Internet Appendix

Generalized Disappointment Aversion and the Variance Term Structure

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Abstract

This appendix provides a detailed description of the data, the numerical methods used to solve different models, model-based asset prices, and additional results not included in the main body of the paper.

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I. Data

A. Consumption, Dividends, and Market Returns

I follow Bansal and Yaron (2004) and construct real per capita consumption growth series (annual, due to the frequency restriction) for the longest sample available, 1930-2016. In the literature, consumption is defined as a sum of personal consumption expenditures on nondurable goods and services. I download the data from the US National Income and Product Accounts (NIPA) as provided by the Bureau of Economic Analysis. I apply the seasonally adjusted annual quantity indexes from Table 2.3.3. (Real Personal Consumption Expenditures by Major Type of Product, Quantity Indexes, A:1929-2016) to the corresponding series from Table 2.3.6. (Real Personal Consumption Expenditures by Major Type of Product, Chained Dollars, A:1995-2016) to obtain real personal consumption expenditures on nondurable goods and services for the sample period 1929-2016. I retrieve mid-month population data from NIPA Table 7.1. to convert real consumption series to per capita terms.

I measure the total market return as the value-weighted return including dividends, and the dividends as the sum of total dividends, on all stocks traded on the NYSE, AMEX, and NASDAQ. The dividends and value-weighted market return data are monthly and are retrieved from the Center for Research in Security Prices (CRSP). To construct the monthly nominal dividend series, I use the CRSP value-weighted returns including and excluding dividends of CRSP common stock market indexes (NYSE, AMEX, NASDAQ, ARCA), denoted by RI_t and RE_t, respectively. Following Hodrick (1992), I construct the price series P_t by initializing $P_0 = 1$ and iterating recursively $P_t = (1 + RI_t)P_{t-1}$. Next, I compute normalized nominal monthly dividends $D_t = (RI_t - RE_t)P_t$. The risk-free return $R_{f,t+1}$ is the 1-month nominal Treasury bill. The nominal annualized dividends are constructed by summing the corresponding monthly dividends within the year. I retrieve the inflation index from CRSP to deflate all quantities to real values.

B. Variance Premium Data

For the variance risk premium, I closely follow Bollerslev, Tauchen, and Zhou (2009), Bollerslev, Gibson, and Zhou (2011), Drechsler and Yaron (2011) and Drechsler (2013). Under the no-arbitrage assumption, the risk-neutral conditional expectation of the return variance is equal to the price of a variance swap, which is a forward contract on the realized variance of the asset. Since the CBOE calculates the VIX index as a measure of the 30-days ahead risk-neutral expectation of the variance of the S&P 500 index, I use the VIX index as a proxy for the risk-neutral expectation of the market's return variation. The VIX is quoted in an annualized standard deviation. Hence, I first take it to a second power to transform it to variance units and then divide it by 12 to obtain monthly frequency. Thus, I obtain a new series defined as $[\text{VIX}]_t^2 = \frac{\text{VIX}_t^2}{12}$. I further use the last available observation of $[\text{VIX}]_t^2$ in a particular month as a measure of the risk-neutral expectation of return variance in that month.

For the objective expectation of return variance, a second component in the variance premium, I calculate a one-step-ahead forecast from a simple regression similar to Drechsler and Yaron (2011) and Drechsler (2013). I first calculate the measure of the realized variance by summing the squared daily log returns on the S&P 500 futures and S&P 500 index obtained from the CBOE. The constructed series are denoted by FUT_t^2 and IND_t^2, respectively. Subsequently, I estimate the following regression:

$$\text{FUT}_{t+1}^2 = \beta_0 + \beta_1 \cdot \text{IND}_t^2 + \beta_2 \cdot [\text{VIX}]_t^2 + \varepsilon_{t+1}. \tag{1}$$

The actual expectation is measured as the one-period ahead forecast and is given by equation (1). I refer to the resulting series as the realized variance and denote it by RV_t. Theoretically, the variance premium should be non-negative in each period. Thus, I truncate the difference between the implied series of $[\text{VIX}]_t^2$ and RV_t from below by 0.

For the empirical strategy above, I obtain the daily data series of the VIX index, S&P 500 index futures, and the S&P 500 index from the CBOE. The main restriction on the length of the constructed monthly variance premium is the VIX index, reported by the CBOE from January 1990. Using high-frequency data would provide a finer estimation precision of the quantities in the variance premium, but my estimates remain largely consistent with the numbers reported by the existing literature.

