33	0.27
25	1.23	0.35	0.78	0.49	0.68	0.96	0.46	0.64	1.61	1.02	0.09
30	1.03	0.23	0.62	0.43	0.62	0.87	0.37	0.57	1.54	0.77	-0.05
35	0.74	0.03	0.62	0.44	0.40	1.02	0.08	0.27	1.23	0.59	-0.21
38	1.13	0.48	0.79	0.60	0.80	1.35	0.43	0.63	1.64	1.02	0.34

TABLE IA19

Performance statistics of all considered portfolios with $\tau = 20\%$

The table compares the portfolios with regard to metrics including the annual Carhart four-factor alpha ($\hat{\alpha}$, in %) with its bootstrap p-value and t-statistic (with use of Newey–West heteroskedasticity and autocorrelation-consistent standard error), the annual standard deviation of the four-factor model residuals ($\hat{\sigma}_{\varepsilon}$, in %), the mean return in excess of the one-month T-bill rate (in %), the annual Sharpe ratio and the annual Information Ratio ($IR = \hat{\alpha}/\hat{\sigma}_{\varepsilon}$). Panel A presents the metrics with use of the portfolios' return from January 1982 to December 2022 whereas Panel B the portfolios' return from January 1982 to December 2019.

Covariate	$\hat{\alpha}$ (p-value)	t-statistic	$\hat{\sigma}_{\varepsilon}$	Mean Return	Sharpe Ratio	IR
Panel A: Whole sample						
R-square	0.70 (0.49)	0.69	4.96	6.58	0.50	0.14
Fund Size	0.12 (0.96)	0.11	4.97	6.05	0.46	0.02
Active Weight	0.50 (0.57)	0.58	4.11	6.85	0.50	0.12
Return Gap	0.29 (0.81)	0.32	4.31	6.74	0.49	0.07
Fund Flow	0.41 (0.67)	0.42	4.57	6.64	0.50	0.09
Expense Ratio	0.90 (0.31)	0.96	4.15	6.92	0.54	0.22
Sharpe	0.08 (0.97)	0.08	4.33	6.43	0.50	0.02
Treynor	0.26 (0.82)	0.27	4.43	6.53	0.50	0.06
Beta	1.16 (0.27)	1.08	5.49	6.63	0.49	0.21
Sigma	0.62 (0.62)	0.56	5.49	6.54	0.48	0.11
$FDR20\%$	-0.03 (0.94)	-0.02	5.27	5.82	0.45	-0.01
Equal Weight	-0.93 (0.02)	-2.36	1.92	6.12	0.48	-0.49
Equal Weight Plus	-0.51 (0.22)	-1.14	2.22	6.30	0.49	-0.23
Panel B: Sample period until December 2019						
R-square	1.13 (0.31)	1.06	5.06	7.02	0.54	0.22
Fund Size	0.48 (0.67)	0.43	5.07	6.45	0.49	0.10
Active Weight	0.79 (0.40)	0.87	4.18	7.16	0.53	0.19
Return Gap	0.60 (0.55)	0.62	4.39	7.05	0.52	0.14
Fund Flow	0.80 (0.49)	0.76	4.65	7.08	0.54	0.17
Expense Ratio	1.35 (0.16)	1.36	4.19	7.38	0.59	0.32
Sharpe	0.43 (0.72)	0.42	4.39	6.85	0.54	0.10
Treynor	0.63 (0.59)	0.61	4.50	6.96	0.54	0.14
Beta	1.64 (0.14)	1.44	5.60	7.07	0.53	0.29
Sigma	1.02 (0.41)	0.86	5.61	6.97	0.52	0.18
$FDR20\%$	0.34 (0.79)	0.30	5.39	6.20	0.48	0.06
Equal Weight	-0.80 (0.03)	-2.00	1.85	6.26	0.50	-0.43
Equal Weight Plus	-0.29 (0.44)	-0.61	2.18	6.62	0.52	-0.13

TABLE IA20

Performance of all considered portfolios in sub-periods

The table displays the performance of the 15 $fFDR10\%$ portfolios corresponding to the 10 underlying covariates and the 5 combined covariates, the $FDR10\%$ and equally weighted portfolios in sub-periods (P1: 1982–1991, P2: 1992–2001, P3: 2002–2011, P4a: 2012–2019 and P4b: 2012–2022) in terms of the annualized alpha (in %) of the whole sub-period, the corresponding t-statistic (with use of Newey–West heteroskedasticity and autocorrelation-consistent standard error) and the annual Sharpe ratio.

	Whole sub-period alpha					Whole sub-period t-statistic					Annual Sharpe Ratio				
Portfolio	P1	P2	P3	P4a	P4b	P1	P2	P3	P4a	P4b	P1	P2	P3	P4a	P4b
R-square	3.45	1.81	1.44	0.58	-2.01	2.71	0.72	1.08	0.16	-0.80	0.67	0.57	0.28	0.84	0.53
Fund Size	2.63	1.49	-0.46	-0.11	-2.40	3.18	0.56	-0.35	-0.03	-0.98	0.67	0.53	0.19	0.78	0.51
Active Weight	3.72	2.17	-0.38	-0.16	-1.39	2.42	0.84	-0.33	-0.14	-1.51	0.65	0.56	0.19	1.10	0.69
Return Gap	3.26	1.77	-1.00	0.05	-1.30	2.46	0.67	-0.70	0.05	-1.43	0.60	0.55	0.15	1.13	0.70
Fund Flow	2.58	1.04	0.12	0.39	-1.90	2.32	0.39	0.13	0.11	-0.78	0.65	0.56	0.23	0.88	0.56
Expense Ratio	4.22	2.34	-0.26	2.87	-0.45	2.03	0.96	-0.28	1.52	-0.26	0.71	0.63	0.20	1.34	0.70
Sharpe	2.23	0.94	0.20	-0.45	-2.40	2.19	0.38	0.24	-0.12	-0.98	0.66	0.65	0.25	0.82	0.53
Treynor	2.16	1.16	0.11	-0.20	-2.28	2.29	0.46	0.13	-0.05	-0.93	0.65	0.63	0.24	0.83	0.53
Beta	3.86	1.19	2.46	0.44	-2.10	1.87	0.40	1.84	0.12	-0.84	0.66	0.42	0.34	0.84	0.53
Sigma	2.58	2.79	0.61	0.34	-2.31	1.91	1.08	0.42	0.09	-0.91	0.58	0.57	0.25	0.80	0.50
OLS	2.14	1.18	0.34	-0.18	-1.26	1.54	0.45	0.33	-0.19	-1.61	0.61	0.61	0.26	1.14	0.71
Ridge	2.98	3.14	-0.34	0.39	-1.05	2.79	1.12	-0.23	0.34	-1.13	0.65	0.55	0.20	1.16	0.72
LASSO	2.47	3.02	-0.49	0.03	-1.33	2.54	1.09	-0.41	0.03	-1.49	0.63	0.55	0.19	1.12	0.69
Elastic Net	2.61	3.63	-0.15	0.06	-1.30	2.32	1.30	-0.14	0.05	-1.49	0.62	0.58	0.21	1.13	0.70
PC 1	1.74	1.76	0.27	1.10	-0.66	1.79	0.71	0.31	0.82	-0.66	0.61	0.65	0.24	1.23	0.75
$FDR10\%$	2.96	1.53	-0.53	-0.54	-2.87	2.52	0.60	-0.38	-0.15	-1.17	0.65	0.52	0.19	0.75	0.48
Equal Weight	-0.52	-1.33	-0.20	-1.35	-1.46	-1.25	-1.58	-0.30	-2.74	-2.61	0.48	0.54	0.23	1.01	0.69
Equal Weight Plus	0.81	-1.07	-0.27	-0.35	-1.42	1.18	-1.14	-0.37	-0.58	-2.24	0.55	0.52	0.21	1.12	0.69

TABLE IA21

Performance comparison of portfolios based on $fFDR$ and sorting

The table shows the portfolios' annual Carhart four-factor alpha (in %) for the period January 1982 to December 2022. The sorting portfolios are those based on covariates (single-sorting) as well as based on both covariates and past alpha (double-sorting).. At the end of each year, for the single-sorting 10% portfolio, funds are sorted by the covariate. Depending on whether the relationship of the covariate and the fund performance is positive or negative, the funds in the top or bottom 10% are chosen to invest in the following year. For the double-sorting 10% portfolio, the funds chosen in the single-sorting 10% are ranked based on the past five-year alpha and then only 10% of the funds in the top are selected. *Note.* As documented in the literature, the R-square and Fund Size (Fund flow, Return Gap and Active Weight) have a negative (positive) effect on the mutual funds' performance. The single- and double-sorting portfolios constructed based on this assumption appear italicized.

