Internet Appendices

Nominal U.S. Treasuries Embed Liquidity Premiums, Too

J. Benson Durham*

* jbd4@columbia.edu.

Appendix A: Overview of the Affine Term Structure Model (ATSM)

As noted in the main text, this study proposed no innovations in the standard ATSM formulae. The underlying model factors follow the familiar mean-reverting process, as in,

(1) $$\mathbf{x}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{x}_t + \boldsymbol{\upsilon}$$

where $\mathbf{x}$, $\boldsymbol{\mu}$, and $\boldsymbol{\upsilon}$ are $k\times 1$ vectors; $\boldsymbol{\Phi}$ is a $k\times 1$ matrix, and the disturbances follow a multivariate normal distribution, $\boldsymbol{\upsilon} \sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma})$. Also, the nominal short rate and market price of risk are affine functions of the state vector, $\mathbf{x}$, that follow, respectively,

(2) $$r_t = \delta_0 + \boldsymbol{\delta}_1'\mathbf{x}_t$$

and

(3) $$\boldsymbol{\lambda}_t = \boldsymbol{\Sigma}^{-\frac{1}{2}}\left(\boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1'\mathbf{x}_t\right)$$

where δ_0 is a scalar, $\boldsymbol{\delta}_1$ and $\boldsymbol{\lambda}_0$ are $k\times 1$ vectors, $\boldsymbol{\lambda}_1$ is a $k\times k$ matrix, and the nominal pricing kernel follows,

(4) $$M_{t+1} = \exp\left[-r_t - \frac{1}{2}\boldsymbol{\lambda}_t'\boldsymbol{\lambda}_t - \boldsymbol{\lambda}_t'\boldsymbol{\Sigma}^{-1/2}\boldsymbol{\upsilon}_{t+1}\right]$$

Given the arbitrage-free pricing relation, the price for any n-maturity bond follows,

(5) $$P_{n,t} = \mathrm{E}_t\left\{M_{t+1}P_{n-1,t+1}\right\}$$

With the usual substitutions, the affine form for log bond prices, with an additional term for measurement error, $u_{n,t}$, follows,

(6) $$\ln P_{n,t} = A_n + \mathbf{B}_n{}'\mathbf{x}_t + u_{n,t}$$

where, following from equations (5) and (6),

(7) $$\mathbf{B}_n{}' = \mathbf{B}_{n-1}{}'(\mathbf{\Phi} - \boldsymbol{\lambda}_1) - \boldsymbol{\delta}_1{}'$$

and

(8) $$A_n = A_{n-1} + \mathbf{B}_{n-1}{}'(\boldsymbol{\mu} - \boldsymbol{\lambda}_0) + \frac{1}{2}\left(\mathbf{B}_{n-1}{}'\mathbf{\Sigma}\mathbf{B}_{n-1} + \sigma^2\right) - \delta_0$$

with the initial conditions $A_0 = 0$ and $\mathbf{B}_0 = \underset{k\times 1}{\mathbf{0}}$. The σ^2 term allows for return fitting errors that are conditionally orthogonal to innovations in the state variables, and therefore this approach follows ACM by embedding pricing errors in the recursions. Also, the *n*-year fitted zero-coupon TIPS yield follows,

(9) $$y_{n,t} = -\frac{1}{n}A_n - \frac{1}{n}\mathbf{B}_n{}'\mathbf{x}_t$$

Again, following Adrian et al. (2013), setting the market price of risk parameters— $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$,— equal to zero produces the "risk-adjusted" bond pricing parameters, $\tilde{A}_n$ and $\tilde{\mathbf{B}}_n$,[1] and the expectation at *t* of average expected short rates through *n*, $\tilde{y}_{n,t}$,[2] follows,

[1] Therefore, with the same initial conditions, the risk-adjusted recursions follow,

$$\tilde{\mathbf{B}}_n{}' = \tilde{\mathbf{B}}_{n-1}{}'\mathbf{\Phi} - \boldsymbol{\delta}_1{}'$$
$$\tilde{A}_n = \tilde{A}_{n-1} + \tilde{\mathbf{B}}_{n-1}{}'\boldsymbol{\mu} + \frac{1}{2}\left(\tilde{\mathbf{B}}_{n-1}{}'\mathbf{\Sigma}\tilde{\mathbf{B}}_{n-1} + \sigma^2\right) - \delta_0$$

[2] Adrian et al. (2013) refer to this quantity as the "risk-neutral" yield.

$$\tilde{y}_{n,t} = -\frac{1}{n}\tilde{A}_n - \frac{1}{n}\tilde{\mathbf{B}}_n{}'\mathbf{x}_t \tag{10}$$

Hence, aggregate expected excess returns on an *n*-period nominal security, aka the "term premium," follows,

$$tp_{n,t} = y_{n,t} - \tilde{y}_{n,t} \tag{11}$$

The ATSM embeds liquidity risk as an observable element in x, as in,

$$x_t = \begin{bmatrix} \mathbf{x}_{t+n}^{PC} \\ x_{t+n}^{LIQ} \end{bmatrix} \tag{12}$$

Therefore, an expression for the expected short rate, equation (2), *n* periods ahead, with further parsing given delineation of the state vector, follows,

$$\begin{aligned} \mathrm{E}_t\left\{r_{t+n}\right\} &= \delta_0 + \begin{bmatrix} \boldsymbol{\delta}_{1:k-1,1} \\ \delta_{k,1} \end{bmatrix}{}'\mathrm{E}_t\left\{\begin{bmatrix} \mathbf{x}_{t+n}^{PC} \\ x_{t+n}^{LIQ} \end{bmatrix}\right\} \\ &= \underbrace{\delta_0 + \boldsymbol{\delta}_{1:k-1,1}{}'\mathrm{E}\left\{\mathbf{x}_{t+n}^{PC}\right\}}_{\text{"frictionless risk-free rate"}} + \delta_{1:k}\mathrm{E}_t\left\{x_{t+n}^{LIQ}\right\} \\ &= \mathrm{E}_t\left\{r_{t+n}^f\right\} + \delta_{1:k}\mathrm{E}_t\left\{x_{t+n}^{LIQ}\right\} \end{aligned} \tag{13}$$

The measure of inflation expectations in the main text comprises to the anticipated nominal frictionless forward rate beginning after five years and ending in 10 years, as in,

$$r_t^* \approx \tilde{f}_{5\to 10,t}^f = 2\tilde{y}_{10,t}^f - \tilde{y}_{5,t}^f \tag{14}$$

Also, for any maturity, the risk-adjusted frictionless yield, $\tilde{y}_{n,t}^f$, is embedded in equation (10), as in,

$$\tilde{y}_{n,t} = \tilde{y}^f_{n,t} + \underbrace{\left(\tilde{y}_{n,t} - \tilde{y}^f_{n,t}\right)}_{\text{risk-adjusted liquidity loading}}$$

(15)
$$-\frac{1}{n}\tilde{A}_n - \frac{1}{n}\tilde{\mathbf{B}}_n{}'\mathbf{x}_t = -\frac{1}{n}\left(\tilde{A}^f_n + \tilde{A}_n - \tilde{A}^f_n\right) - \frac{1}{n}\begin{bmatrix}\tilde{\mathbf{B}}_{n,1:k-1}\\ \tilde{B}_{n,k}\end{bmatrix}'\begin{bmatrix}\mathbf{x}^{PC}_t\\ x^{LIQ}_t\end{bmatrix}$$
$$= \underbrace{-\frac{1}{n}\tilde{A}^f_n - \frac{1}{n}\tilde{\mathbf{B}}_{n,1:k-1}{}'\mathbf{x}^{PC}_t}_{\text{risk-adjusted frictionless yield}} \underbrace{-\frac{1}{n}\left(\tilde{A}_n - \tilde{A}^f_n\right) - \frac{1}{n}\tilde{B}_{n,k}{}'x^{LIQ}_t}_{\text{risk-adjusted liquidity loading}}$$

where

$$-\frac{1}{n}\tilde{A}^f_n = -\frac{1}{n}\left(\tilde{A}^f_{n-1} + \boldsymbol{\mu}_{1:k-1}{}'\tilde{\mathbf{B}}_{n-1,1:k-1} - \boldsymbol{\delta}_0 + \frac{1}{2}\tilde{\mathbf{B}}_{n-1,1:k-1}{}'\underset{(1:k-1)\times(1:k-1)}{\boldsymbol{\Sigma}}\tilde{\mathbf{B}}_{n-1,1:k-1}\right)$$

Note that $\tilde{y}^f_{n,t}$ reflects the complete system dynamics, including its liquidity or "dirty" components. To be more precise, $-\frac{1}{n}\tilde{\mathbf{B}}_{n,1:k-1}$ in equation (15), or the first $k-1$ full rows of $\frac{1}{n}\boldsymbol{\delta}_1{}'\left(\mathbf{I}-\boldsymbol{\Phi}^n\right)\left(\mathbf{I}-\boldsymbol{\Phi}\right)^{-1}$, which also appears in $-\frac{1}{n}\tilde{A}^f_n$, is clearly informed by parameters related to dynamics of the LF. Namely, these include the first four, or $k-1$, entries of the last column of $\boldsymbol{\Phi}$, or $\underset{1:k-1,k}{\boldsymbol{\Phi}}$, which represent the effects of one-period lagged values of the LF on each frictionless factor.

