Online Appendix for

Inferring Aggregate Market Expectations from the Cross-Section of Stock Prices

Journal of Financial and Quantitative Analysis

Turan G. Bali, D. Craig Nichols, and David Weinbaum*

* Bali, turan.bali@georgetown.edu, Georgetown University McDonough School of Business; Nichols, dcnichol@syr.edu, Syracuse University Whitman School of Management; Weinbaum (corresponding author), dweinbau@syr.edu, Syracuse University Whitman School of Management..

Appendix A. Derivation of Equation (9)

The proof of equation (9) starts from the unnumbered equation on page 156 of Ohlson (1999),

$$P_t = b_t + \alpha_1 x_t^a + \alpha_2 x_{2t} + \beta \upsilon_t.$$

The above equation is stated using the notation in Ohlson (1999), thus b_t denotes book value at date t, x_t^a denotes abnormal earnings, and x_{2t} denotes transitory earnings. Rearranging to separate core and transitory earnings,

$$P_t = b_t + \alpha_1(x_{1t} + x_{2t} - rb_{t-1}) + \alpha_2 x_{2t} + \beta \upsilon_t,$$

$$P_t = b_t + \alpha_1(x_{1t} - rb_{t-1}) + (\alpha_1 + \alpha_2) x_{2t} + \beta \upsilon_t,$$

where, continuing with the notation from Ohlson (1999), x_{1t} denotes core earnings and r is the cost of equity. At this point, the first two expressions on the right hand side are familiar from Ohlson (1995). Continuing, $k = r\alpha_1$, so $\alpha_1 = k/r$, and

$$P_t = b_t + \frac{k}{r}(x_{1t} - rb_{t-1}) + (\alpha_1 + \alpha_2) x_{2t} + \beta \upsilon_t,$$

$$P_t = b_t + k\left(\frac{x_{1t}}{r} - \frac{rb_{t-1}}{r}\right) + (\alpha_1 + \alpha_2) x_{2t} + \beta \upsilon_t,$$

$$P_t = b_t + k\frac{x_{1t}}{r} - k\frac{rb_{t-1}}{r} + (\alpha_1 + \alpha_2) x_{2t} + \beta \upsilon_t,$$

$$P_t = b_t + k\frac{x_{1t}}{r} - kb_{t-1} + (\alpha_1 + \alpha_2) x_{2t} + \beta \upsilon_t.$$

Using the clean surplus relation,

$$P_t = b_t + k\frac{x_{1t}}{r} - k(b_t - x_t + d_t) + (\alpha_1 + \alpha_2) x_{2t} + \beta \upsilon_t,$$

$$P_t = b_t + k\frac{x_{1t}}{r} - k(b_t - x_{1t} - x_{2t} + d_t) + (\alpha_1 + \alpha_2) x_{2t} + \beta \upsilon_t,$$

$$P_t = b_t + k\frac{x_{1t}}{r} - k(b_t - x_{1t} + d_t) + (\alpha_1 + \alpha_2 + k) x_{2t} + \beta \upsilon_t,$$

$$P_t = b_t + k\frac{x_{1t}}{r} - kb_t + kx_{1t} - kd_t + (\alpha_1 + \alpha_2 + k) x_{2t} + \beta \upsilon_t,$$

$$P_t = (1-k)b_t + k\frac{x_{1t}}{r} + kx_{1t} - kd_t + (\alpha_1 + \alpha_2 + k) x_{2t} + \beta \upsilon_t,$$

$$P_t = (1-k)b_t + \alpha_1 x_{1t} + kx_{1t} - kd_t + (\alpha_1 + \alpha_2 + k) x_{2t} + \beta \upsilon_t.$$

Furthermore, $\alpha_1 + k = k\varphi$, where $\varphi = \frac{1+r}{r}$, therefore

$$P_t = (1-k)b_t + k\varphi x_{1t} - kd_t + (\alpha_1 + \alpha_2 + k)x_{2t} + \beta v_t,$$

$$P_t = (1-k)b_t + k(\varphi x_{1t} - d_t) + (\alpha_1 + \alpha_2 + k)x_{2t} + \beta v_{t,}$$

which yields equation (9), with the appropriate change in notation, and replacing r with R_t.[1]