Portfolio	R-square	Fund Size	Active Weight	Return Gap	Fund Flow	Expense Ratio	Sharpe	Treynor	Beta	Sigma
Panel A: Performance of $fFDR10\%$ and $fFDR20\%$ portfolios										
$fFDR10\%$	0.84	0.20	0.73	0.31	0.33	0.94	0.17	0.24	1.11	0.34
$fFDR20\%$	0.70	0.12	0.50	0.29	0.41	0.90	0.08	0.26	1.16	0.62
Panel B: Assuming a positive effect of the covariate on performance of the fund										
Single sort 10%	-1.08	-0.73	*-0.95*	*-1.90*	*-1.18*	-2.56	-0.10	-0.45	-2.29	-2.60
Double sort 10%	-0.67	0.07	*1.13*	*-1.81*	*-0.55*	-2.35	-0.04	-0.40	-1.98	-0.40
Single sort 20%	-1.22	-0.74	*-0.90*	*-1.60*	*-0.84*	-1.89	-0.33	-0.48	-1.98	-1.88
Double sort 20%	-1.20	-0.04	*0.25*	*-1.04*	*-0.43*	-1.04	-0.21	-0.38	-0.71	-0.43
Panel C: Assuming a negative effect of the covariate on performance of the fund										
Single sort 10%	*-1.27*	*-1.01*	-1.25	-1.14	-1.31	-0.58	-2.15	-2.49	0.12	-0.52
Double sort 10%	*-2.51*	*-0.48*	-1.60	-0.53	-0.69	0.94	1.10	0.24	-0.34	0.38
Single sort 20%	*-1.11*	*-1.00*	-1.18	-1.18	-1.29	-0.46	-1.67	-1.66	0.00	-0.71
Double sort 20%	*-0.92*	*-0.11*	-1.18	-0.12	-0.66	-0.02	0.51	-0.33	-0.08	-0.26

TABLE IA22

Performance of $FDR10\%$ portfolios conditional on each of covariates

Covariate	Quintile	$\hat{\alpha}$ ($p-$value)	t-statistic	$\hat{\sigma}_{\varepsilon}$	Mean Return	Sharpe Ratio	IR	#Funds
R-square	1	0.02 (0.97)	0.02	6.98	5.19	0.46	0.00	15
	2	2.50 (0.08)	1.36	8.65	8.26	0.55	0.29	9
	3	-0.25 (0.79)	-0.31	4.62	6.51	0.48	-0.05	7
	4	0.03 (0.99)	0.03	5.26	6.05	0.43	0.01	6
	5	-1.27 (0.24)	-1.25	6.46	5.98	0.41	-0.20	12
Fund Flow	1	-0.33 (0.78)	-0.35	7.17	6.25	0.47	-0.05	12
	2	0.05 (0.99)	0.04	6.01	6.82	0.48	0.01	8
	3	0.18 (0.92)	0.16	6.27	6.20	0.48	0.03	7
	4	-0.53 (0.52)	-0.61	5.21	6.15	0.46	-0.10	10
	5	0.17 (0.88)	0.13	6.85	5.06	0.39	0.03	9
Active Weight	1	3.20 (0.45)	0.55	26.95	6.75	0.32	0.12	14
	2	-1.69 (0.10)	-1.75	5.19	4.85	0.37	-0.32	10
	3	0.81 (0.41)	0.91	5.24	7.13	0.50	0.16	7
	4	0.38 (0.77)	0.31	7.31	6.59	0.47	0.05	6
	5	0.36 (0.71)	0.39	5.54	6.17	0.47	0.06	6
Return Gap	1	1.30 (0.45)	0.79	7.99	6.04	0.45	0.16	7
	2	0.46 (0.60)	0.57	4.51	7.38	0.54	0.10	9
	3	-0.73 (0.52)	-0.59	6.51	6.02	0.43	-0.11	8
	4	-0.64 (0.47)	-0.73	4.78	6.77	0.49	-0.13	9
	5	-1.48 (0.23)	-1.23	6.94	4.70	0.34	-0.21	8
Fund Flow	1	-0.88 (0.57)	-0.59	7.98	4.16	0.33	-0.11	7
	2	-0.21 (0.85)	-0.13	7.27	5.90	0.42	-0.03	11
	3	-0.11 (0.88)	-0.10	6.09	6.62	0.50	-0.02	13
	4	0.23 (0.82)	0.26	4.84	6.37	0.49	0.05	8
	5	0.38 (0.75)	0.34	5.70	6.16	0.47	0.07	14
Expense Ratio	1	-0.47 (0.67)	-0.39	5.79	5.23	0.43	-0.08	11
	2	-0.72 (0.42)	-0.87	4.84	5.40	0.43	-0.15	10
	3	1.59 (0.40)	0.70	10.81	6.52	0.44	0.15	8
	4	0.36 (0.77)	0.31	5.88	6.34	0.47	0.06	6
	5	-0.70 (0.62)	-0.53	7.77	5.07	0.36	-0.09	10
Sharpe Ratio	1	1.03 (0.65)	0.45	12.94	5.51	0.36	0.08	2
	2	-1.47 (0.38)	-0.85	8.19	3.97	0.30	-0.18	10
	3	-0.33 (0.72)	-0.31	6.31	6.77	0.46	-0.05	22
	4	0.93 (0.38)	0.89	6.19	7.02	0.49	0.15	33
	5	0.73 (0.72)	0.33	10.40	6.05	0.43	0.07	58
Treynor Ratio	1	0.33 (0.84)	0.15	11.57	2.34	0.24	0.03	2
	2	1.59 (0.42)	0.77	8.69	8.11	0.51	0.18	8
	3	-0.27 (0.83)	-0.21	6.40	7.02	0.46	-0.04	18
	4	-0.40 (0.66)	-0.45	4.66	6.55	0.46	-0.08	32
	5	0.02 (0.98)	0.02	5.39	5.79	0.46	0	53
Beta	1	-0.63 (0.44)	-0.67	4.64	4.23	0.47	-0.14	8
	2	-0.09 (0.93)	-0.08	6.12	6.65	0.51	-0.01	6
	3	-0.65 (0.52)	-0.55	6.60	6.05	0.43	-0.10	11
	4	0.81 (0.40)	0.82	5.41	8.47	0.54	0.15	9
	5	2 (0.47)	0.59	15.53	6.40	0.36	0.13	9
Sigma	1	-0.18 (0.79)	-0.25	3.77	5.02	0.47	-0.05	11
	2	-0.09 (0.90)	-0.11	4.92	5.67	0.47	-0.02	7
	3	-0.93 (0.21)	-1.27	4.49	5.35	0.43	-0.21	8

	4	0.34 (0.80)	0.31	6.01	7	0.51	0.06	7
	5	2.02 (0.46)	0.60	16.29	5.74	0.35	0.12	17

TABLE IA23

Comparison of portfolios' performances for varying time lengths of investing: restricted data

We consider 10 portfolios including ten $fFDR10\%$ portfolios and the $FDR10\%$ portfolios of BSW. We compare the average alphas (annualized, in $\%$) of the portfolios that are kept for periods of exactly n consecutive years. For more details, refer to Table 3 of the main paper.

n	R-square	Fund Size	Active Weight	Return Gap	Fund flow	Expense Ratio	Sharpe	Treynor	Beta	Sigma	$FDR10\%$
Panel A: Whole sample											
5	1.30	0.96	1.06	0.08	0.82	0.85	0.48	0.61	1.42	0.74	0.28
10	1.17	0.82	0.90	-0.05	0.75	0.58	0.45	0.58	1.44	0.43	0.09
15	1.23	0.75	0.91	0.02	0.70	0.41	0.44	0.53	1.40	0.32	-0.02
20	1.40	0.86	1.10	0.22	0.80	0.45	0.55	0.62	1.42	0.45	0.15
25	1.35	0.77	1.04	0.15	0.73	0.40	0.54	0.61	1.35	0.38	0.13
30	1.09	0.63	0.79	-0.07	0.66	0.29	0.50	0.58	1.24	0.27	-0.03
35	1.03	0.67	0.61	-0.16	0.72	0.35	0.56	0.65	1.23	0.36	-0.08
40	0.91	0.57	0.47	-0.19	0.59	0.26	0.43	0.50	1.08	0.36	-0.06
41	0.99	0.63	0.50	-0.07	0.66	0.33	0.40	0.47	1.14	0.47	0.05
Panel B: Sample period until December 2019											
5	1.22	0.85	1.23	0.17	0.70	0.76	0.34	0.48	1.36	0.60	0.25
10	1.22	0.85	1.06	0.08	0.74	0.60	0.43	0.56	1.51	0.42	0.21
15	1.45	0.97	1.22	0.29	0.85	0.63	0.56	0.65	1.53	0.51	0.32
20	1.56	1.09	1.37	0.46	0.95	0.65	0.66	0.73	1.61	0.61	0.46
25	1.34	0.84	1.07	0.19	0.78	0.44	0.53	0.61	1.45	0.41	0.24
30	1.08	0.67	0.83	-0.02	0.70	0.31	0.48	0.57	1.36	0.30	0.04
35	1.02	0.68	0.75	-0.08	0.69	0.31	0.47	0.55	1.32	0.36	-0.07
38	1.43	1.04	0.79	0.19	1.06	0.73	0.77	0.85	1.63	0.85	0.43

TABLE IA24

Performance of $fFDR\tau$ portfolios with combined covariates: restricted data

The table displays the average n-year alpha of the $fFDR10\%$ portfolios using the covariates given by the first principal component (PC 1), the OLS, ridge, LASSO and elastic net (see descriptions in Figure 5 of the main manuscript). The average n-year alpha (annualized, in %) of each portfolio is calculated as described in Table 3 of the main manuscript.

n	OLS	Ridge	LASSO	Elastic Net	PC 1
Panel A: Whole sample					
5	0.58	0.84	0.84	0.64	0.62
10	0.67	1.00	0.93	0.72	0.75
15	0.73	1.08	1.00	0.79	0.85
20	0.98	1.26	1.15	0.96	1.01
25	0.90	1.19	1.03	0.84	0.95
30	0.77	1.01	0.84	0.65	0.83
35	0.79	0.92	0.82	0.59	0.84
40	0.56	0.74	0.74	0.39	0.62
41	0.62	0.81	0.87	0.47	0.63
Panel B: Sample period until December 2019					
5	0.72	1.00	1.01	0.78	0.76
10	0.82	1.24	1.17	0.95	0.90
15	1.00	1.42	1.33	1.13	1.09
20	1.18	1.61	1.49	1.30	1.20
25	0.96	1.38	1.21	1.03	1.00
30	0.88	1.17	1.03	0.81	0.91
35	0.81	1.05	0.96	0.68	0.87
38	0.88	1.14	1.19	0.76	0.91

TABLE IA25

Comparison of portfolios' performance for varying time lengths of investing: portfolios of unprofitable funds

We consider 11 portfolios including the equal weight minus (EW^-), the $FDR^-10\%$ and the $fFDR^-10\%$ with the ten individual covariates. The table compares the average alphas (annualized, in $\%$) of portfolios that are kept in periods of exactly n consecutive years. For more details, refer to Table 3 of the main paper.