Appendix B: Curve Fitting Errors and Maturity

As mentioned in the main text, one possible alternative approach to construct a LF would be to incorporate maturity-specific liquidity proxies, in turn to afford a liquidity-adjusted yield curve, maturity by maturity, to compare alongside the "dirty" term structure.[3] A provisional exploration of this issue comprises estimates of the relation between each of the NSS-VRP-KR fitting errors, $e_{i,t}^{NSS-VRP-KR}$, and some function of the maturity, as in,

$$e_{i,t}^{NSS-VRP-KR} = \hat{\alpha}_t^{NSS-VRP-KR} + \hat{\beta}_t^{NSS-VRP-KR} f\left(m_{i,t}\right) + \varepsilon_{i,t}^{NSS-VRP-KR} \tag{16}$$

where $\hat{\beta}_t^{NSS-VRP-KR}$ is a scalar or vector that captures the cross-sectional relation at observation t between noise and maturity for each fitting method. Also, $f\left(m_{i,t}\right)$ is one of four possible functional forms—either a linear, log, quadratic, or cubic—of the number of days to maturity for the individual bond. Briefly, sturdy relations between maturity per se and underlying pricing error, to form the bases for a maturity-specific liquidity proxy, seem largely absent from the data, again including daily estimates of equation (16) from January 2, 1987, through September 1, 2023.

For example, as Graphs A–D in Appendix B Figure 1 indicate, the adjusted R-squared values based on equation (16) across each curve fitting method and functional form of maturity are quite low, as well as volatile over the sample, which suggests inconsistent and modest differentiation in liquidity across the term structure. Moreover, based on the corresponding p values of F statistics illustrated Graph E, the coefficients in $\hat{\beta}_t^{NSS-VRP-KR}$ are jointly statistically significant at best for less than 25% of all sample days, in the case of the cubic functional form using the NSS fitting errors. However, that this preliminary analysis using fitting errors does not readily recommend a maturity-

[3] An anonymous referee made this helpful suggestion.

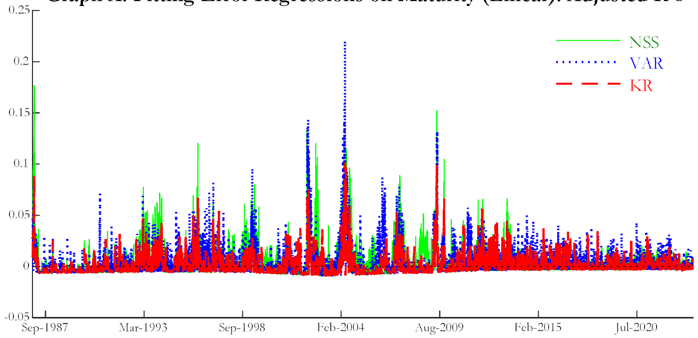
specific approach does not preclude the possibly that other metrics might uncover varying liquidity conditions across preferred habitats along the curve. Given the dearth of literature that explicitly accounts for liquidity premiums in arbitrage-free models of nominal Treasuries, further analysis along these lines seems would be instructive.

Appendix B FIGURE 1

Fitting Errors as Functional Forms of Maturity

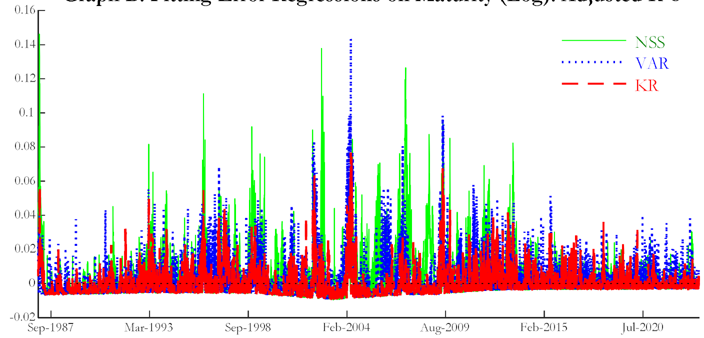
Graphs A–D of Appendix B Figure 1 show the adjusted R-squared values based on equation (16) across each curve fitting method and functional form of maturity—linear, log, quadratic, and cubic. Graph E reports the corresponding p values of the F statistics.

Graph A. Fitting Error Regressions on Maturity (Linear): Adjusted R^2s



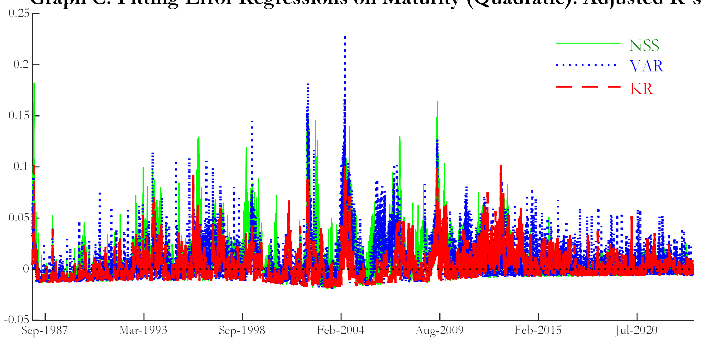
Sample averages: NSS (0.0056), VRP (0.0037), and KR (0.0016).

Graph B. Fitting Error Regressions on Maturity (Log): Adjusted R^2s



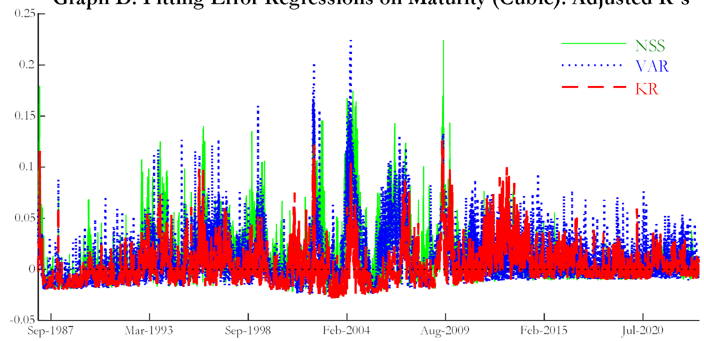
Sample averages: NSS (0.0053), VRP (0.0025), and KR (0.00024).

Graph C. Fitting Error Regressions on Maturity (Quadratic): Adjusted R^2s

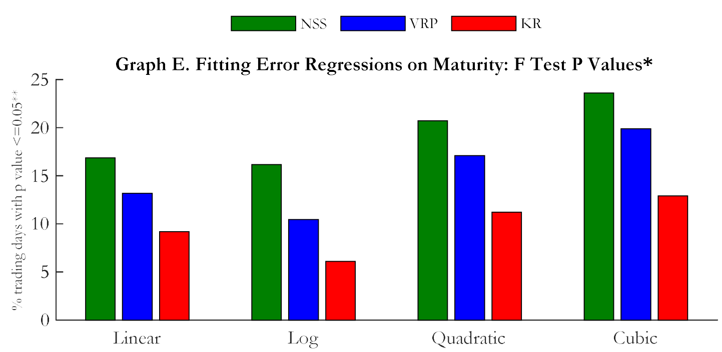


Sample averages: NSS (0.01), VRP (0.0072), and KR (0.0024).

Graph D. Fitting Error Regressions on Maturity (Cubic): Adjusted R^2s



Sample averages: NSS (0.015), VRP (0.0099), and KR (0.0038).



Graph E. Fitting Error Regressions on Maturity: F Test P Values*

*Sample: (02-Jan-1987-01-Sep-2023). **Total number of days=9165.