Appendix B. Variance Decomposition

Cochrane (2011) provides a variance decomposition that shows that discount rate variation accounts for all of the variation in dividend yields. Starting from the Campbell and Shiller (1988) linearization of the one-period return in equation (13), solving forward iteratively, subtracting the current dividend, and ignoring the constant (thus treating all variables as deviations from their means) yields the following expression for the dividend yield:

$$d_t - p_t \approx \sum_{j=1}^{m} \rho^{j-1} r_{t+j} - \sum_{j=1}^{m} \rho^{j-1} \Delta d_{t+j} + \rho^m (d_{t+m} - p_{t+m}),$$

which is equation (1) in Cochrane (2011). It follows that, if one runs regressions of long run returns ($\sum_{j=1}^{m} \rho^{j-1} r_{t+j}$), long run dividend growth ($\sum_{j=1}^{m} \rho^{j-1} \Delta d_{t+j}$), and future weighted dividend yields ($\rho^m (d_{t+m} - p_{t+m})$), all on current dividend yields, the slope coefficients must add up to one. The coefficients can therefore be interpreted as proportions of dividend yield variation attributable to each source. Cochrane (2011 pg. 1050) finds that "all price-dividend ratio volatility corresponds to variation in expected returns."

Panel A of Table A1 replicates Cochrane's (2011) analysis over our sample period. We begin by estimating a simple VAR that relates one-year ahead returns, dividend growth, and dividend yields to current dividend yields, and then use the one-year coefficients to infer long-run coefficients at $m = 15$ and $m \to \infty$. Like Cochrane (2011), for ease of interpretation, we use dividend growth implied by equation (13), so that the coefficients sum up to one exactly (using

---

[1] As we note in Section II, although in our framework R_t varies over time, at each time t it is a fixed constant: in equations (5) and (6), R_t is the same for all future periods τ. Ohlson's (1995, 1999) logic therefore applies in our setting without modification.

actual dividend growth produces similar results). In Panel B, we then replace the one-year ahead market return with our measure of expected market return R_t, again using equation (13) and our measure of expected returns R_t to infer expected dividend growth. In untabulated results, we find that actual dividend growth has a 94 percent correlation with this newly constructed measure of expected dividend growth (the correlation between actual dividend growth and Cochrane's measure based on realized returns is 30 percent). Both variance decompositions strongly confirm Cochrane's finding that essentially all variation in dividend yields corresponds to variation in discount rates.[2]

Appendix C. Additional In-Sample Predictive Regressions

The fifth regression in each set of results in Panel A of Table 3 omits the ending discount rate and thus represents a univariate in-sample predictive regression at the monthly frequency. In this Appendix, we show the results of additional in-sample predictive regressions: we consider univariate and bivariate regressions based on both monthly and annual horizons, and we control for the full set of 21 alternative discount rate measures and predictive variables described in Section II. We use linear regressions where the dependent variable is the return on the CRSP value-weighted index in excess of the one-month Treasury bill rate at frequencies of one month and one year. We rely on monthly observations.

Since we work in intervals of one and 12 months, but estimate regressions at the monthly frequency, our research design employs overlapping information for the 12-month analysis, introducing moving average effects. To adjust for this, the reported *t*-statistics are based on Hodrick (1992) standard errors. In the context of predictive regressions with overlapping observations, Ang and Bekaert (2007) show that the standard error correction in Hodrick (1992)

[2] In untabulated results, we also consider variance decompositions that employ the earnings yield. We find that these are more difficult to interpret because earnings yield variation corresponds to a mix of variation in earnings growth and discount rates.

provides more conservative test statistics than those based on Newey and West (1987) or other commonly employed standard errors.

Table A2 reports the results of monthly predictive regressions in Panel A and annual predictive regressions in Panel B. Each panel shows the results of univariate regressions on the left and bivariate regressions on the right. Note that the univariate regressions in Panel A of Table A2 do not match the corresponding regressions reported in Panel A of Table 3 exactly. This is because Table 3 loses one observation, as it also shows results from regressions that employ changes in discount rates.

The results in Table A2 indicate that the forecasting ability of ExR_t dominates that afforded by the alternative predictors. While certain predictive variables perform well at one of the two horizons in these in-sample tests, except for LTG_t, none comes close to the performance of ExR_t in both monthly and annual predictive regressions, and ExR_t also performs well in the bivariate specifications. The predictability is statistically significant and economically large. For example, we find that, at the annual horizon, our measure predicts future returns on a nearly one-to-one basis. That is, when our measure is 100 basis points above its average, returns for the next year are higher by 96 basis points, on average.