C. Options Data for the Skew Premium and Implied Volatility Skew

The empirical strategy and key definitions of the skew risk premium are in line with Bakshi, Kapadia, and Madan (2003) and Kozhan, Neuberger, and Schneider (2013). For the empirical analysis of the skew risk premium and implied volatility surface, I use European options written on the S&P 500 index and traded on the CBOE. The options data set covers the period from January 1996 to December 2016 and is from OptionMetrics. Options data elements include the type of options (call/put) along with the contract's variables (strike price, time to expiration, Greeks, Black-Scholes implied volatilities, closing spot prices of the underlying) and trading statistics (volume, open interest, closing bid and ask quotes), among other details. The empirical estimates of the conditional skew risk premium are computed in line with Kozhan et al. (2013). The empirical strategy consists of calculating fixed and floating legs for the skew swap, which correspond to the risk-neutral and physical expectations of the return skewness. For a detailed description of the methodology, see Kozhan et al. (2013).

To construct the empirical implied volatility curves, I first compute the moneyness for each observed option using the daily S&P 500 index on a particular trading day. I filter out all data entries with non-standard settlements. I use the remaining observations to construct the implied volatility surface for a range of moneyness and maturities. In particular, I follow Christoffersen and Jacobs (2004) and perform polynomial extrapolation of volatilities in the maturity time and strike prices. This strategy makes use of all available options and not only those with a specific maturity time. The fitted values are further used to construct the implied volatility curves.

II. Representative Agent's Maximization Problem

A representative agent starts with an initial wealth denoted by W_0. Each period t, the agent consumes C_t consumption goods and invests in N assets traded on the competitive market. Denote the fraction of the total t-period wealth W_t invested in the i-th asset with gross real return $R_{i,t+1}$ by $\omega_{i,t}$. Then, the agent's budget constraint in period t takes the form:

$$W_{t+1} = (W_t - C_t)R^{\omega}_{t+1} \tag{2}$$

$$\sum_{i=1}^{N} \omega_{i,t} = 1 \ \text{ and } \ R^{\omega}_{t+1} = \sum_{i=1}^{N} \omega_{i,t} R_{i,t+1}. \tag{3}$$

The agent chooses $\{C_t, \omega_{1,t}, ..., \omega_{N,t}\}$ in period t to maximize the utility subject to equations (2)-(3).

The Bellman equation becomes:

$$J_t = \max_{C_t, \omega_{1,t}, \ldots, \omega_{N,t}} \left\{ (1-\beta) C_t^\rho + \beta \left[\mathcal{R}_t(J_{t+1}) \right]^\rho \right) \right\}^{1/\rho}$$

subject to equations (2)-(3). I guess the optimal value function of the form $J_t = \phi_t W_t$. Using this conjecture of J_t and the form of $\mathcal{R}_t$, I rewrite the Bellman equation as:

$$\phi_t W_t = \max_{C_t, \omega_{1,t}, \ldots, \omega_{N,t}} \left\{ (1-\beta) C_t^\rho + \beta \left[\mathbb{E}_t \Big[(\phi_{t+1} W_{t+1})^\alpha \mathcal{K}(\phi_{t+1} W_{t+1}) \Big] \right]^{\rho/\alpha} \right\}^{1/\rho},$$

$$\mathcal{K}(x) = \frac{1 + \theta \mathbb{I}\{x \leqslant \delta \mathcal{R}_t(x)\}}{1 + \theta \delta^\alpha \mathbb{E}_t \Big[\mathbb{I}\{x \leqslant \delta \mathcal{R}_t(x)\} \Big]}.$$

Note that the function $\mathcal{K}$ defined above is homogeneous of degree zero.