Appendix C: Robustness Checks on Model Factor Constructions and Liquidity Effects

Given economically meaningful loadings on liquidity outlined in the main text, further examination of the factor construction, which again follows the precedent for TIPS in AACMY, is warranted. To start, Graph A in Appendix C Figure 1 shows the factor excess return loadings, and Graphs B–F include plots of two sets of sensitivities, the coefficients on the factors from the regressions of excess returns on state-variable innovations and lagged factors, alongside the recursive pricing parameters used to generate fitted yields.[4] The two estimates are very similar indeed, and as detailed in ACM, narrow differences between the excess return regression coefficients and the imputed recursive pricing parameters implies that the model satisfactorily replicates the dynamics of yields and returns. But to focus on impact of liquidity, as for yields, the return loadings are greater in absolute than all other factors (about 0.03 for both metrics across maturities), save the level (0.114 and 0.112), including the slope of the term structure (0.015 and 0.016).

Also, returning to the preliminary motivation, the solid blue line in Graph A of Appendix C Figure 2 shows the coefficients from regressions of yields across the term structure on the corresponding LF for nominals, as described previously and as shown in Graph F of Figure 6. The corresponding estimates for TIPS, using the LF from Durham (2023), follows the dotted black line. Although the corresponding R-squared values for the nominal (univariate) regressions are lower, about 0.07 compared to 0.18 on average, the coefficients on the nominal LF, which imply a 63-basis-point increase on average across five- to 10-year yields for a one standard error increase in the LF, clearly rival those for TIPS for corresponding maturities, which average about 60 basis points.

[4] The ACM pricing parameters ("beta," the diamonds in the charts) follow,

$$rx_{n-1,t+1} = \boldsymbol{\beta}_{n-1,t}{}'\boldsymbol{\lambda}_0 - \frac{1}{2}\left(\boldsymbol{\beta}_{n-1,t}{}'\boldsymbol{\Sigma}\boldsymbol{\beta}_{n-1,t} + \sigma^2\right) + e_{t+1}^{(n-1)} + \underbrace{\boldsymbol{\beta}_{n-1,t}{}'\boldsymbol{\lambda}_1}_{\text{"beta"}}\mathbf{x}_t + \boldsymbol{\beta}_{n-1,t}{}'\boldsymbol{\upsilon}_{t+1}$$

where rx is excess return, and the remaining parameters are as defined in the Appendix A.

As such, the base motivation for including a LF on this score is as compelling for nominals as for TIPS, even though the affine-model literature nearly ubiquitously presumes nominals are frictionless, in stark contrast to the universal prior that TIPS embed a sizeable liquidity premium.

To assess further the size and robustness of the LF, another tack considers other observable variables across dozens of regressions that address possible specification bias. These include survey measures of anticipated real GDP growth, inflation, budget deficits, measures of survey dispersion around those projections, as well as measures of how Treasuries contribute to wider portfolio risk, including GARCH-based models of return volatility and M-GARCH-based measures of return correlations and betas with respect to equity returns.[5] The motivation for including liquidity as an observable factor in an affine model becomes more compelling, insofar as the LF is consistently robust and economically significant with the expected sign, which would be problematic for the prevailing frictionless prior in the ATSM literature.

More specifically, this analysis comprises time-series regressions of the form,

$$y_{m,t} = \varphi_0^{(n)} + \varphi_{LF}^{(n)} LF_t + \boldsymbol{\varphi}_t^{(n)\prime} \mathbf{x}_t^{(n)} + \varepsilon_{m,t}^{(n)} \tag{17}$$

where $y_{m,t}$ is the yield on an *m*-maturity bond at time *t*, and *m* includes seven maturity points— 2, 3, 5, 7, 10, 20, and 30 years; *LF* is the contemporaneous LF; $\mathbf{x}^{(n)}$ is the n^{th} of *N* possible specifications of control variables or alternative factors,[6] and $\varepsilon_{m,t}^{(n)}$ is the corresponding error term. Briefly, as noted

[5] See Durham (2020) for a similar analysis of the impact of Treasury bond "beta" relative to other factors in a similar empirical (regression) framework.

[6] As listed in Appendix C Table 1 in the case of 10-year yields, each of the (N = 96) total specifications includes one-year-ahead mean expected real GDP growth, CPI inflation, and budget deficits from Consensus Economics (CE) surveys, along with the LF. Also, alternative specifications include every possible linear combination across four sets of alternative proxies, largely following specification searches in the "extreme bound analysis" literature (e.g., Leamer (1983), Levine and Renelt (1992)). The first includes two measures related to macroeconomic uncertainty, including the standard deviations of CE forecasts of growth and inflation. The second set that includes six variables captures measures of wider financial market stress that possibly capture "flight-to-quality" episodes, alternatively including GARCH-based

in Appendix C Table 1 in the case of the 10-year maturity, the coefficients on LF are statistically significant with at least 95 percent confidence in about 86 percent of the 96 alternative regressions, and the estimates formally pass "extreme bound analysis" (EBA) "robustness" criteria across alternative specifications, as outlined in Sala-I-Martin (1997).[7] Moreover, as shown in Graph B of Appendix C Figure 2, the magnitude of the weighted-average estimated coefficients is sizeable, ranging from about 8 to 21 basis points across the yield curve per a one-standard-error LF shock, a range of coefficients that compares meaningfully to even the most robust factors. Therefore, again, that the LF appears robust to several other common correlates lends further motivation for incorporating liquidity premiums in arbitrage-free models of nominal yields.

All in all, each of these results suggest that not only TIPS but also nominal bonds embed priced, time-varying liquidity risks, notably in this arbitrage-free framework, and therefore common ATSMs of nominal yields may be imprecisely specified.

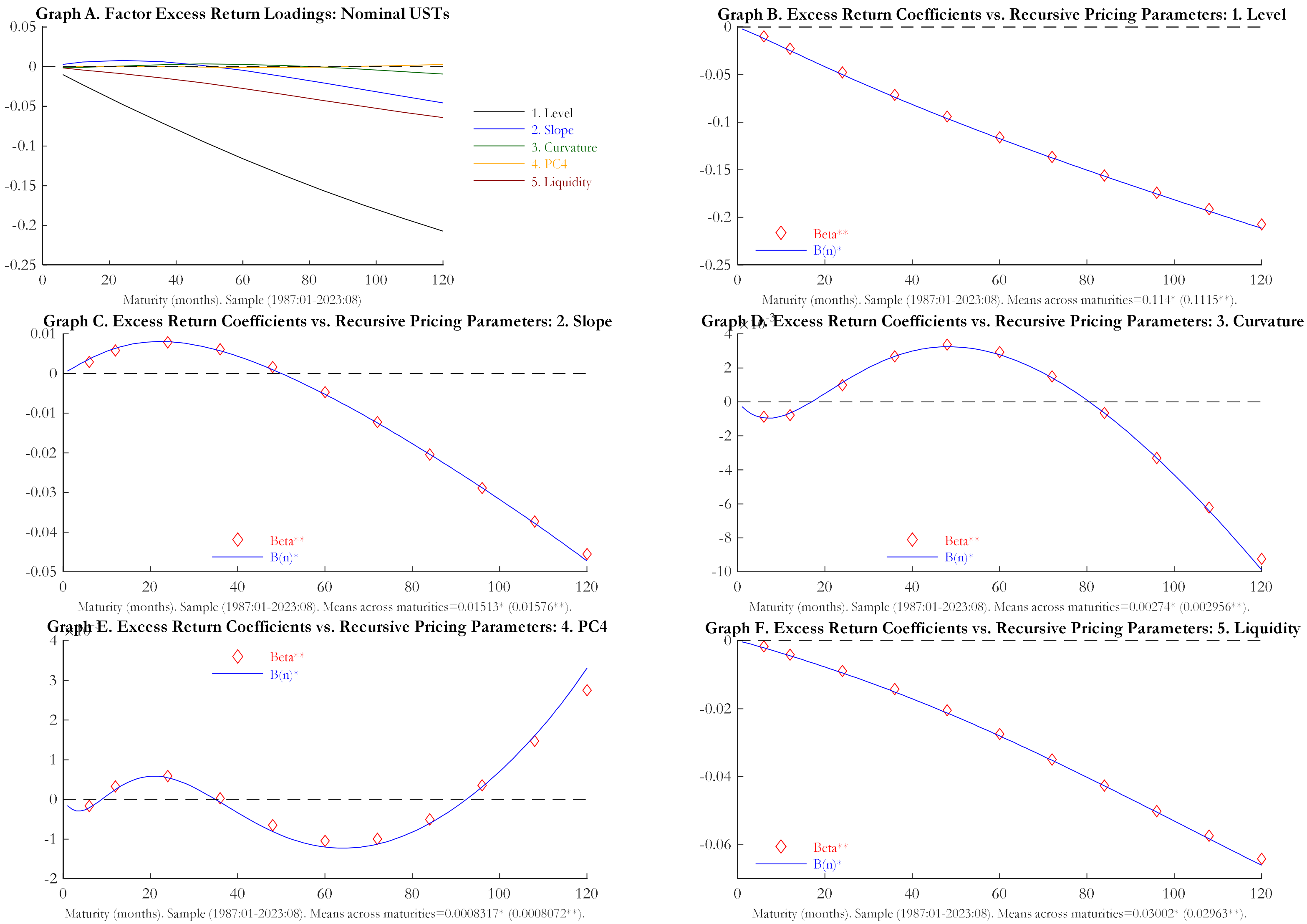
U.S. equity market return volatility and its squared value, as well as dynamic measures of covariance risks relative to 10 S&P 500 sectors as well as selected global asset classes—commodities (BCOM Index), emerging market bonds (JPEIGLBL Index), government bonds (SBWGL Index), emerging market stocks (MXEF Index), and developed market stocks (MXWO Index)—alternatively based on average pairwise correlations as well as the relative magnitude of the first dynamic principal component of returns, in terms of the percentage of variance explained. The third set includes measures related to interest rate uncertainty, including the standard deviation of CE survey forecasts of 3-month and 10-year yields, as well as GARCH-based return volatility of 1- and 10-year Treasury bonds, alternatively. Finally, the fourth set of variables addresses the hedging value of nominal Treasuries, alternatively including M-GARCH-based dynamic betas or correlation coefficients relative to local equity market returns, largely following Durham (2020). (The regression models are sampled at the monthly CE survey frequency, that is, at the estimated time each poll was taken.)