The results featuring the index of long-term expected earnings growth LTG_t are of particular interest. First, we confirm the Bordalo et al. (2022) finding that LTG_t is a strong predictor of market returns in sample. Moreover, while we find that LTG_t adds substantial predictive power to the regression, we find that it does so in a way that is essentially orthogonal to ExR_t. This is consistent with the low correlations between LTG_t and ExR_t in Table 2, and with results in Bordalo et al. (2022) that indicate that LTG_t measures market expectations of long-run fundamentals rather than long-run discount rates.

Appendix D. Out-of-Sample Graphical Diagnostic Test

In Figure A1, we plot the difference between the cumulative sum of squared errors from the mean return benchmark model and the cumulative sum of squared errors from ExR_t and the various other predictive variables we consider, focusing on annual returns that we evaluate monthly. Goyal and Welch (2003) propose this graphical diagnostic tool as a simple way to examine the performance of a predictive variable over time. As Goyal and Welch (2003) and Welch and Goyal (2008) point out, while the units on such plots are difficult to interpret intuitively, two features are important: (*i*) each month a positive value indicates that the predictive variable under consideration has outperformed the benchmark model up to that point, and (*ii*) each month a positive slope indicates that the variable had a lower forecasting error than the benchmark model over that 12-month period.

Figure A1 shows that ExR_t performs well in this framework: the plot exhibits a drift that is clearly upward, albeit irregularly so. The implied cost of capital ICC_t and the Kelly and Pruitt (2013) discount rate measure also do well. However, the other predictive variables, such as the traditional valuation ratios (the dividend yield, the earnings-to-price ratio, and the book-to-market ratio) and the LTG_t index, perform poorly in this graphical test: the curves do not reach the end of the sample in positive territory, and they often exhibit prolonged periods of downward drift.

Table A1. Variance Decomposition

This table reports results from regressions of weighted long-run returns, dividend growth (Δd_t), and future dividend yields (dp_t), all on current dividend yields. Panel A constructs weighted long-run returns from actual one-year ahead returns (RET_t) and Panel B uses the current expected return (R_t) estimated from a first stage Theil-Sen regression of prices on earnings, book values, and dividends. The columns labeled Estimate, *t*-statistic, and *R*-square report results of a VAR that relates one-year ahead returns, dividend growth, and dividend yields to current dividend yields. The columns labeled "m = 15" and "m = infinity" use the one-year coefficients to infer long-run coefficients. We report *t*-statistics based on Newey-West (1987) standard errors. The sample period is 1976 to 2018.

Panel A. Realized Returns

DV=	Estimate	*t*-statistic	R-square	m = 15	m = infinity
RET_{t+1}	0.104	3.47	8.80%	0.89	1.19
Δd_{t+1}	0.017	0.64	0.30%	-0.14	-0.19
dp_{t+1}	0.951	35.66	91.20%	0.25	0.00

Panel B. Expected Returns

DV=	Estimate	*t*-statistic	R-square	m = 15	m = infinity
R_t	0.086	8.62	46.70%	0.73	0.98
Δd_{t+1}	-0.002	-0.06	0.00%	0.02	0.02
dp_{t+1}	0.951	35.66	91.20%	0.25	0.00

Table A2. Predictive Regressions

This table reports results from regressions of one- and 12-month ahead returns on return predictors. The dependent variable is the CRSP value-weighted index return in excess of the one-month T-Bill return over the next one or twelve months. ExR denotes the expected excess return from a first stage Theil-Sen estimation of prices on earnings, book values, and dividends. ICC denotes the implied cost of capital from Li et al. (2013). RIV denotes implied cost of capital from the residual income valuation model in Gode and Mohanram (2003). OJN denotes the implied cost of capital from the Ohlson and Juettner-Nauroth (2005) model from Gode and Mohanram (2003). KP denotes the three-pass return predictor from Kelly and Pruitt (2013). LTG is value-weighted long-term EPS growth forecast for the S&P 500. CAPE is Shiller's cyclically-adjusted P/E ratio. The remaining variables are from Welch and Goyal (2008) and collected from Amit Goyal's website. DP denotes the logged dividend to price ratio. DY denotes the logged dividend yield. EP denotes the logged earnings to price ratio. DE denotes the logged dividend payout ratio. SVAR denotes stock return variance. BM denotes the book-to-market ratio. NTIS denotes net equity expansion. TBL denotes the three month treasury yield. LTY denotes the long term yield. DFY denotes the default yield spread. DFR denotes the default return spread. INFL denotes inflation. IK denotes the investment to capital ratio. LTR denotes the long term rate of return. TMS denotes the term spread. The sample period is 1976 to 2018, except for LTG, which starts in 1982. We report *t*-statistics based on Hodrick (1992) standard errors.