n	R-square	Fund Size	Active Weight	Return Gap	Fund flow	Expense Ratio	Sharpe	Treynor	Beta	Sigma	EW^-	$FDR^-10\%$
5	-3.64	-3.91	-2.42	-2.74	-2.87	-3.51	-2.22	-2.09	-3.61	-3.96	-1.41	-3.91
10	-3.51	-3.82	-2.39	-2.65	-2.64	-3.35	-2.02	-1.91	-3.48	-3.82	-1.31	-3.83
15	-3.28	-3.49	-2.23	-2.36	-2.34	-3.02	-1.69	-1.60	-3.15	-3.54	-1.11	-3.54
20	-3.11	-3.23	-2.14	-2.17	-2.15	-2.77	-1.49	-1.43	-2.94	-3.32	-0.97	-3.29
25	-3.16	-3.28	-2.14	-2.20	-2.18	-2.81	-1.53	-1.45	-2.99	-3.35	-0.95	-3.36
30	-3.40	-3.59	-2.34	-2.47	-2.43	-3.12	-1.76	-1.68	-3.28	-3.67	-1.05	-3.65
35	-3.77	-3.95	-2.65	-2.87	-2.85	-3.53	-2.18	-2.08	-3.71	-4.15	-1.22	-3.97
40	-3.87	-3.98	-2.90	-2.99	-3.08	-3.69	-2.41	-2.32	-3.91	-4.28	-1.42	-3.98
41	-3.76	-3.86	-2.79	-2.91	-3.08	-3.57	-2.38	-2.28	-3.92	-4.17	-1.35	-3.93

TABLE IA26

Performance statistics of the portfolios of unprofitable funds with $\tau = 10\%$

The table compares the portfolios with regard to metrics including the annual Carhart four-factor alpha ($\hat{\alpha}$, in %) with its bootstrap p-value and t-statistic (with use of Newey–West heteroskedasticity and autocorrelation-consistent standard error), the annual standard deviation of the four-factor model residuals ($\hat{\sigma}_{\varepsilon}$, in %), the mean return in excess of the one-month T-bill rate (in %), the annual Sharpe ratio and the annual Information Ratio ($IR = \hat{\alpha}/\hat{\sigma}_{\varepsilon}$).

Covariate	$\hat{\alpha}$ (p-value)	t-statistic	$\hat{\sigma}_{\varepsilon}$	Mean Return	Sharpe Ratio	IR
R-square	-3.76 (< 0.01)	-5.14	3.37	3.49	0.31	-1.12
Fund Size	-3.86 (< 0.01)	-5.33	3.11	3.51	0.31	-1.24
Active Weight	-2.79 (< 0.01)	-4.59	3.16	4.56	0.37	-0.88
Return Gap	-2.91 (< 0.01)	-4.06	3.33	4.50	0.37	-0.87
Fund Flow	-3.08 (< 0.01)	-4.51	3.21	4.24	0.35	-0.96
Expense Ratio	-3.57 (< 0.01)	-4.88	3.40	3.56	0.31	-1.05
Sharpe	-2.38 (< 0.01)	-3.48	3.01	4.81	0.39	-0.79
Treynor	-2.28 (< 0.01)	-3.52	2.90	4.99	0.40	-0.78
Beta	-3.92 (< 0.01)	-5.20	3.93	3.88	0.33	-1.00
Sigma	-4.17 (< 0.01)	-5.06	3.59	2.96	0.28	-1.16
$FDR^{-}10\%$	-3.93 (< 0.01)	-5.33	3.46	3.35	0.30	-1.14
Equal Weight	-0.93 (0.02)	-2.36	1.92	6.12	0.48	-0.49
Equal Weight Minus	-1.35 (< 0.01)	-3.07	2.08	5.86	0.46	-0.65

FIGURE IA1

Comparison of FDR^+ and $fFDR^+$.

The graphs show the differences between the two procedures with respect to their null proportion estimations and rejection rules. Panels A and B show that π_0 is estimated as a fixed number in the FDR^+ (see (2)) but as a step function in the $fFDR^+$ (see Appendix A). Panel C shows the rejection rules of the FDR^+ and $fFDR^+$: the former selects all the funds corresponding to the points on the left of the vertical green dashed line which consists of all funds with positive estimated alphas and p-values less than 0.008, whereas the latter all the funds corresponding to the points below the horizontal red dashed line which consists of all funds with estimated q-value (see (8)) less than 0.45. Panel D shows the distribution of the selected funds in Panel C with respect to the p-value and the covariate z. In Panels C and D, only funds with positive estimated alpha are shown as ultimately both methods select funds from this set. The solid green points represent funds selected by the FDR^+, whereas the red circles the funds selected by the $fFDR^+$; the green points with a red ring are the commonly selected funds.

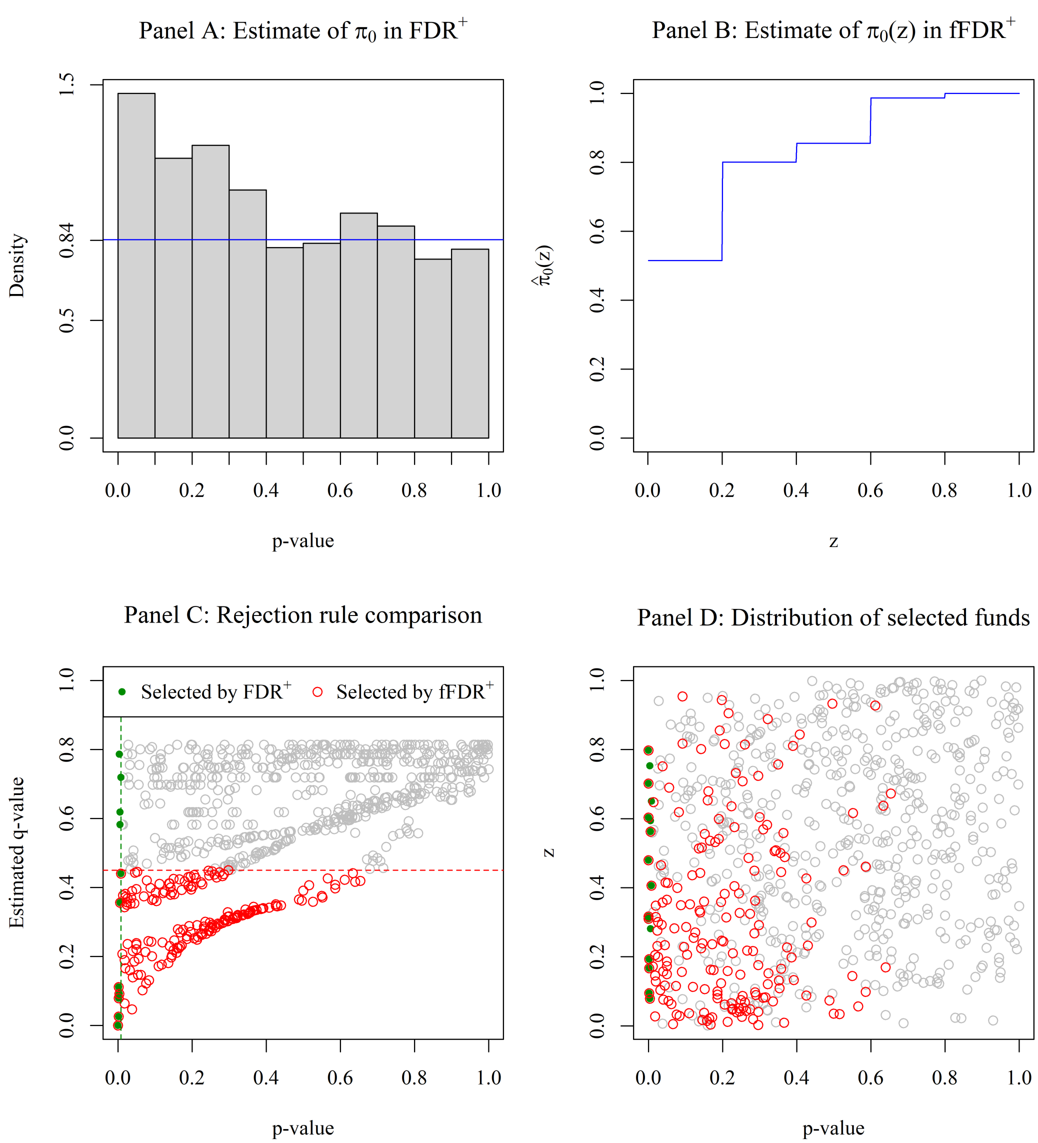


FIGURE IA2

Joint density function $f(p, z)$

The graph shows the heatmap of the density function $f(p, z)$.

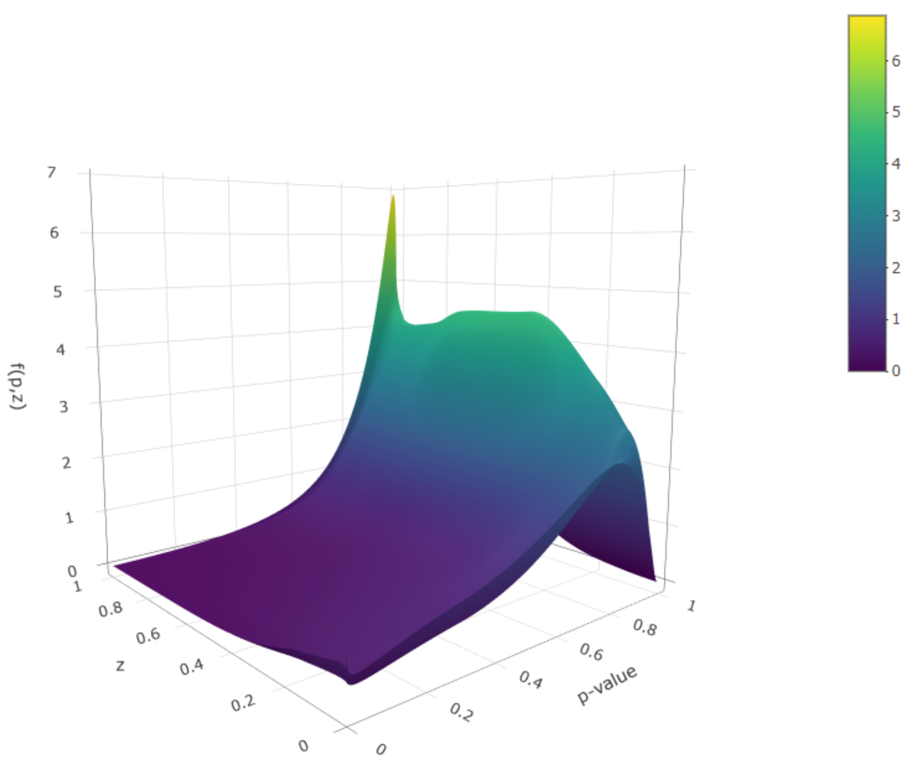


FIGURE IA3

Variance of falsely classified fund ratio: discrete distribution of α

The graphs show the gap in variance of the falsely classified fund ratio of the FDR^+ over the $fFDR^+$. The simulated data are balanced panels with cross-sectional independence.