[7] As listed in Appendix C Table 1, the probability that $\varphi_{LF} <= 0$ is less than 1 percent. This assessment is based on the estimated cumulative density function (CDF) of the N estimates, following an assumed Gaussian distribution of coefficient standard errors or otherwise ("W-A CDF [Gaussian]" and "W-A CDF," respectively), as described in Sala-I-Martin (1997).

Appendix C FIGURE 1

Model Factor Excess Return Loadings

Graph A in Appendix Figure 1 shows the loadings from the excess return regressions. The diamonds in Graphs F–F, "beta" or $\beta_{n-1,t}'\lambda_1$, also capture the response to contemporaneous shocks, factor by factor, alongside the corresponding ATSM-implied loadings in Graph A.

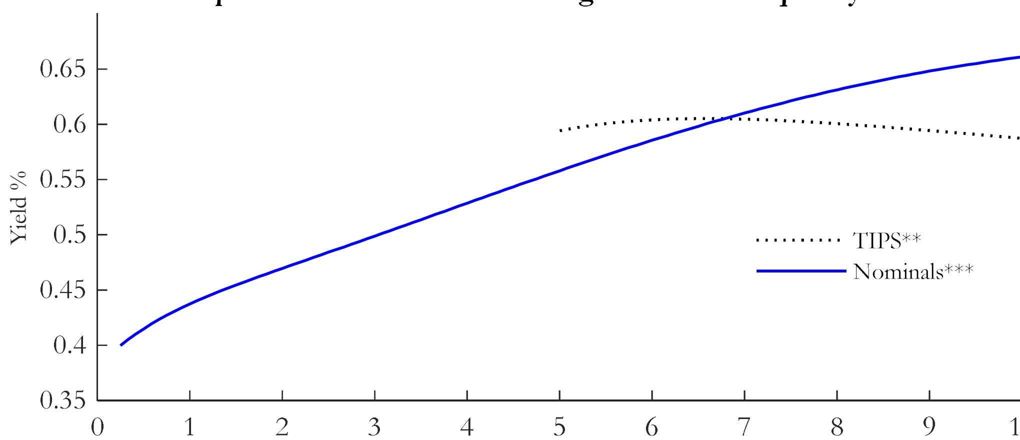


Appendix C FIGURE 2

Liquidity Loadings Sensitivity: TIPS and Regression Analyses

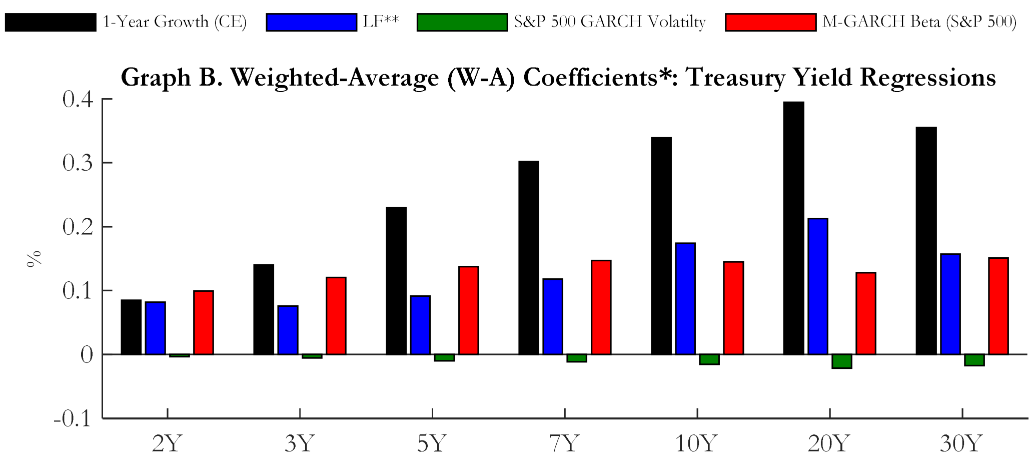
Graph A of Appendix C Figure 2 shows the coefficients from regressions of yields across the term structure on the corresponding LF for nominals, as shown in Graph F of Figure 6 in the main text. Graph B of Appendix C Figure 2 shows the magnitudes of the weighted-average coefficients for selected independent variables, including the LF, following alternative regression specifications as in equation (17).

Graph A. OLS Betas*: Yields Regressions on Liquidity Factor



Maturity (years). *Response to 1-standard-error increase in factor. **Based on Durham (2023). Sample (07/1999-09/2023) Mean R^2=0.18. *** Sample (01/1987-08/2023). Mean R^2 (>= 5 years)=0.071. Mean across maturites >= 5 years (nominals)=0.6% (0.63%).

Graph B. Weighted-Average (W-A) Coefficients*: Treasury Yield Regressions



*Based 1-standard-error shocks on average across (96) models of Treasury yields (02/1996-09/2023).
**Min. (max.) coefficient=7.59bps (21.3bps).

Appendix C TABLE 1

Extreme Bound Analysis of LF Robustness

Appendix C Table 1 shows, for the case of the 10-year maturity, summary statistics for 96 alternative regressions following alternative specifications as in equation (17), namely EBA "robustness" criteria as outlined in Sala-I-Martin (1997).

10-Year U.S. Treasury Yield
Coefficients: Level Regressions
Feb 1996-Sep 2023
Observations per Regression (332)