Panel A. Univariate and bivariate regressions of monthly aggregate returns

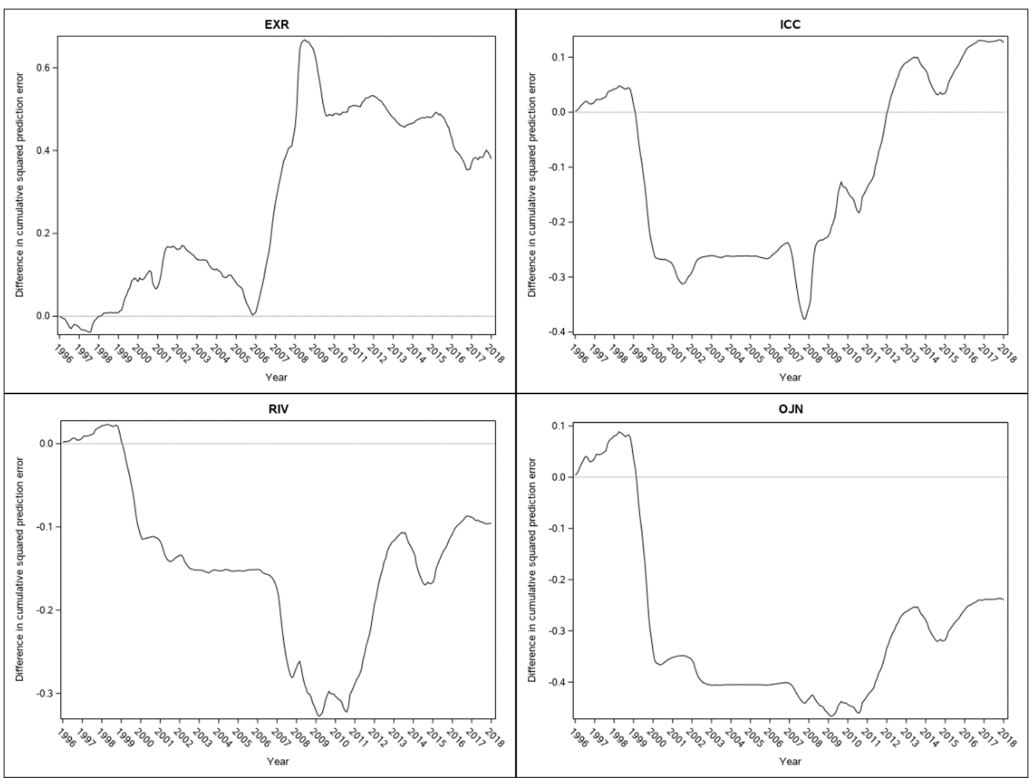
		CONSTANT	ExR	*X*	ADJ RSQ	CONSTANT	ExR	*X*	ADJ RSQ
	ExR	-0.001	0.106		0.71%				
		(-0.15)	(2.22)						
X=	ICC	0.000		0.105	0.16%	-0.007	0.105	0.102	0.85%
		(-0.06)		(1.25)		(-1.10)	(2.19)	(1.24)	
	RIV	0.002		0.073	0.04%	-0.006	0.110	0.082	0.81%
		(0.34)		(1.00)		(-0.97)	(2.28)	(1.13)	
	OJN	0.002		0.072	-0.07%	-0.005	0.107	0.074	0.64%
		(0.25)		(0.73)		(-0.72)	(2.22)	(0.76)	
	KP	0.032		-0.463	0.34%	0.016	0.088	-0.274	0.68%
		(1.85)		(-1.48)		(0.73)	(1.58)	(-0.76)	
	DP	0.024		0.005	0.04%	0.001	0.105	0.000	0.52%
		(1.38)		(1.03)		(0.03)	(1.82)	(0.05)	
	DY	0.025		0.005	0.08%	0.003	0.101	0.001	0.52%
		(1.47)		(1.11)		(0.15)	(1.77)	(0.17)	
	EP	0.014		0.003	-0.12%	0.000	0.106	0.000	0.52%
		(0.98)		(0.52)		(-0.00)	(1.97)	(0.03)	
	CAPE	0.011		0.000	-0.02%	-0.003	0.107	0.131	0.70%
		(1.99)		(-0.87)		(-0.78)	(2.21)	(0.96)	
	DE	0.009		0.003	-0.14%	0.000	0.106	0.000	0.52%
		(1.34)		(0.40)		(-0.06)	(2.24)	(0.01)	
	SVAR	0.009		-1.038	1.17%	0.001	0.129	-1.194	2.28%
		(4.55)		(-2.38)		(0.28)	(2.70)	(-2.86)	
	BM	0.005		0.003	-0.17%	0.001	0.134	-0.007	0.67%
		(1.33)		(0.36)		(0.27)	(2.22)	(-0.80)	
	NTIS	0.006		-0.019	-0.19%	-0.001	0.118	-0.083	0.65%
		(3.08)		(-0.18)		(-0.24)	(2.21)	(-0.72)	
	TBL	0.009		-0.057	0.03%	0.002	0.129	-0.095	1.08%
		(2.90)		(-1.02)		(0.64)	(2.53)	(-1.61)	
	LTY	0.010		-0.052	-0.07%	0.005	0.132	-0.106	0.99%
		(2.10)		(-0.79)		(1.08)	(2.49)	(-1.48)	
	DFY	0.005		0.086	-0.19%	0.001	0.112	-0.178	0.55%
		(0.93)		(0.16)		(0.17)	(2.39)	(-0.32)	
	DFR	0.006		0.260	0.59%	0.000	0.105	0.256	1.28%
		(3.27)		(1.31)		(-0.13)	(2.15)	(1.31)	
	INFL	0.005		0.282	-0.14%	-0.001	0.105	0.083	0.52%
		(2.02)		(0.45)		(-0.19)	(2.06)	(0.12)	
	IK	0.035		-0.807	0.17%	0.035	0.119	-1.014	1.08%
		(1.46)		(-1.20)		(1.48)	(2.44)	(-1.51)	
	LTR	0.006		0.109	0.42%	-0.001	0.105	0.107	1.12%
		(2.81)		(1.57)		(-0.36)	(2.24)	(1.54)	
	TMS	0.003		0.129	-0.01%	-0.003	0.107	0.131	0.70%
		(0.97)		(0.94)		(-0.78)	(2.21)	(0.96)	
	LTG	0.040		-0.003	0.90%	0.033	0.167	-0.003	2.09%
		(2.34)		(-1.92)		(1.92)	(2.16)	(-2.05)	