The Return on the Aggregate Consumption Claim Asset. I further conjecture that the consumption C_t is homogeneous of degree one in wealth at the optimum, that is $C_t = b_t W_t$. Then, I obtain the Bellman equation:

$$\phi_t^\rho = \left\{ (1-\beta) \left(\frac{C_t}{W_t} \right)^\rho + \beta \left(1 - \frac{C_t}{W_t} \right)^\rho \left[\mathbb{E}_t \Big[(\phi_{t+1} R_{t+1}^\omega)^\alpha \mathcal{K}(\phi_{t+1} R_{t+1}^\omega) \Big] \right]^{\rho/\alpha} \right\} \tag{4}$$

or equivalently

$$\phi_t^\rho = \left\{ (1-\beta) b_t^\rho + \beta \left(1 - b_t \right)^\rho y_t^* \right\} \tag{5}$$

$$y_t^* = \left[\mathbb{E}_t \Big[(\phi_{t+1} R_{t+1}^\omega)^\alpha \mathcal{K}(\phi_{t+1} R_{t+1}^\omega) \Big] \right]^{\rho/\alpha}.$$

Taking the FOC of the right side of a simplified Bellman equation (4) with respect to C_t, I find:

$$(1-\beta)\left(\frac{C_t}{W_t}\right)^{\rho-1} = \beta\left(1-\frac{C_t}{W_t}\right)^{\rho-1} y_t^*.$$

or using the notations:

$$(1-\beta)b_t^{\rho-1} = \beta(1-b_t)^{\rho-1}y_t^*. \tag{6}$$

Solving for y_t^* from the last equation and substituting it into equation (5), I deduce:

$$\phi_t = (1-\beta)^{\frac{1}{\rho}} b_t^{\frac{\rho-1}{\rho}} = (1-\beta)^{\frac{1}{\rho}}\left(\frac{C_t}{W_t}\right)^{\frac{\rho-1}{\rho}}$$

Shifting one period ahead the formula for ϕ_t and substituting ϕ_{t+1} into equation (6), I obtain:

$$(1-\beta)C_t^{\rho-1} = \beta(W_t-C_t)^{\rho-1}\left[\mathbb{E}_t\left[(1-\beta)^{\alpha/\rho}\left(\frac{C_{t+1}}{W_{t+1}}\right)^{\alpha\frac{\rho-1}{\rho}}\left(R_{t+1}^{\omega}\right)^{\alpha}\mathcal{K}\left(\phi_{t+1}R_{t+1}^{\omega}\right)\right]\right]^{\rho/\alpha}.$$

Then, I rewrite the equation above as:

$$C_t^{\rho-1} = \beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{\frac{W_{t+1}}{(W_t-C_t)}}\right)^{\alpha\frac{\rho-1}{\rho}}\left(R_{t+1}^{\omega}\right)^{\alpha}\mathcal{K}\left(\left(\frac{C_{t+1}}{\frac{W_{t+1}}{W_t-C_t}}\right)^{\frac{\rho-1}{\rho}}R_{t+1}^{\omega}\right)\right]^{\rho/\alpha}.$$

and derive the asset pricing restriction for the return on the total wealth R_{t+1}^{ω} :

$$\mathbb{E}_t\left[\left\{\underbrace{\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1}R_{t+1}^{\omega}\right)^{1/\rho}}_{z_{t+1}}\right\}^{\alpha}\mathcal{K}\left(\underbrace{\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1}R_{t+1}^{\omega}\right)^{1/\rho}}_{z_{t+1}}\right)\right]^{1/\alpha} = 1.$$

Define R^c_{t+1} the return on the consumption endowment. In equilibrium, $R^c_{t+1} = R^\omega_{t+1}$ and, as in Routledge and Zin (2010), using the definition of the certainty equivalent and the function $\mathcal{K}$, the return R^c_{t+1} should satisfy the equation:

$$\mathcal{R}_t(z_{t+1}) = 1, \quad z_{t+1} = \left(\beta \left(\frac{C_{t+1}}{C_t}\right)^{\rho-1} R^c_{t+1}\right)^{1/\rho}. \tag{7}$$

Rewriting R^c_{t+1} in the form:

$$R^c_{t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{\frac{W_{t+1}}{C_{t+1}}}{\frac{W_t}{C_t} - 1} \cdot \frac{C_{t+1}}{C_t} = \frac{\xi_{t+1}}{\xi_t - 1} \cdot \frac{C_{t+1}}{C_t},$$

the wealth-consumption ratio $\xi_t = \frac{W_t}{C_t}$ can be found from the equation:

$$\mathbb{E}_t \left[\beta^{\frac{\alpha}{\rho}} \left(\frac{C_{t+1}}{C_t}\right)^{\alpha} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1}\right)^{\frac{\alpha}{\rho}} \cdot \mathcal{K}(z_{t+1})\right] = 1.$$