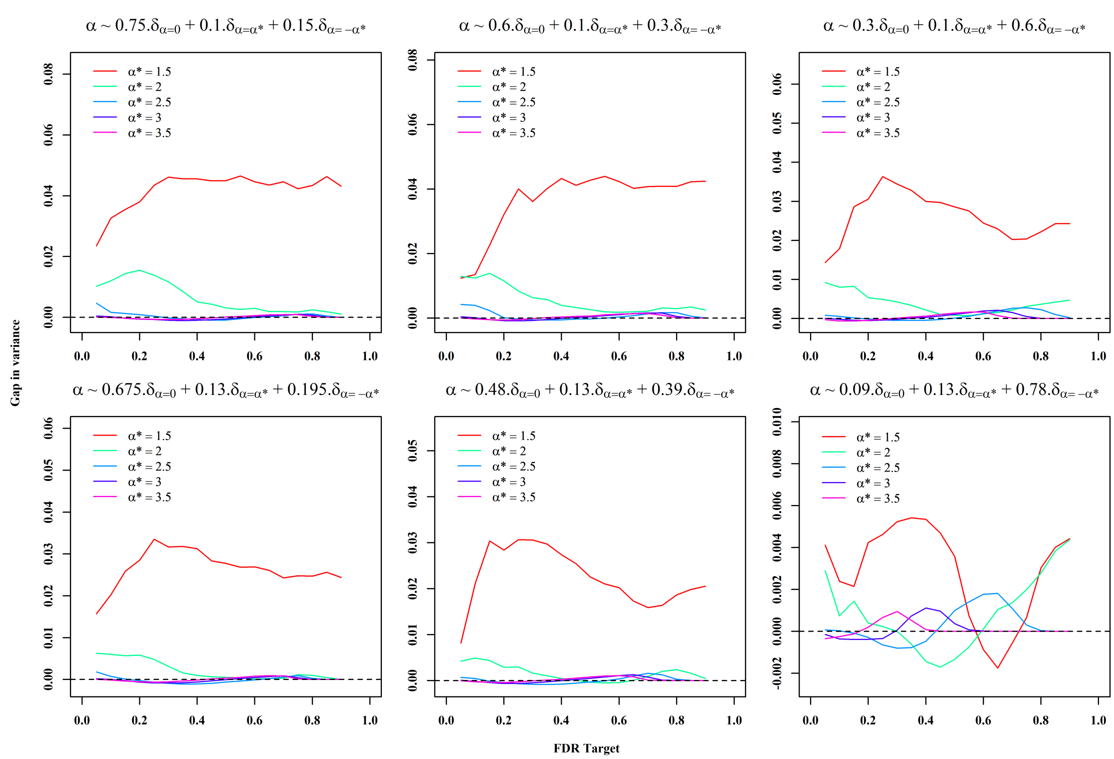


FIGURE IA4

Variance of falsely classified fund ratio: mixed discrete and normal α

The graphs show the gap in variance of the falsely classified fund ratio of the FDR^{+} over the $fFDR^{+}$. The simulated data are balanced panels with cross-sectional independence.

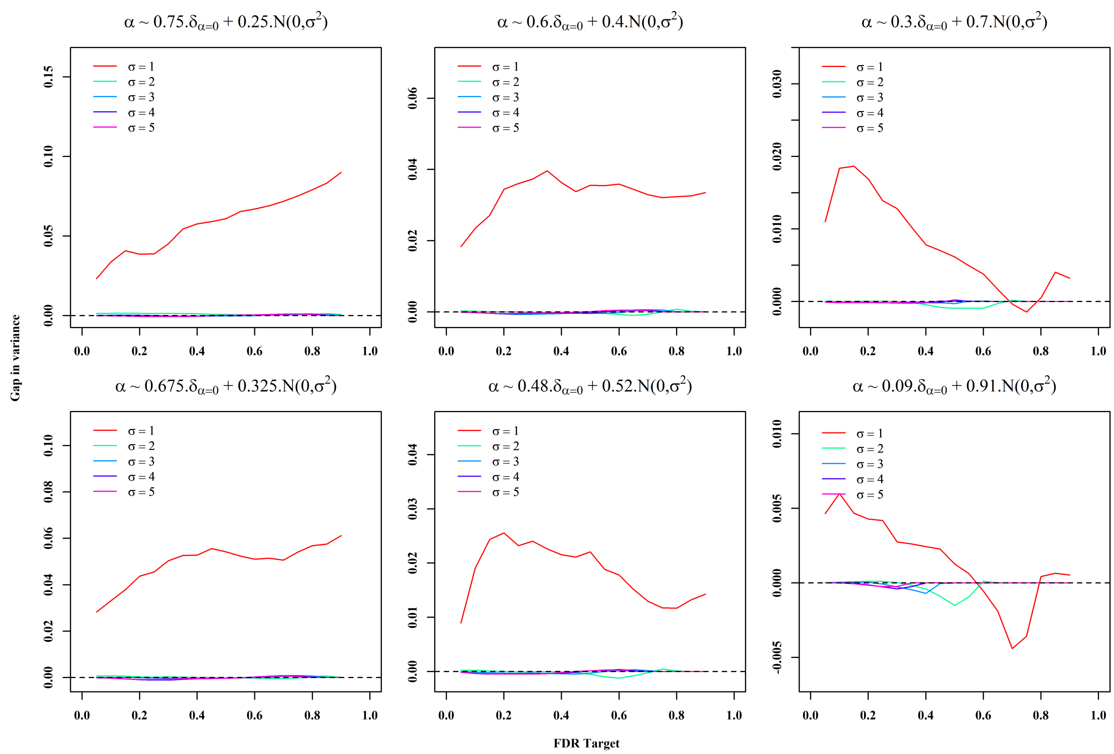


FIGURE IA5

Variance of falsely classified fund ratio: two normal distributions of α

The graphs show the gap in variance of the falsely classified fund ratio of the FDR^+ over the $fFDR^+$. The simulated data are balanced panels with cross-sectional independence.

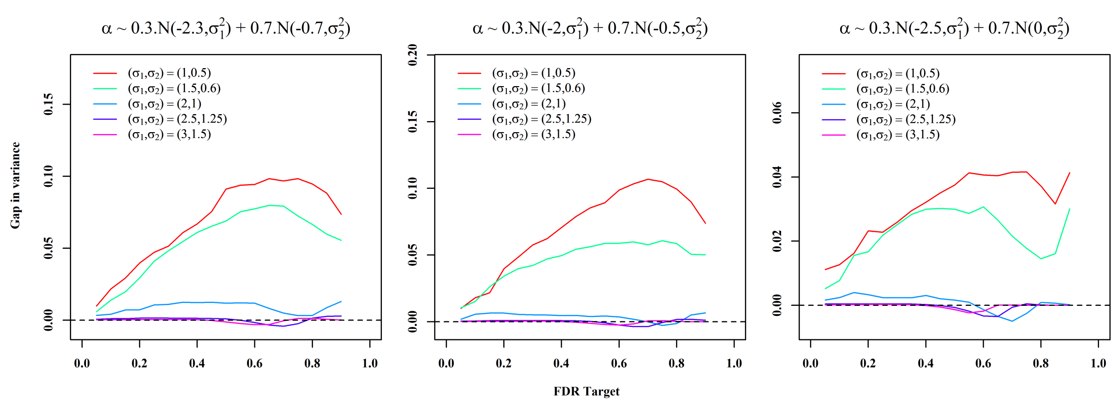


FIGURE IA6

Performance of $fFDR^{+}$ for discrete distribution of α

The graphs show the performance of the $fFDR^{+}$ in terms of FDR control when alphas are drawn from a discrete distribution. The simulated data are balanced panels with cross-sectional dependence.