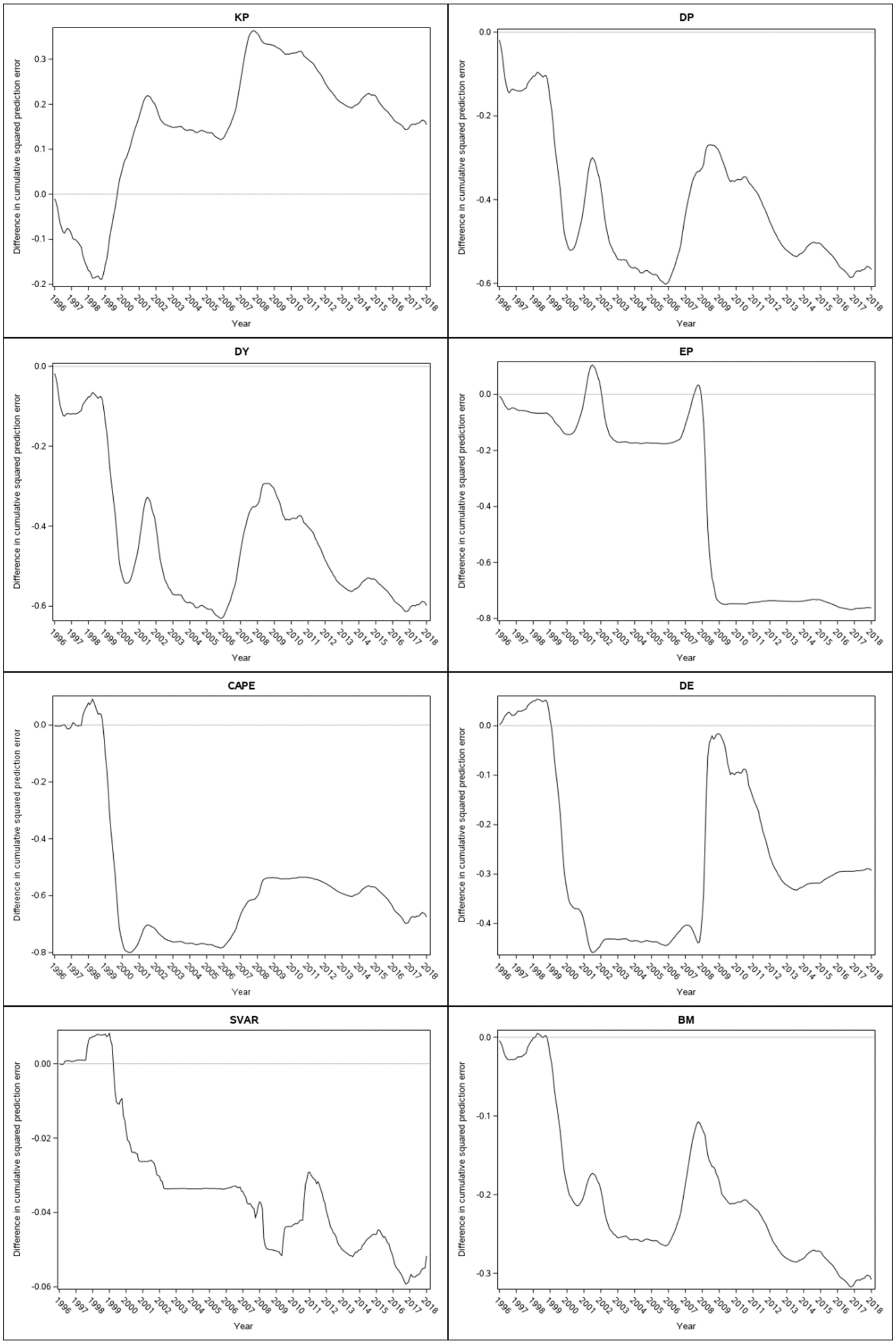
Panel B. Univariate and bivariate regressions of annual aggregate returns