Independent Variable	W-A Beta	Min. Beta	Max. Beta	Ave. N-W p value	Max. N-W p value	% Significant	W-A CDF	W-A CDF (Gaussian)	Ave. R^2	# Regs.
3-Month Short Rate (CE)	0.90	0.78	1.03	0.00	0.00	100%	0.000	0.000	0.83	96
1-Year Inflation (CE)	-0.28	-0.37	-0.22	0.00	0.00	100%	1.000	1.000	0.83	96
1-Year Growth (CE)	0.34	0.25	0.41	0.00	0.00	100%	0.000	0.000	0.83	96
LF	**0.17**	**0.08**	**0.34**	**0.02**	**0.22**	**86%**	**0.009**	**0.001**	**0.83**	**96**
1-Year Budget Surplus (% GDP) (CE)	0.34	0.20	0.44	0.00	0.01	100%	0.000	0.000	0.83	96
1-Year Growth Std. Dev. (CE)	0.02	-0.03	0.14	0.48	0.99	23%	0.134	0.170	0.83	48
1-Year Inflation Std. Dev. (CE)	0.05	0.05	0.18	0.10	0.35	46%	0.024	0.027	0.83	48
S&P 500 GARCH Volatilty	-0.02	-0.21	0.03	0.22	0.85	44%	0.139	0.969	0.83	16
% Explained First Dynamic PC (S&P 500 Sectors)	0.03	0.09	0.27	0.01	0.06	94%	0.001	0.000	0.83	16
Ave. A-DCC M-GARCH Coef. (S&P 500 Sectors)	0.03	0.10	0.23	0.00	0.03	100%	0.000	0.000	0.83	16
% Explained First Dynamic PC (Global Assets)	-0.01	-0.10	0.02	0.33	0.86	13%	0.134	0.859	0.82	16
Ave. A-DCC M-GARCH Coef. (Global Assets)	-0.01	-0.12	0.02	0.36	0.86	13%	0.132	0.846	0.82	16
S&P 500 GARCH Vol. Squared	-0.02	-0.24	-0.03	0.12	0.56	75%	0.157	0.995	0.83	16
3-Month Short Rate Std. Dev. (CE)	0.10	0.40	0.43	0.00	0.00	100%	0.000	0.000	0.83	24
10-Year Yield Std. Dev. (CE)	0.10	0.35	0.42	0.00	0.00	100%	0.000	0.000	0.84	24
UST Bond Return GARCH Volatility	0.09	0.31	0.44	0.00	0.00	100%	0.000	0.000	0.82	24
UST 1-Year Bond Return GARCH Volatility	0.08	0.24	0.39	0.00	0.00	100%	0.000	0.000	0.81	24
M-GARCH Correlation (S&P 500)	0.17	0.26	0.41	0.00	0.00	100%	0.000	0.000	0.83	48
M-GARCH Beta (S&P 500)	0.14	0.24	0.32	0.00	0.00	100%	0.000	0.000	0.83	48

Appendix D: Additional Analyses of Cyclicality

As noted in the main text, a reasonable suspicion about the reported finding that liquidity rather than frictionless term premia account for counter-cyclicality is that the outlying spike in estimated liquidity premiums during the GFC primarily produces the result. To address this issue, consider three additional analyses.

1. Structural Break Tests

One possible addendum, which does not omit the outlying yet nonetheless informative instance of the GFC, is to consider whether the positive coefficient of the recession dummy in the liquidity premium regressions is robust across structural breaks, namely also significant before the episode. To start, the specification below evaluates such a break before and after 2008, given a simple indicator function, $I_{t \geq 01/2008}$, as in,

(18) $$\begin{bmatrix} tp^{f}_{10Y,t} \\ lp_{10Y,t} \end{bmatrix} = \alpha + \boldsymbol{\beta}'\mathbf{x}_t + \boldsymbol{\beta}^{t \geq 01/2008}{}'\mathbf{x}_t I_{t \geq 01/2008} + \varepsilon_t$$

Briefly, as noted in the last column in the upper panel of Appendix D Table 1, the conditional expectation of the liquidity premium prior to 2008 is about 11 basis points greater during recessions ($\hat{\beta}_{lp} = 0.115$), which increases to about 79 basis points ($\hat{\beta}_{lp} + \hat{\beta}^{t \geq 01/2008}_{lp}$) for observations after 2007. True, the magnitude of the estimated relation changes. Still, the sign is consistently positive and robust across breaks, in accordance with generally counter-cyclical liquidity premiums. At the same time, the results for the frictionless term premium are not consistent across the 2008 break, indicative of ambiguous cyclicality. For example, the statistically significant results suggest counter-cyclicality before the break $\hat{\beta}_{tp^f} = 1.07$, but afterwards the estimates ($\hat{\beta}_{tp^f} + \hat{\beta}^{t \geq 01/2008}_{tp^f}$) indicate pro-cyclicality by about the same margin, with frictionless term premiums about 99 basis points lower during recessions. These results are broadly consistent with predominate perceptions of supply (demand) shocks before (after) the break.

To further assess the robustness of nominal liquidity premium counter-cyclicality, again based on recessions, an alternative follows equation (18) but relaxes the assumption of a known structural break over the 404-month sample and randomly tests for breaks between observations l and $T-l$, where $l=80$ months into the sample, and $T=404$ months, for a total of 245 possible breaks, and therefore 245 estimates of $\hat{\beta}$ and $\hat{\beta}^{t\geq mm/yyyy}$, in turn for tp^{KW}, tp^{ACM}, tp^{f}, and lp, again for the 10-year maturity point. Graphs A and B of Appendix D Figure 1 summarize the results, for the subset of breaks (again, for each component of yields) that result in R-squared statistics equal to at least the 75th percentile of all R-squared values among all random assumptions. All in all, to corroborate the results in the upper panel of Appendix D Table 1, Graph B indicates that only the liquidity premium produces results that are largely robust to alternative assumptions about structural changes in the relation, with the same positive coefficient on the recession dummy. Each of the 61 estimates of $\hat{\beta}_{lp}$ and $\hat{\beta}_{lp}^{t\geq mm/yyyy}$ is positive; most are statistically significant, and the weighed-average coefficients (based on R-squared values) are comparable in magnitude to those reported in upper panel of Appendix D Table 1. Finally, the estimates for frictionless term premiums exhibit decreasing (increasing) perceptions of supply (demand) shocks over the sample, as Graph A suggests.

2. Band-Spectrum Regressions on Expected Real GDP Growth

To further assess robustness, continuous readings on one-year-ahead anticipated real GDP growth, as opposed to dichotomous recession indicators, comprise another valid benchmark to assess cyclicality. Even so, the GFC nevertheless looms large on the empirics. Similar in spirit to evaluating structural breaks, an instructive objective is to use a methodology that assesses robustness to this outlying sharp, notably as well as relatively brief, increase in the estimated liquidity premium. Again, the key question is whether the modestly lived liquidity shock during the worst of the GFC drives the overall relation. Confidence in liquidity premium counter-cyclicality should increase, insofar as lower-frequency movements in the liquidity premiums corresponding with lower-frequency variation in real anticipated GDP growth, as opposed to a relation confined to higher-frequency shocks, or perhaps noise, in both series.

To move from the time to the frequency domain, Graphs A and B of Appendix D Figure 2 show the spectral decompositions of the relevant series, based on high-, medium-, and low-frequency variation (HF, MF, and LF), defined as less than or equal to 24 months, at least 24 and up to 60 months, and cycles that last at least 60 months, respectively. Following band-spectrum regressions (Engle (1974), Chaudhuri and Lo (2015)), the relation between liquidity premiums and real GDP growth can be assessed separately across these frequency bands, all else equal and following the specifications with the same control variables as in the specification in the main text but substituting the NBER recession dummy with growth. As the lower panel of Appendix D Table 1 suggests, consistent with counter-cyclicality, a one-standard-error increase in real GDP growth corresponds with not only a 19-basis-point decrease in nominal liquidity premiums at the HF band but also about an 18-basis-point decrease at the MF band between 24 and 60 months, both of which are safely statistically significant. Briefly, that these relations appear robust, within longer time scales between two and five years, boosts confidence that inferences of counter-cyclical liquidity premiums are not wholly sensitive to the sharp and relatively transitory shock during the worst of the GFC.

3. M-GARCH-Based Dynamic Betas and Correlations

Naturally, there are few full business cycles to evaluate, given this sample from the late 1980s, whether based on outright recessions or real GDP growth. Yet another approach to this issue, more broadly, is to consider time-varying return correlations and betas with the yardstick risky asset class, the S&P 500. Appendix D Figure 3 shows estimates of these quantities based on an asymmetric GARCH model (Cappiello, Engle, and Sheppard, (2006)) and synthetic weekly returns on nominal USTs that owe to the nominal liquidity premium component (the red histograms in Graphs A and C and dotted lines in Graphs B and D) and the frictionless term premium (blue) component of yields. Strictly speaking, the average correlation coefficients and betas on the liquidity premium components of returns are, although not large, positive—0.07 and 0.04, respectively. Moreover, the estimates are positive for 100 percent of all sample weeks, which broadly suggest counter-cyclicality and are consistent with intuition that the absolute hedging value of nominal USTs deteriorates amid greater aversion to liquidity shocks.[8]

Furthermore, by contrast, the average beta and correlation for the estimated component of bond returns that owe to revisions in the estimated frictionless term premium are negative, approximately -0.05 to -0.06, but not as consistently so—each is less than zero for only about 72 percent of weeks in the sample. Moreover, as Graphs B and D indicate, these overall results belie likely structural changes in the correlation and betas, respectively, of nominal USTs with stocks over the sample (Durham, (2020)). Visual inspection suggests that these measures have been more commonly positive from the mid-2000s or so, indicative of frictionless term premium pro-cyclicality and the hedging value of the nominal-default-risk-free asset class in later periods. In sum, these results cannot owe entirely to the outlying GFC episode and imply a consistent distinction in the compensation investors demand for different components of nominal USTs, even during more normal times.