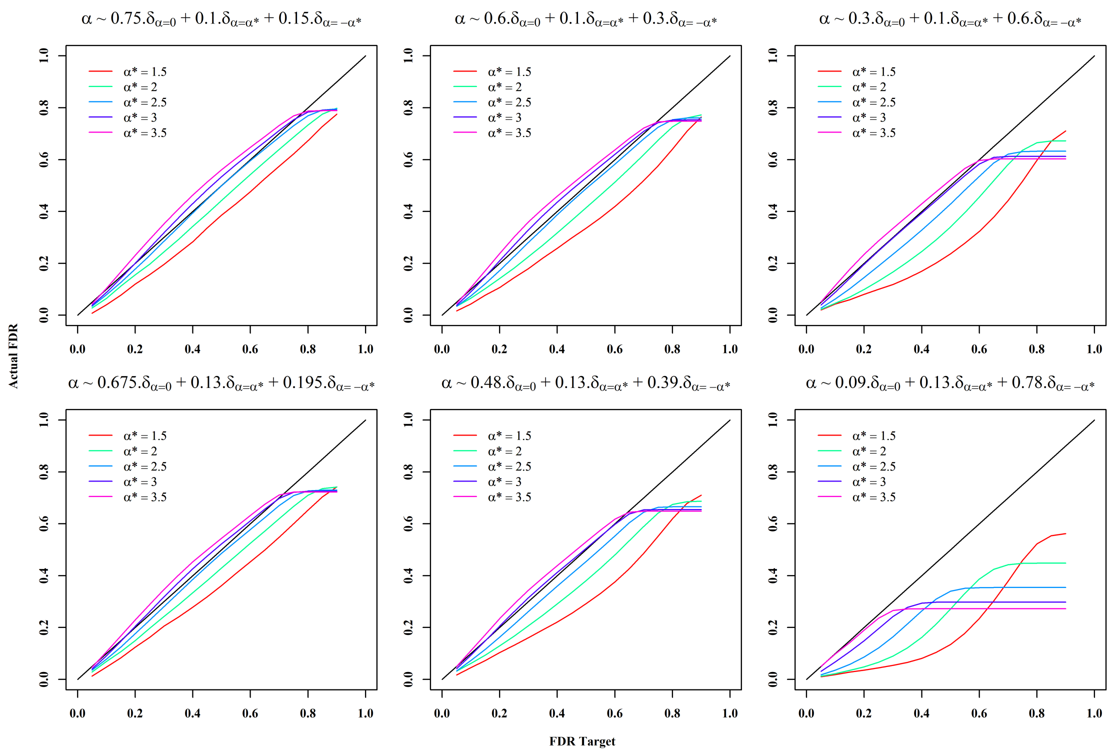


FIGURE IA7

Performance of $fFDR^{+}$ for discrete and normal distribution mixture of α

The graphs show the performance of the $fFDR^{+}$ in terms of FDR control when alphas are drawn from a mixture of discrete and normal distributions. The simulated data are balanced panels with cross-sectional dependence.

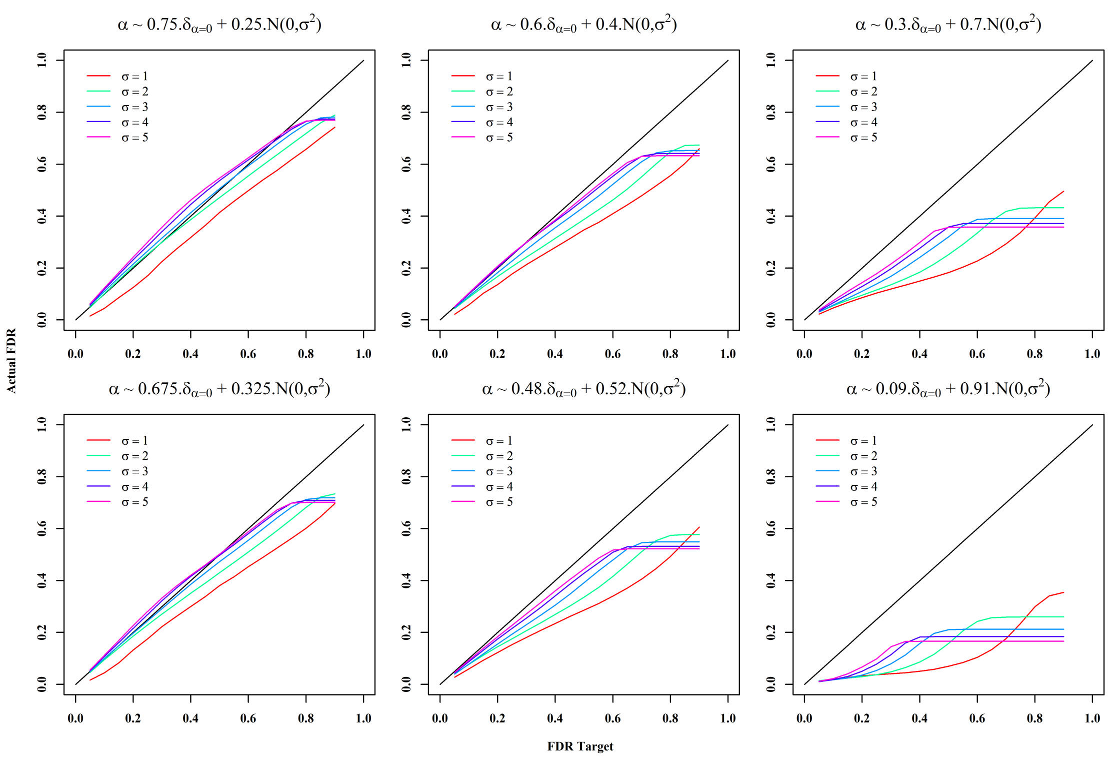


FIGURE IA8

Performance of $fFDR^{+}$ for continuous distribution of α

The graphs show the performance of the $fFDR^{+}$ in terms of FDR control when alphas are drawn from a continuous distribution which is a mixture of two normals. The simulated data are balanced panels with cross-sectional dependence.

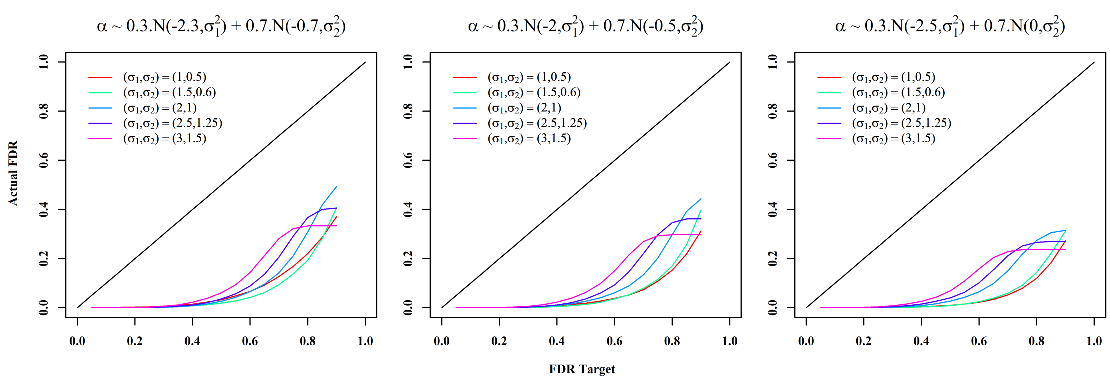
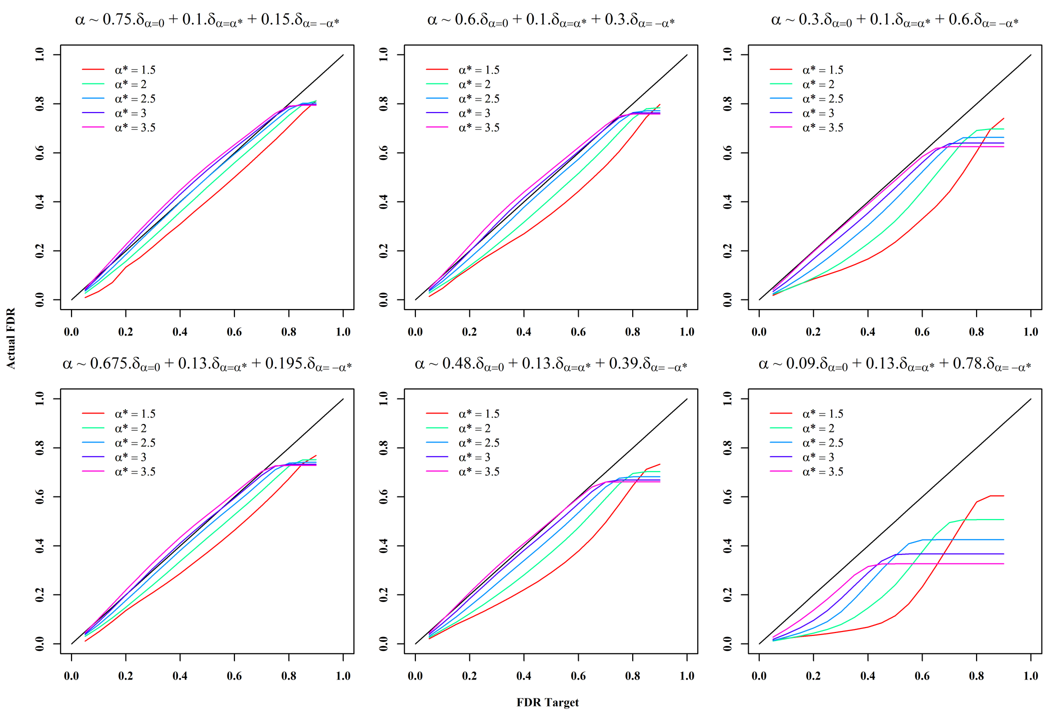


FIGURE IA9

Performance of $fFDR^{+}$ for discrete α with unbalanced panel data

The graphs show the performance of the $fFDR^{+}$ in terms of FDR control when alphas are drawn from the discrete distribution with unbalanced panel data.