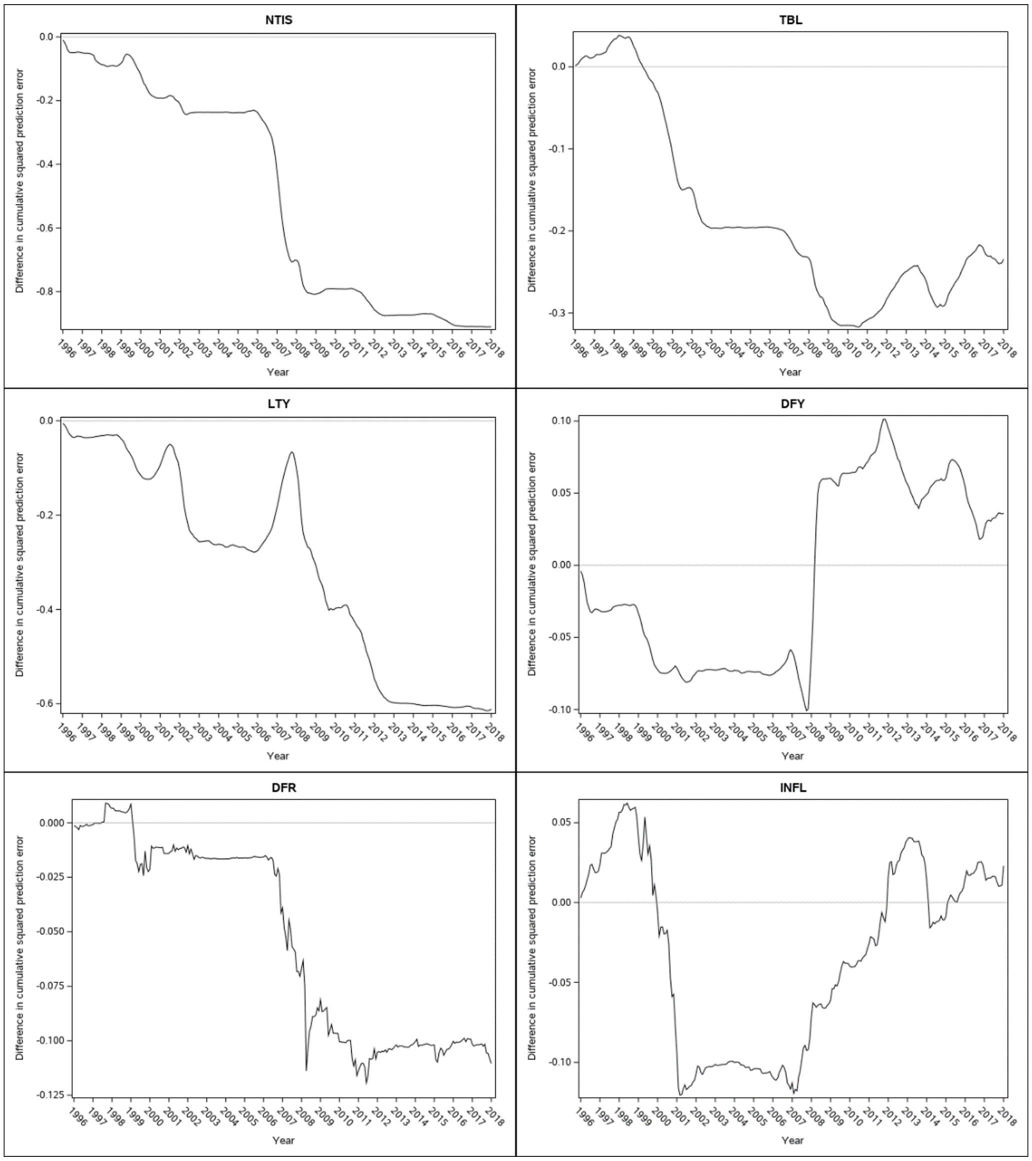
		CONSTANT	ExR	*X*	ADJ RSQ	CONSTANT	ExR	*X*	ADJ RSQ
	ExR	0.021	0.959		4.95%				
		(0.57)	(2.02)						
X=	ICC	-0.001		1.311	3.71%	-0.060	0.947	1.289	8.54%
		(-0.01)		(1.42)		(-0.89)	(1.98)	(1.40)	
	RIV	0.028		0.875	2.18%	-0.041	1.002	0.956	7.59%
		(0.50)		(1.13)		(-0.68)	(2.13)	(1.25)	
	OJN	0.006		1.189	2.12%	-0.056	0.965	1.206	7.15%
		(0.08)		(1.10)		(-0.73)	(2.03)	(1.12)	
	KP	0.324		-4.350	3.11%	0.181	0.780	-2.679	5.84%
		(1.62)		(-1.21)		(0.80)	(1.53)	(-0.70)	
	DP	0.318		0.064	2.67%	0.139	0.800	0.029	5.22%
		(1.53)		(1.14)		(0.54)	(1.44)	(0.45)	
	DY	0.315		0.064	2.61%	0.139	0.804	0.030	5.24%
		(1.52)		(1.12)		(0.55)	(1.47)	(0.46)	
	EP	0.137		0.019	0.11%	0.013	0.968	-0.003	4.77%
		(0.94)		(0.37)		(0.08)	(2.05)	(-0.05)	
	CAPE	0.146		-0.003	2.53%	0.066	0.810	-0.002	5.45%
		(2.31)		(-1.07)		(0.74)	(1.52)	(-0.53)	
	DE	0.135		0.067	1.72%	0.061	0.870	0.043	5.53%
		(2.49)		(1.13)		(0.82)	(1.71)	(0.69)	
	SVAR	0.077		1.988	0.16%	0.020	0.943	0.847	4.83%
		(3.44)		(0.62)		(0.56)	(1.93)	(0.26)	
	BM	0.059		0.051	0.52%	0.027	1.070	-0.030	4.94%
		(1.28)		(0.57)		(0.58)	(2.02)	(-0.29)	
	NTIS	0.082		-0.020	-0.19%	0.019	1.045	-0.586	5.24%
		(3.29)		(-0.02)		(0.52)	(2.15)	(-0.47)	
	TBL	0.103		-0.467	0.83%	0.045	1.153	-0.800	7.56%
		(2.97)		(-0.73)		(1.04)	(2.46)	(-1.27)	
	LTY	0.093		-0.160	-0.11%	0.053	1.107	-0.612	5.88%
		(1.78)		(-0.22)		(0.95)	(2.29)	(-0.82)	
	DFY	0.032		4.610	1.41%	-0.001	0.878	2.542	5.22%
		(0.60)		(0.96)		(-0.02)	(1.72)	(0.50)	
	DFR	0.082		0.396	-0.07%	0.021	0.957	0.366	4.87%
		(3.59)		(0.71)		(0.58)	(2.03)	(0.66)	
	INFL	0.097		-5.203	1.14%	0.035	1.081	-7.257	7.28%
		(3.93)		(-1.23)		(0.89)	(2.34)	(-1.84)	
	IK	0.417		-9.302	3.18%	0.417	1.096	-11.214	9.58%
		(1.55)		(-1.22)		(1.56)	(2.34)	(-1.50)	
	LTR	0.078		0.572	1.00%	0.017	0.954	0.559	5.91%
		(3.40)		(2.75)		(0.47)	(2.03)	(2.71)	
	TMS	0.035		2.188	3.43%	-0.027	0.963	2.200	8.44%
		(0.83)		(1.48)		(-0.59)	(2.03)	(1.50)	
	LTG	0.470		-0.031	9.61%	0.382	2.010	-0.033	23.59%
		(2.37)		(-1.88)		(1.92)	(3.91)	(-2.02)	