[8] These estimates of either the dynamic conditional correlation or beta of the liquidity premium component of returns vis-à-vis the S&P 500 do not appear to be sensitive to periods when liquidity premiums were outright positive during the sample, such as during the GFC.

.

Appendix D TABLE 1

Premia Cyclicality: Further Regression Robustness Checks

The top panel of Appendix D Table 1 reports the results of the regressions following equation (18). The lower panel shows the results of band-spectrum regressions (Engle (1974)) of liquidity premium estimates on macroeconomic variables, including one-year expected real GDP growth.

Regression Results
Monthly Observations
January 1990-August 2023
*** p-value <=0.1 ** p-value <=0.05 *** p-value <= 0.01 (Based on Newey-West standard errors.)**

	Frictionless Term Premium	Liquidity Premium
Intercept	1.99***	-0.38***
Inflation Survey Dispersion	-0.186	-0.0863*
Growth Survey Dispersion	0.296	0.14***
NBER Recession Dummy	**1.07****	**0.115****
Inflation Survey Dispersion * 2008 Dummy	0.0758	0.364***
Growth Survey Dispersion * 2008 Dummy	-0.377	-0.187***
NBER Recession Dummy * 2008 Dummy	**-2.06*****	**0.67*****
R-Squared	0.123	0.606
Observations	404	404

Band-Spectrum Regressions: 10-Year Nominal U.S. Treasury Affine-Model Decompositions
Monthly Observations
October 1989-August 2023
*** p-value <=0.1 ** p-value <=0.05 *** p-value <= 0.01**
HIGH Freq. (HF): < (24) months. MEDIUM Freq. (MF): >= (24) months & <= 5 years. LOW Freq. (LF): > 5 years.

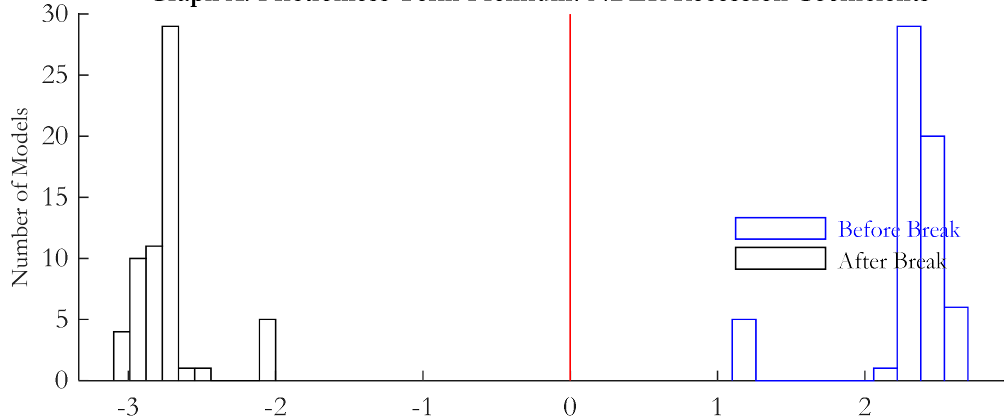
	Frequency Band:	HF	MF	LF
Liquidity Premium	Intercept	-0.0355***	1.7****	-0.0354
Liquidity Premium	Inflation Survey Dispersion	-0.0128*	0.161**	0.383***
Liquidity Premium	Growth Survey Dispersion	-0.109**	-0.0745**	-0.0385
Liquidity Premium	**1-Year Survey Expected Growth**	**-0.189*****	**-0.178*****	**-0.0372**
Liquidity Premium	R-Squared	0.214	0.427	0.652
Liquidity Premium	Observations	406	406	406

Appendix D FIGURE 1

Premia Cyclicality and Alternative Structural Breaks

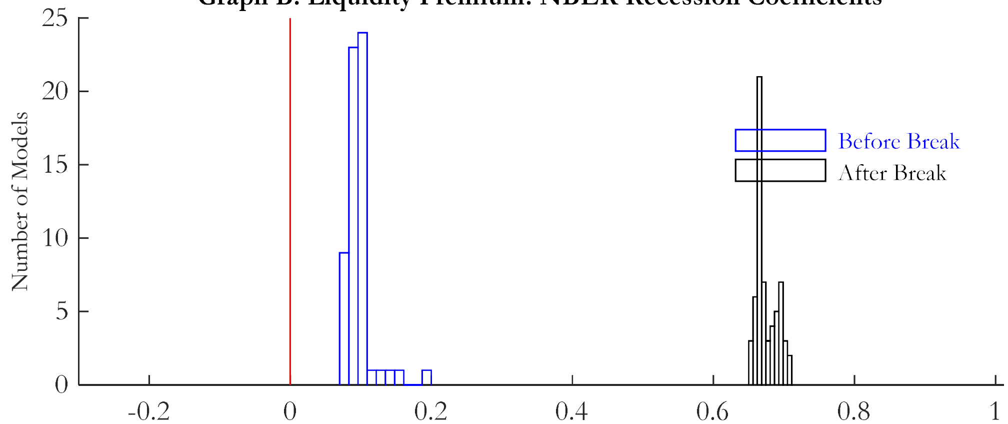
Graphs A and B of Appendix Figure 1 show the coefficients on the NBER recession dummy, broadly following equation (18), but for different structural break assumptions with the frictionless term premium and the liquidity premium estimates, respectively, on the left-hand side as the dependent variable.

Graph A. Frictionless Term Premium: NBER Recession Coefficients



Coefficients %. Break range (08/1996-08/2002) >= the (75th) percentile of R^2s given all (245) breaks (08/1996) to (12/2016). Max R^2=0.19
W-A coefficient before (after) =2.29(-2.73). Max. N-W p-value before (after) break across (61) regressions=0.0309(0.00259).
Fraction statistically significant with 95% (90%) confidence before [after]=1(1) [1(1)].

Graph B. Liquidity Premium: NBER Recession Coefficients



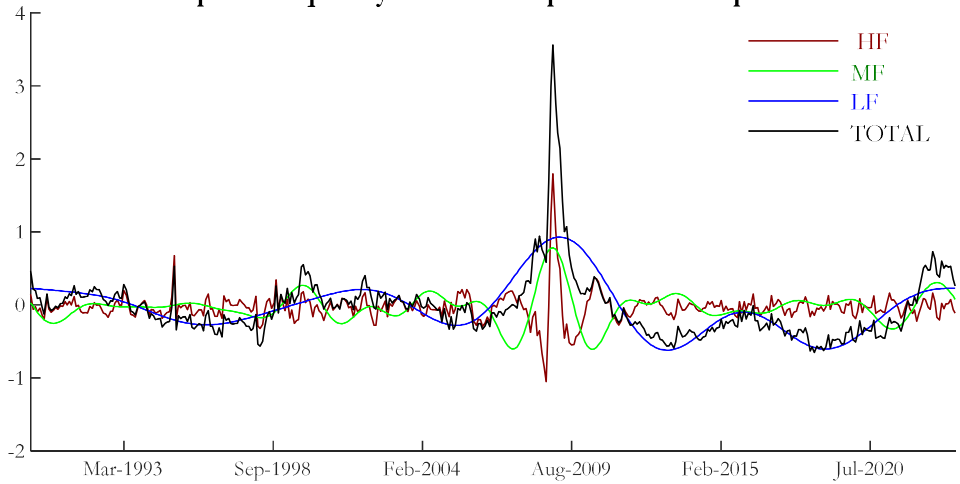
Coefficients %. Break range (04/2002-05/2008) >= the (75th) percentile of R^2s given all (245) breaks (08/1996) to (12/2016). Max R^2=0.61
W-A coefficient before (after) =0.096(0.676). Max. N-W p-value before (after) break across (61) regressions=0.104(0.0261).
Fraction statistically significant with 95% (90%) confidence before [after]=0.31(0.95) [1(1)].