Panel A: Cross-sectional Independent Data



Panel B: Cross-sectional Dependent Data

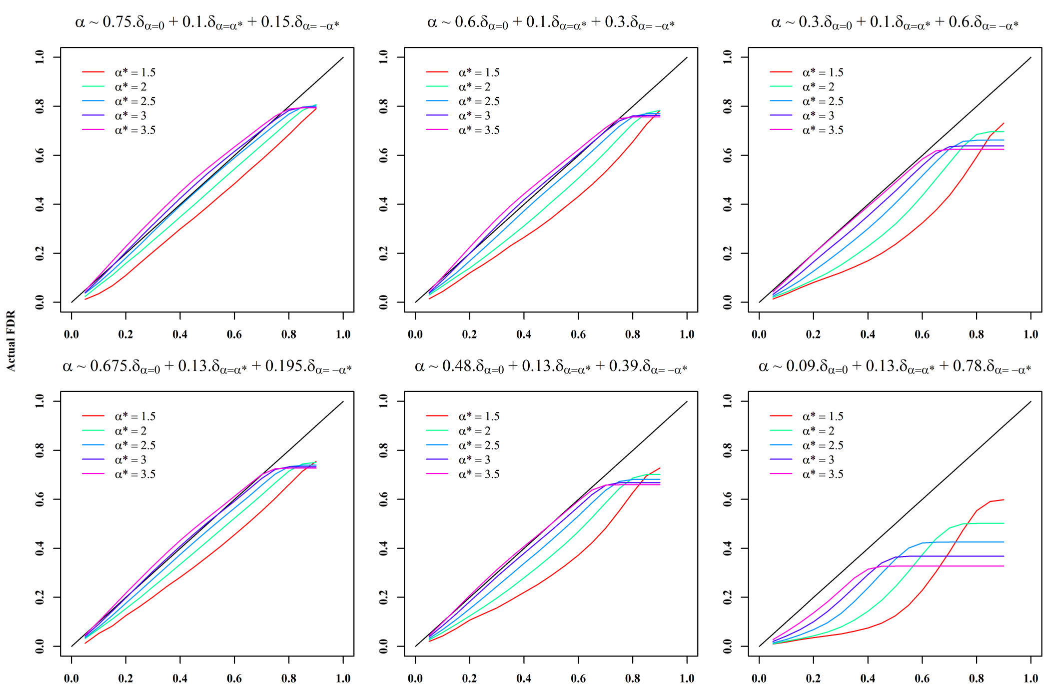
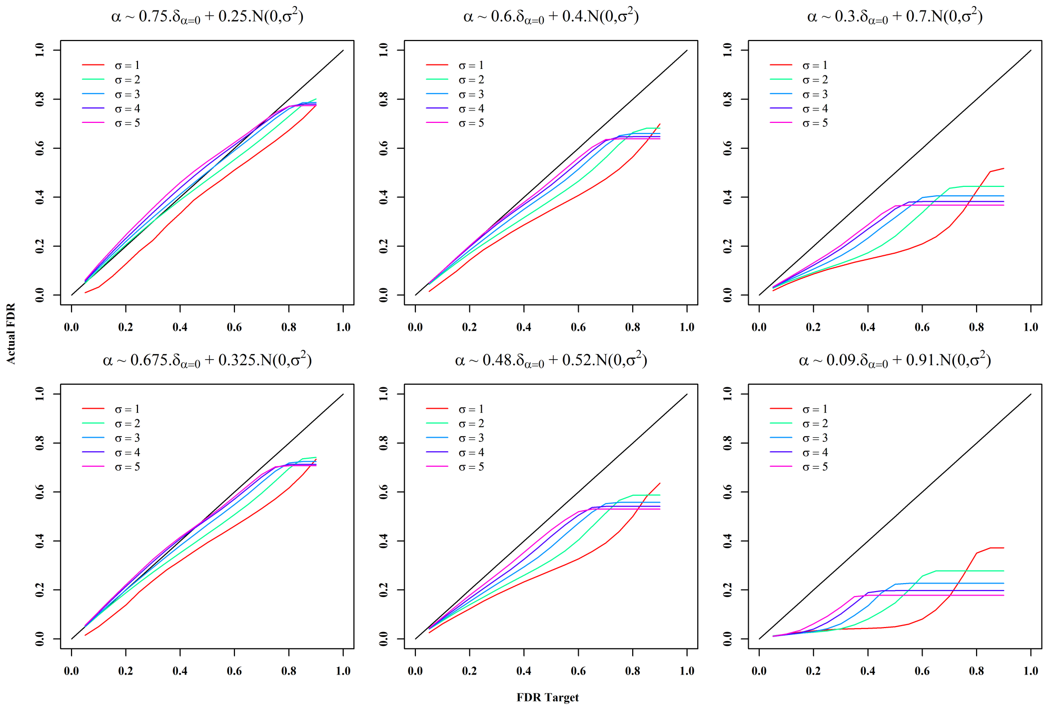


FIGURE IA10

Performance of $fFDR^{+}$ for discrete-normal α with unbalanced panel data.

The graphs show the performance of the $fFDR^{+}$ in terms of FDR control when alphas are drawn from the discrete-normal distribution with unbalanced panel data.

Panel A: Cross-sectional Independent Data



Panel B: Cross-sectional Dependent Data

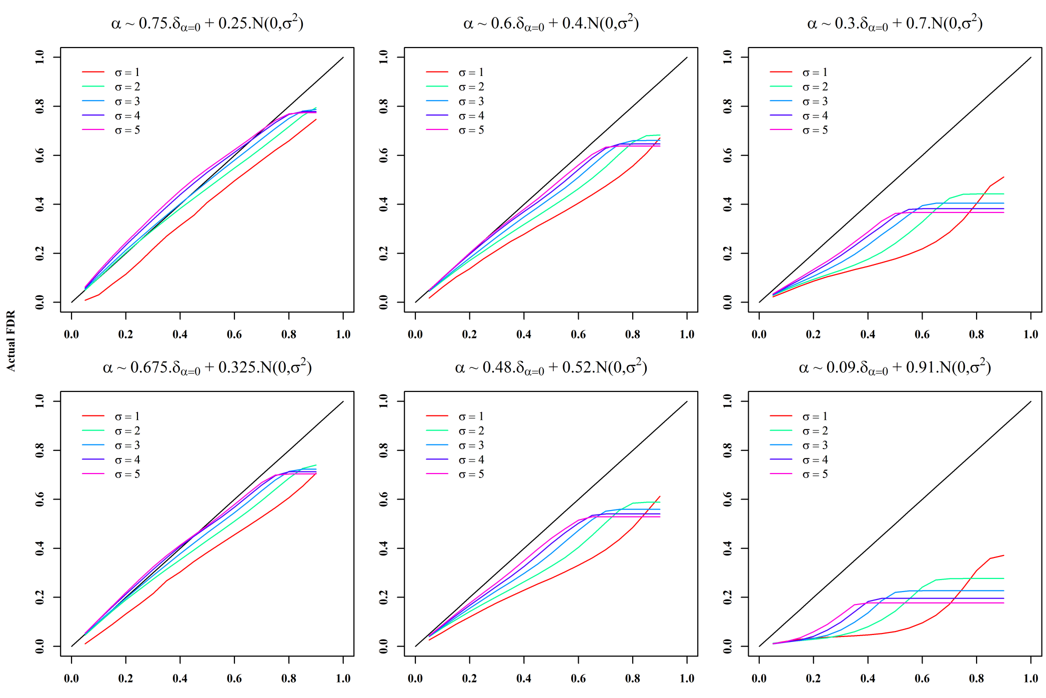
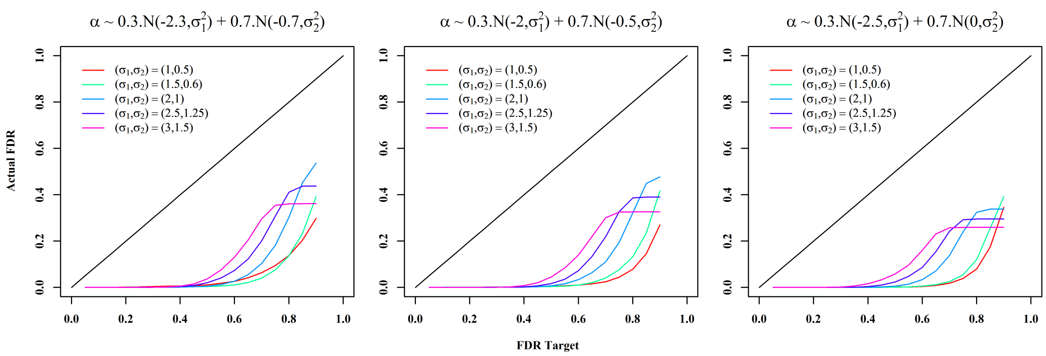


FIGURE IA11

Performance of $fFDR^+$ for two-normal α with unbalanced panel data

The graphs show the performance of the $fFDR^+$ in terms of FDR control when alphas are drawn from the mixture of two normal distributions with unbalanced panel data.

Panel A: Cross-sectional Independent Data



Panel B: Cross-sectional Dependent Data

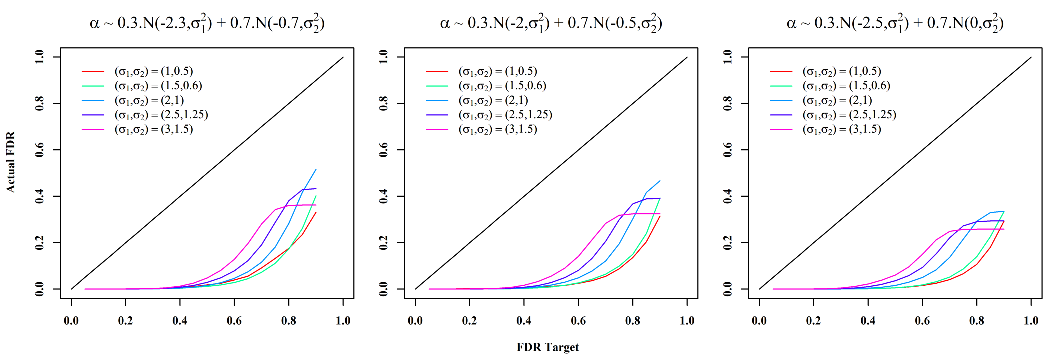
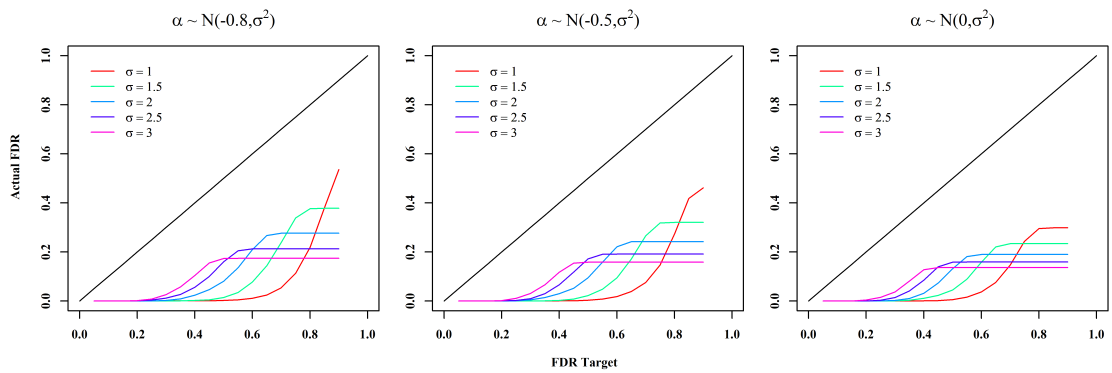


FIGURE IA12

Performance of $fFDR^{+}$ for single normal α with balanced panel data

The graphs show the performance of the $fFDR^{+}$ in terms of FDR control when alphas are drawn from the single normal distribution with balanced panel data.