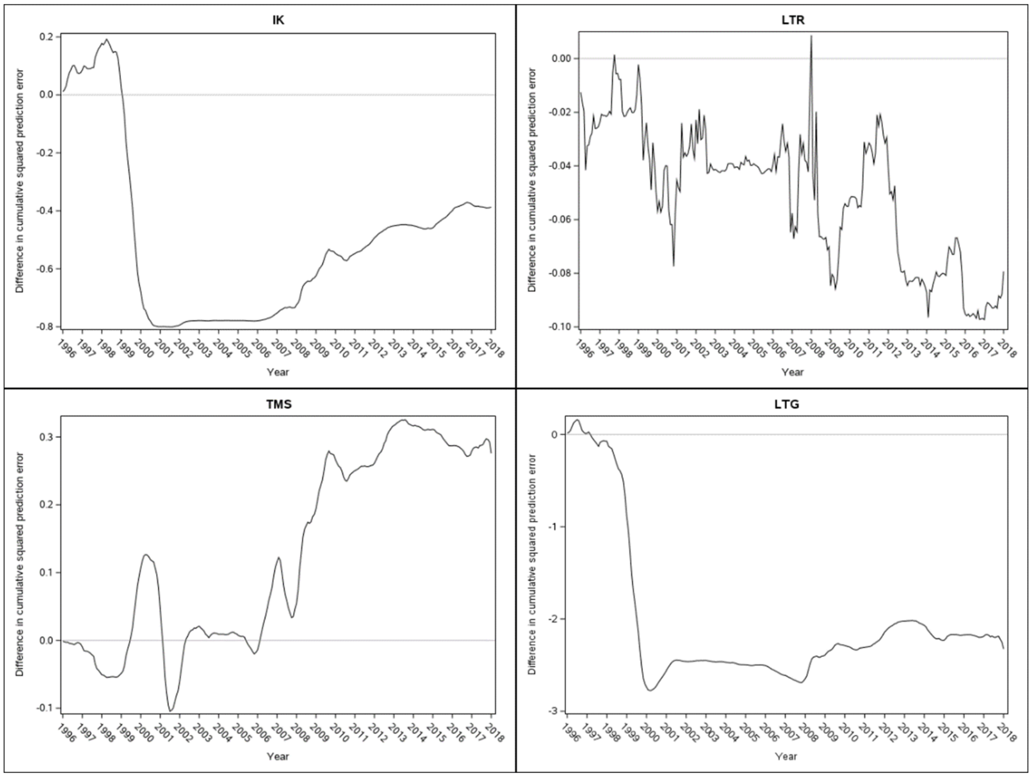
Figure A1. Cumulative difference in prediction errors for equity premium predictors