Appendix D FIGURE 2

Spectral Analysis of Key Underlying Series

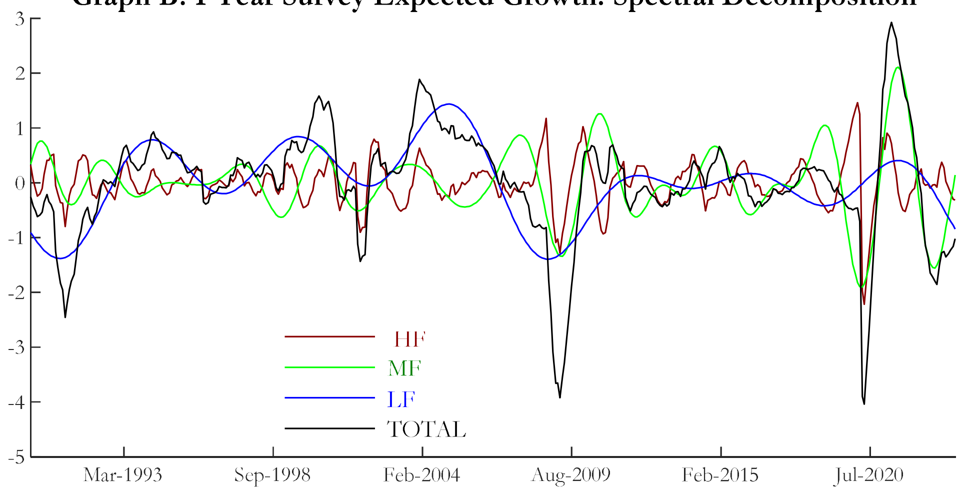
Graphs A and B of Appendix D Figure 2 shows the spectral decompositions of the estimated liquidity premium and one-year survey expected GDP growth, respectively, over the sample.

Graph A. Liquidity Premium: Spectral Decomposition



HIGH freq. (HF): < (24) months. MEDIUM freq. (MF): >= (24) months & <= 5 years. LOW freq (LF): > 5 years.

Graph B. 1-Year Survey Expected Growth: Spectral Decomposition



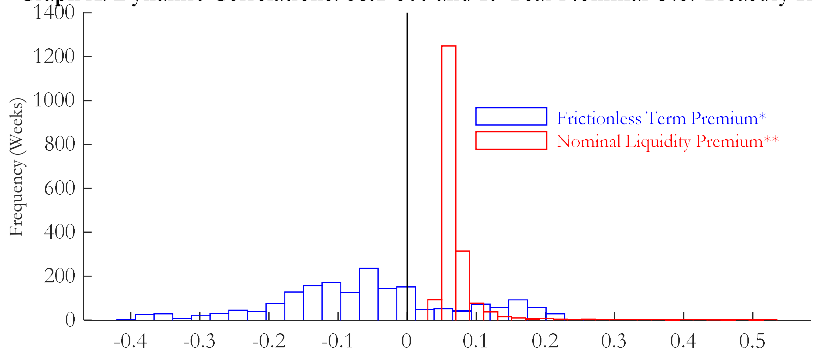
HIGH freq. (HF): < (24) months. MEDIUM freq. (MF): >= (24) months & <= 5 years. LOW freq (LF): > 5 years.

Appendix D FIGURE 3

Nominal UST Premia and Hedging Value

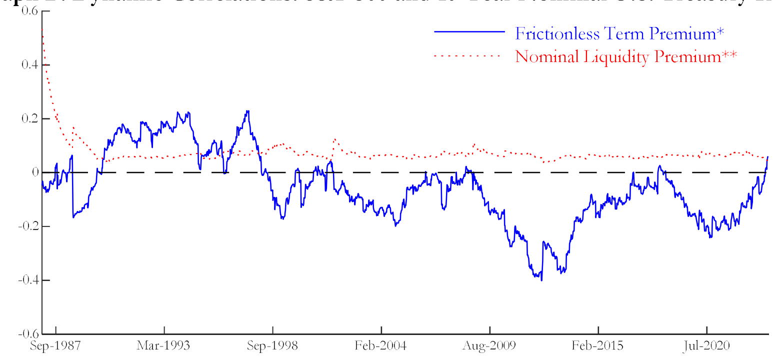
Appendix D Figure 3 shows dynamic correlations and betas, based on an asymmetric M-GARCH (Cappiello et al. (2006)), of weekly S&P 500 returns over the full sample and synthetic returns on the estimated components of nominal UST yields that owe to the frictionless term premium (the blue histograms in Graphs A and C and solid lines in the Graphs B and D, respectively) and the nominal liquidity premium (red histograms and dotted red lines).

Graph A. Dynamic Correlations: S&P 500 and 10-Year Nominal U.S. Treasury Returns



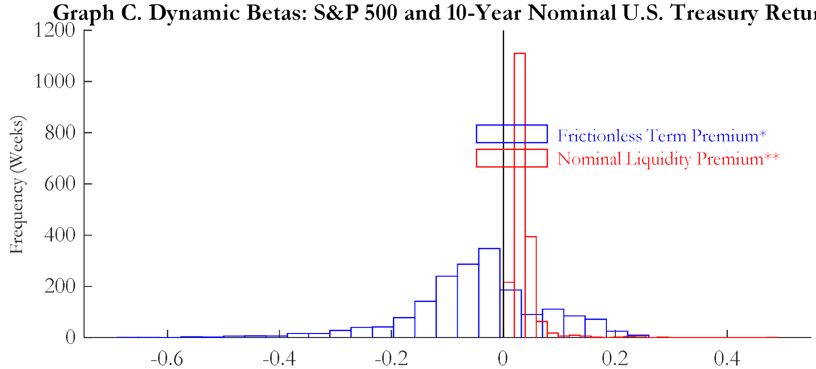
Based on an asymmetric M-GARCH of synthetic weekly returns. (09-Jan-1987-01-Sep-2023).
*Mean(standard deviation)[% < 0]=-0.058(0.13)[72%]. **Mean(standard deviation)[% < 0]=0.073(0.044)[0%].

Graph B. Dynamic Correlations: S&P 500 and 10-Year Nominal U.S. Treasury Returns



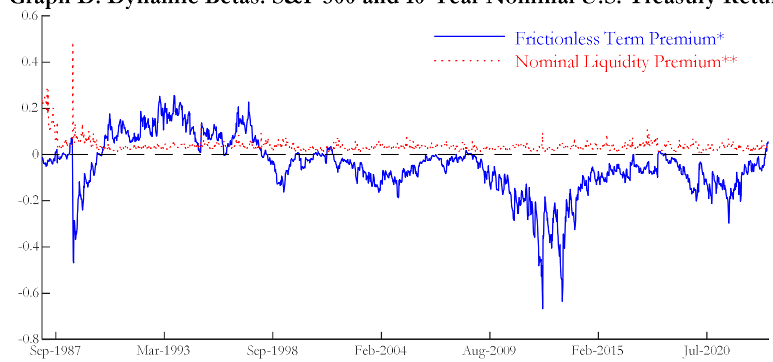
Based on an asymmetric M-GARCH of synthetic weekly returns. (09-Jan-1987-01-Sep-2023).
Expected value (Correlations) | liquidity premium <=0 (>0) =0.064 (0.11). Indicator N-W p value=0.00017.

Graph C. Dynamic Betas: S&P 500 and 10-Year Nominal U.S. Treasury Returns



Based on an asymmetric M-GARCH of synthetic weekly returns. (09-Jan-1987-01-Sep-2023).
*Mean(standard deviation)[% < 0]=-0.049(0.12)[72%]. **Mean(standard deviation)[% < 0]=0.037(0.027)[0%].

Graph D. Dynamic Betas: S&P 500 and 10-Year Nominal U.S. Treasury Returns



Based on an asymmetric M-GARCH of synthetic weekly returns. (09-Jan-1987-01-Sep-2023).
Expected value (Betas) | liquidity premium <=0 (>0) =0.033 (0.051). Indicator N-W p value=0.013.

Appendix E: Robustness Checks on Central Bank Asset Purchase Effects

Additional sensitivity analysis addresses the inherently subjective selection of dates listed in Table 7 in the main text. To start, Thornton (2017) argues that this empirical approach neglects statistical significance. Toward this end, consider simple time-series regressions of daily changes in each estimated component of yields over the sample, following,

$$\Delta y_t = \alpha + \beta_I I_{t\in\mathbf{D}} + \boldsymbol{\beta}'\mathbf{x}_t + \varepsilon_t \tag{19}$$

where y alternatively references the relevant estimated components of yields; $I_{t\in\mathbf{D}}$ is an indicator function equal to one if date t is within the set of days, **D**, listed in Table 7; and **x** includes the surprise component of key macroeconomic data releases over the sample as controls,[9] from February 5, 2008, through September 1, 2023. Statistically insignificant estimates of β_I for any component should vitiate confidence in the cumulative effects listed in Table 7. Indeed, the last row of Appendix E Table 1 indicates that LSAPs largely had no aggregate, statistically significant effect on either frictionless term or liquidity premiums. However, the modest estimated 3-basis-point declines in average frictionless anticipated rates over two and ten years are robust, again consistent with a LSAP signaling channel.[10]

Furthermore, an alternative identification strategy stems from Gürkaynak et al. (2005) and extends to "asset purchase surprises." Generally following, say, Husted et al. (2020) or Swanson (2021), the asset purchase surprise is the daily change in ten-year yields that is orthogonal to the

[9] The standardized surprise component refers to the actual data release minus the Bloomberg survey median, divided by the corresponding survey standard deviation for non-farm payrolls, initial claims, ISM manufacturing, ISM non-manufacturing, consumer confidence, the University of Michigan survey, capacity utilization, initial jobless claims, retail sales, and the consumer price index.