Panel A: Cross-sectional Independent Data



Panel B: Cross-sectional Dependent Data

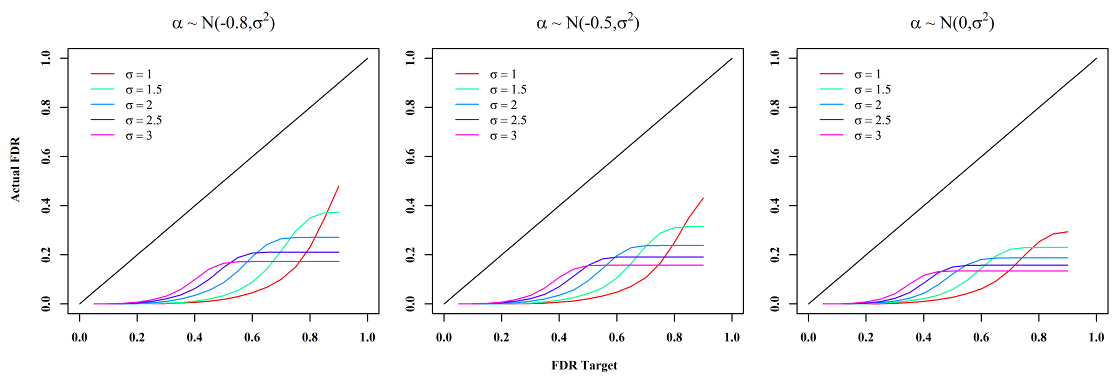
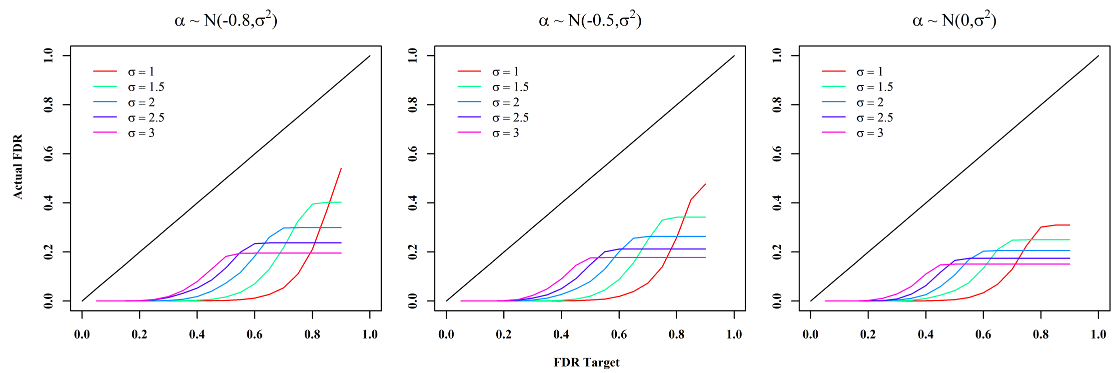


FIGURE IA13

Performance of $fFDR^{+}$ for single normal α with unbalanced panel data

The graphs show the performance of the $fFDR^{+}$ in terms of FDR control when alphas are drawn from the single normal distribution with unbalanced panel data.

Panel A: Cross-sectional Independent Data



Panel B: Cross-sectional Dependent Data

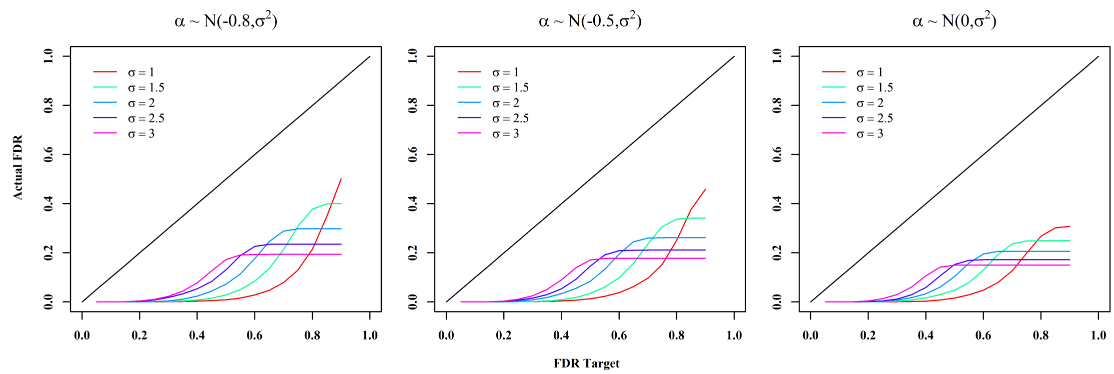


FIGURE IA14

Performance comparison between the FDR^{+} and the $fFDR^{+}$

The graphs compare the performance of the $fFDR^{+}$ in terms of FDR control and power when the generated data are based on a real data sample.

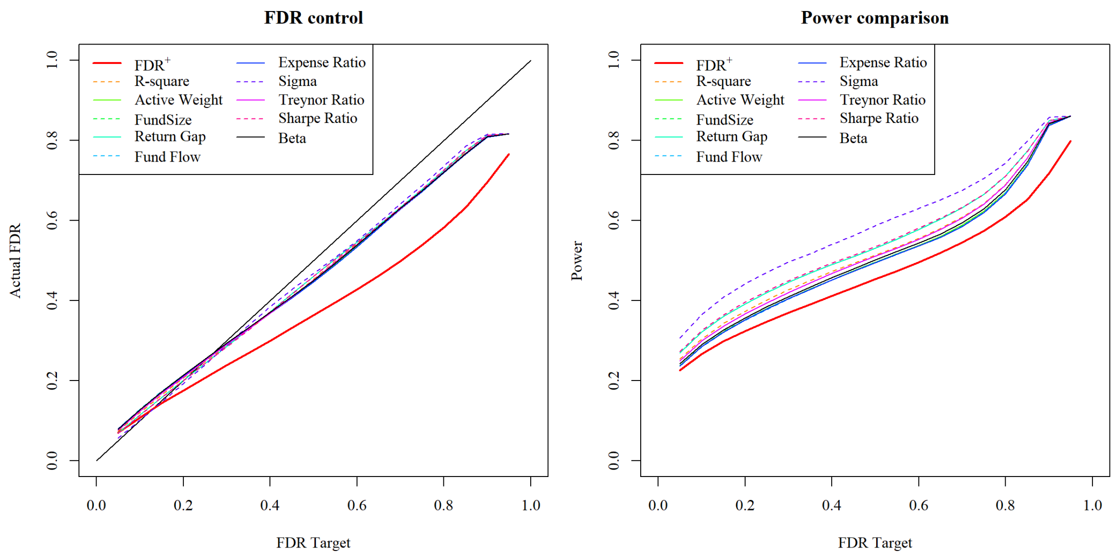


FIGURE IA15

Evolution of wealth

The graph plots the evolution of 1 dollar invested at the beginning of 1982 in the ten $FDR10\%$ portfolios corresponding to the ten covariates, the $fFDR10\%$, the Equal Weight and Equal Weight Plus portfolios.

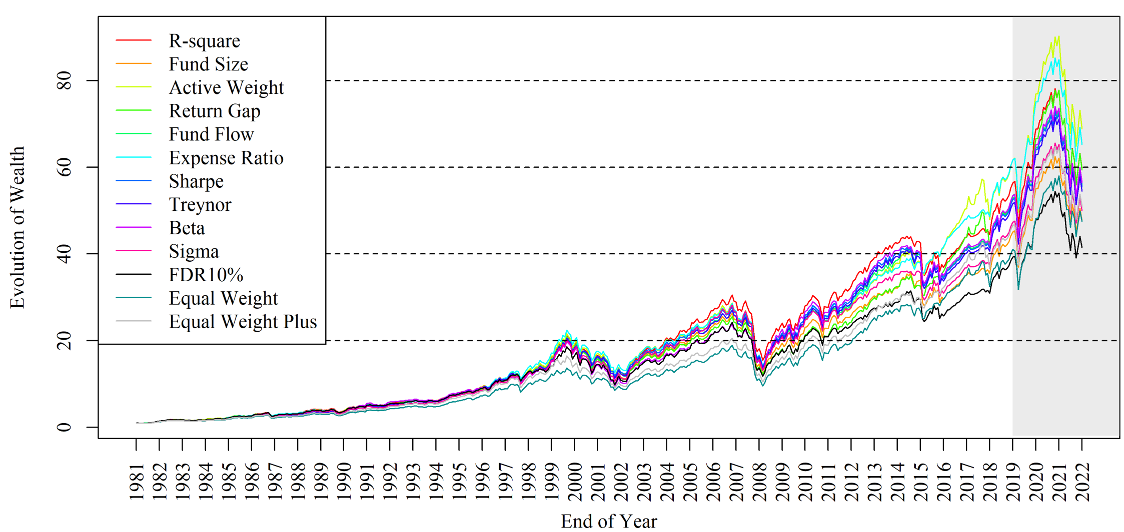


FIGURE IA16

Evolution of wealth of $fFDR\tau$ portfolios with combined covariates

The graph plots the evolution of 1 dollar invested at the beginning of 1982 in the ten $FDR10\%$ portfolios corresponding to the ten covariates, $fFDR10\%$, Equal Weight and Equal Weight Plus portfolios.