Cumulative squared prediction errors for the prevailing average less the cumulative squared prediction error for the forecasting model for alternative return predictors for monthly forecasts of annual returns for 1996 to 2018. ExR denotes the expected excess return from a first stage Theil-Sen estimation of prices on earnings, book values, and dividends. ICC denotes the implied cost of capital from Li et al. (2013). RIV denotes implied cost of capital from the residual income valuation model in Gode and Mohanram (2003). OJN denotes the implied cost of capital from the Ohlson-Jeuttner Nauroth model from Gode and Mohanram. KP denotes the three-pass return predictor from Kelly and Pruitt (2013). LTG is value-weighted long-term EPS growth forecast for the S&P 500. CAPE is Shiller's cyclically-adjusted P/E ratio. The remaining variables are from Welch and Goyal (2008) and collected from Amit Goyal's website. DP denotes the logged dividend to price ratio. DY denotes the logged dividend yield. EP denotes the logged earnings to price ratio. DE denotes the logged dividend payout ratio. SVAR denotes stock return variance. BM denotes the book-to-market ratio. NTIS denotes net equity expansion. TBL denotes the three month treasury yield. LTY denotes the long term yield. DFY denotes the default yield spread. DFR denotes the default return spread. INFL denotes inflation. IK denotes the investment to capital ratio. LTR denotes the long term rate of return. TMS denotes the term spread.




KP
DP
DY
EP
CAPE
DE
SVAR
BM
Difference in cumulative squared prediction error
Year


NTIS
TBL
LTY
DFY
DFR
INFL
Difference in cumulative squared prediction error
Year



IK
LTR
TMS
LTG
Difference in cumulative squared prediction error
Year