[10] Dates specific to some studies are robust. For example, the more recent instances under focus in Rebucci, Hartley and Jiménez (2022) imply a significant 3-basis point (13-basis-point) increase (decrease) in frictionless term (liquidity) premiums owing to LSAPs during the pandemic.

simultaneous "forward guidance" shock, defined as the change in the estimated frictionless average short rate over two years, following this model, that is orthogonal to the first difference in subsequent month fed funds futures rates (i.e., "FF2 Comdty"). The estimation sample for these shocks is the union of dates among those listed in Table 7 and all FOMC dates as well as intermeeting moves. The upper panel of Appendix E Table 2 (upper panel) lists the dates that include the largest asset purchase easing surprises, at least one standard-error less than the average estimate. Briefly, cumulative changes in yield components across these days largely follows the substantive results in Table 7, with net responses of -58, -179, and 8 basis points for the ten-year frictionless expected short rate, the frictionless term premium, and the liquidity premium, respectively. These results again suggest that LSAPs largely work through signaling and frictionless risk premiums, rather than liquidity premiums per se.[11]

Furthermore, this inference is largely statistically robust, again following regressions like equation (19), but by replacing **D** with the days listed in the upper panel. On average, following the first column of the lower panel of Appendix E Table 2, all else equal, asset-purchase easings correspond with safely statistically significant 7- and 6-basis-point drops in two- and 10-year average frictionless anticipated short rates, respectively, and an 18-basis-point decline in 10-year frictionless term premiums. However, the estimated 1-basis-point increase in liquidity premiums is not statistically different from zero, nor incidentally, are the corresponding effects of similarly defined asset-purchase tightenings,[12] for any estimated component of nominal yields. Yet all in all, if anything these model decompositions suggests that LSAPs generally have larger effects on frictionless term premiums, and even on anticipated frictionless short rates, than on liquidity

[11] That said, note also that the decline in the liquidity premium after the March 15, 2020, FOMC announcement is robust to this alternative method.

[12] These estimates also correspond to (19), but where **D** comprises dates for which orthogonalized increase in 10-year yields, relative to the contemporaneous forward-guidance surprise, is at least one (positive) standard deviation from the mean response.

premiums. Perhaps intuitively, the unrobust empirical results for liquidity premiums reflects corresponding theoretical ambiguity. The sheer magnitude of asset purchases, which strictly remove free float after all, neither obviously lower immediate trading costs nor reduce frictions that cause deviations from the law of one price.

Appendix E Table 1

Robustness Checks on LSAP Effects: Statistical Significance

Appendix E Table 1 reports the results following equation (19), which largely follows Thornton (2017).

Conditional Regression Coefficients
Daily Responses (Basis Points)
3899 Observations (02/05/2008--09/01/2023)
*** p-value <=0.1 ** p-value <=0.05 *** p-value <= 0.01**

Sample:	**10-Year Yields**	**2-Year Frictionless Expected Rate**	**10-Year Frictionless Expected Rate**	**10-Year Frictionless Premium**	**10-Year Liquidity Premium**
Gagnon et al. (2011)	-11	-3	-3	-3	-4
Wright (2012)	-9*	-3	-3	-8	0
Krishnamurthy and Vissing-Jorgensen (2012)	-21**	-4	-4	-19**	1
D'Amico et al. (2014)	-13	-2	-2	-13	0
Swanson (2021)	-7	-7**	-5**	-17***	10**
Rebucci et al. (2022)	-16***	1***	-2***	3**	-12***
Total (selected days across cited studies above)	**-5***	**-3***	**-3****	**-4**	**0**

Appendix E Table 2

Robustness Checks on LSAP Effects: Alternative Identification Strategies

The top panel of Appendix E Table 2 lists the dates of the largest asset purchase easing surprises, based on the daily change in 10-year yields that is orthogonal to both the "target" and "forward guidance" shocks, given the union of dates listed in Table 7, FOMC days, and intermeeting moves. The lower panel of Appendix E Table 2 reports coefficients from regressions that resemble equation (19) but replace **D** with the dates listed in the top panel (and identified correspondingly in the second column for "asset purchase tightening" dates).

Identification: Reponses to 'Asset Purchase' Monetary Policy Announcements
Daily Changes (Basis Points)
(Days in sample with >= one-standard-error 'asset purchase' easing surprise)

Changes:	Surprise: Asset Purchase	Surprise: Target	Surprise: Forward Guidance	10-Year Yields	2-Year Frictionless Expected Rate	**10-Year Frictionless Expected Rate**	**10-Year Frictionless Premium**	**10-Year Liquidity Premium**
11/25/2008	-19	-5	-1	-22	-5	**-4**	**-24**	**4**
12/1/2008	-13	3	-14	-19	-14	**-11**	**-29**	**14**
12/16/2008	-28	-17	10	-27	3	**0**	**-18**	**-6**
3/18/2009	-45	-1	-9	-49	-11	**-11**	**-42**	**0**
9/21/2010	-9	0	-5	-13	-6	**-6**	**-15**	**4**
9/21/2011	-12	2	12	-10	11	**9**	**-26**	**5**
9/18/2013	-10	0	-12	-16	-13	**-11**	**-13**	**2**
3/3/2020	-11	-30	-9	-16	-21	**-13**	**-11**	**5**
3/16/2020	-25	-6	4	-26	0	**-3**	**0**	**-17**
6/15/2022	-10	2	-10	-15	-11	**-8**	**0**	**-4**
Total (cumulative all listed days)	-183	-51	-35	-213	-67	**-58**	**-179**	**8**

Conditional Regression Coefficients: 'Asset Purchase' Identification
Daily (Basis Points)
3899 Observations (02/05/2008--09/01/2023)
*** p-value <=0.1 ** p-value <=0.05 *** p-value <= 0.01**

Component:	**Asset Purchase Easing**	**Asset Purchase Tightening**
10-Year Yields	-21***	9***
2-Year Frictionless Expected Rate	-7**	-4
10-Year Frictionless Expected Rate	-6***	-2
10-Year Frictionless Premium	-18***	6
10-Year Liquidity Premium	1	3

Additional Appendices References

Cappiello, L., R. F. Engle, and K. Sheppard. "Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns," *Journal of Financial Econometrics*, 4 (2006), 537–572.

Chaudhuri, S., and A. W. Lo. "Spectral Analysis of Stock-Return Volatility, Correlation, and Beta," IEEE Signal Processing and Signal Processing Educational Workshop (SP/SPE), 232–236 (2015).

D'Amico, S., C. Li, J. S. Sears, and M. E. Cahill. "The Term-Premium Effect of the Federal Reserve's Asset Purchase Announcements," Working Paper (2014).

Durham, J. B. "U.S. Treasury Bond Betas: 1961–2019," *Journal of Fixed Income*, 29 (2020), 20–47.

Engle, R. F. "Band Spectrum Regression," *International Economic Review*, 15 (1974), 1–11.

Husted, L, J. Rogers, and B. Sun. "Monetary Policy Uncertainty, *Journal of Monetary Economics*, 115 (2020), 20–36.

Leamer, E. E. "Let's Take the Con Out of Econometrics," *American Economic Review*, 73 (1983), 31–43.

Levine, R. and D. Renelt. "A Sensitivity Analysis of Cross-Country Growth Regressions," *American Economic Review*, 82 (1992), 942–963.

Rebucci, A., J. S. Hartley and D. Jiménez. "An Event Study of COVID-19 Central Bank Quantitative Easing in Advanced and Emerging Economies," NBER Working Paper Series, no. 27339 (2022).

Sala-I-Martin, X. "I Just Ran Two Million Regressions," *American Economic Review*, 87 (1997), Papers and Proceedings of the Hundred and Fourth Annual Meeting of the American Economic Association, 178–183.

Thornton, D. L. "Effectiveness of QE: An Assessment of Event-Study Evidence," *Journal of Macroeconomics*, 52 (2017), 56–74.