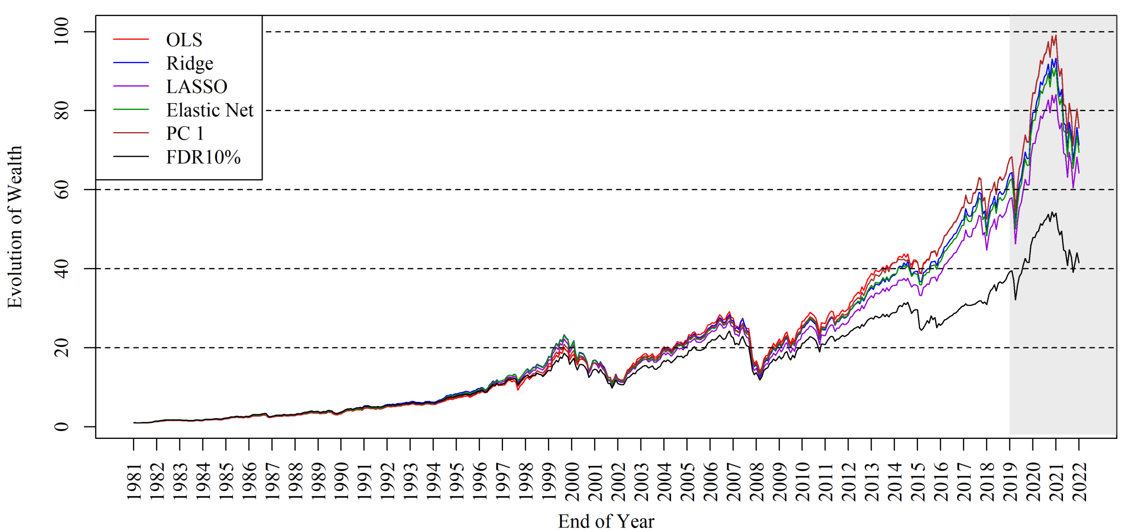


FIGURE IA17

Alpha evolution of $fFDR20\%$ and $FDR20\%$ portfolios over time

The graph presents the evolution of annualized alpha of the ten $fFDR20\%$ portfolios corresponding to the ten covariates, the $FDR20\%$ of BSW and the two equally weighted portfolios.

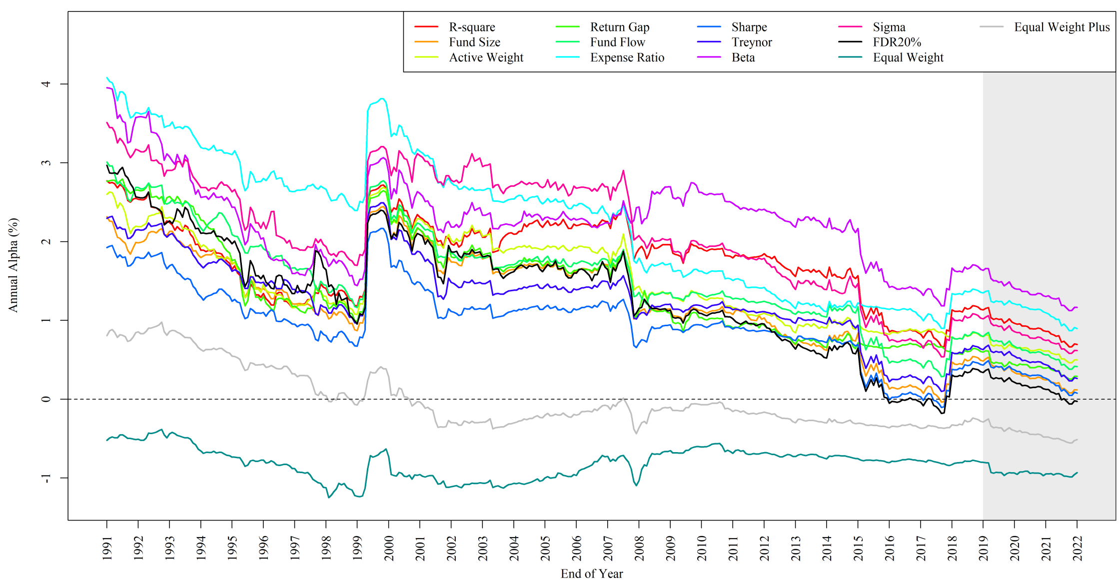


FIGURE IA18

Alpha evolution of $fFDR\tau$ portfolios under alternative coviariates' proxy.

The graph presents the evolution of annualized alpha (in %) of the ten $fFDR10\%$ portfolios (corresponding to ten covariates), the portfolio $FDR10\%$ of BSW and the two equally weighted portfolios. The proxy of a covariate (except the R-square and the four covariates obtained from the asset pricing models) is its average realizations in the five years in-sample period.

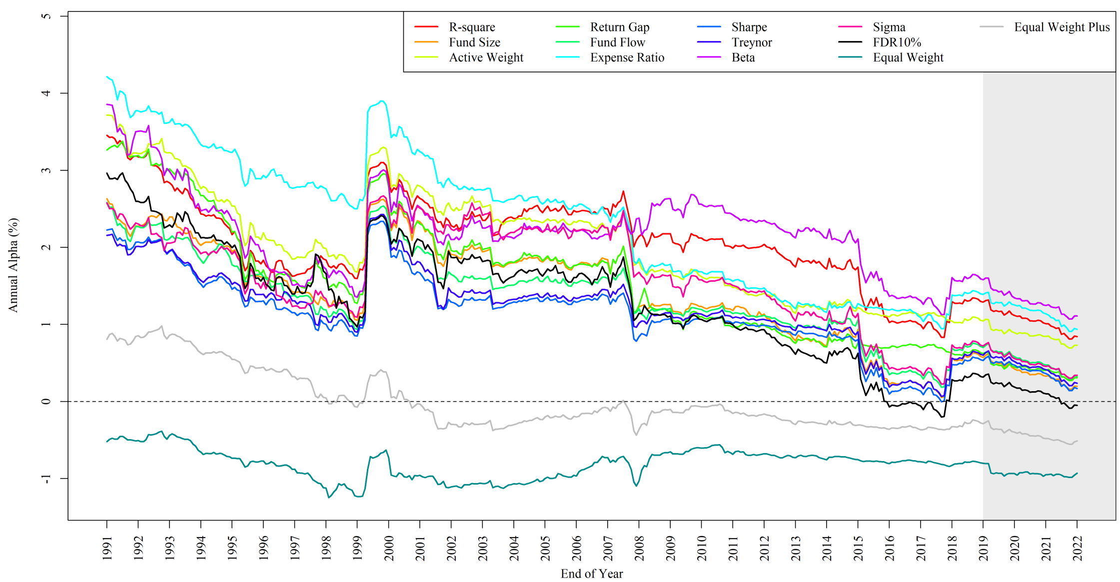


FIGURE IA19

Alpha evolution of $fFDR10\%$ and $FDR10\%$ portfolios over time

The graph presents the evolution of annualized alphas (in %) of the ten $fFDR10\%$ portfolios corresponding to the ten covariates, the portfolio $FDR10\%$ of BSW and the two equally weighted portfolios.

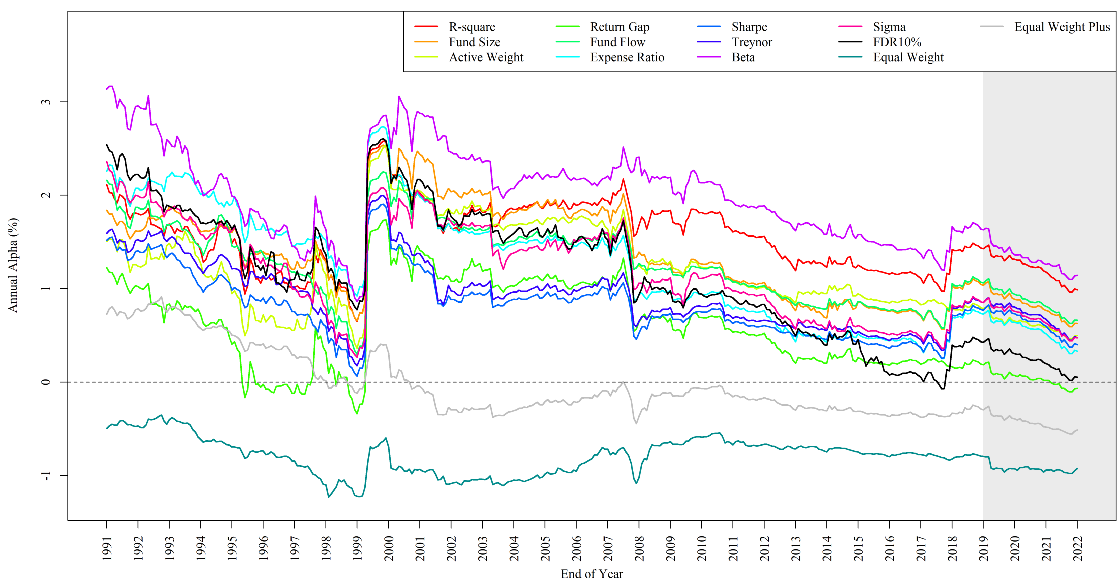


FIGURE IA20

Alpha evolution of $fFDR10\%$ portfolios with combined covariates

The graph shows the alpha evolution of the $fFDR10\%$ portfolios with each using a covariate obtained from either the principal component method or regression method; for the former, the covariate is the first principal component (PC 1) of the five covariates, whereas for the latter the new covariate is a linear combination of the five underlying covariates with the weights obtained based on one of the OLS, LASSO, Ridge and elastic net regressions.

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