The Return on the Aggregate Dividend Asset. Following Routledge and Zin (2010), the portfolio problem for the obtained values ϕ_{t+1} reads as follows:

$$\max_{\omega_{1,t},\ldots,\omega_{N,t}} \mathcal{R}_t(\phi_{t+1} R^\omega_{t+1}),$$

subject to the constraints $\sum_{i=1}^{N} \omega_{i,t} = 1$ and $R^\omega_{t+1} = \sum_{i=1}^{N} \omega_{i,t} R_{i,t+1}$. Taking the FOC with respect to the weight $\omega_{i,t}$, I derive:

$$\mathbb{E}_t \left[\phi^\alpha_{t+1} (R^\omega_{t+1})^{\alpha-1} [1 + \theta \mathbb{I}(\phi_{t+1} R^\omega_{t+1} < \delta \mathcal{R}_t)] R_{i,t+1}\right] = 0.$$

Taking the difference between the i-th and j-th FOCs, I thus obtain:

$$\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha-1}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)](R_{i,t+1}-R_{j,t+1})\right]=0.$$

Multiplying the last equation by $\omega_{j,t}$ and summing over j, I further obtain:

$$\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha-1}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]R_{i,t+1}\underbrace{\sum_{j=1}^{N}\omega_{j,t}}_{=1}\right]=$$

$$=\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha-1}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]\underbrace{\sum_{j=1}^{N}R_{j,t+1}\omega_{j,t}}_{=R_{t+1}^{\omega}}\right]$$

$$\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha-1}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]R_{i,t+1}\right]=$$

$$=\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]\right]. \tag{8}$$

Following Epstein and Zin (1989), it is straightforward to show that $\phi_{t+1}=\frac{z_{t+1}}{R_{t+1}^{\omega}}$ holds in equilibrium. Using these equilibrium conditions and the definition of $\mathcal{R}_t$, I have:

$$\mathbb{E}_t\left[\phi_{t+1}^{\alpha}(R_{t+1}^{\omega})^{\alpha}[1+\theta\mathbb{I}(\phi_{t+1}R_{t+1}^{\omega}<\delta\mathcal{R}_t)]\right]=\mathbb{E}_t\left[z_{t+1}^{\alpha}[1+\theta\mathbb{I}(z_{t+1}<\delta\mathcal{R}_t)]\right]=$$

$$\mathbb{E}_t\left[1+\theta\delta^{\alpha}\mathbb{I}(z_{t+1}<\delta\underbrace{\mathcal{R}_t(z_{t+1})}_{=1})]\right]\underbrace{\mathcal{R}_t(z_{t+1})^{\alpha}}_{=1}=\mathbb{E}_t\left[1+\theta\delta^{\alpha}\mathbb{I}(z_{t+1}<\delta]\right]. \tag{9}$$

Combining equations (8)-(9) and using the equilibrium condition $R_{t+1}^{c}=R_{t+1}^{\omega}$, I finally obtain the asset pricing restriction for the gross return $R_{i,t+1}$:

$$\mathbb{E}_t\left[\frac{z_{t+1}^{\alpha}(R_{t+1}^{c})^{-1}(1+\theta\mathbb{I}(z_{t+1}<\delta)R_{i,t+1}}{1+\theta\delta^{\alpha}\mathbb{E}_t\left[\mathbb{I}(z_{t+1}<\delta)\right]}\right]=1, \tag{10}$$

Moreover, the pricing kernel M_{t+1} is:

$$M_{t+1}=\frac{z_{t+1}^{\alpha}(R_{t+1}^{c})^{-1}(1+\theta\mathbb{I}(z_{t+1}<\delta))}{1+\theta\delta^{\alpha}\mathbb{E}\left[\mathbb{I}(z_{t+1}<\delta)\right]}.$$

Rewriting $R_{i,t+1}$ in the form:

$$R_{i,t+1}=\frac{P_{i,t+1}+D_{i,t+1}}{P_{i,t}}=\frac{\frac{P_{i,t+1}}{D_{i,t+1}}+1}{\frac{P_{i,t}}{D_{i,t}}}\cdot\frac{D_{i,t+1}}{D_{i,t}}=\frac{\lambda_{t+1}+1}{\lambda_t}\cdot\frac{D_{i,t+1}}{D_{i,t}},$$

the price-dividend ratio of the i-th asset $\lambda_t=\frac{P_{i,t}}{D_{i,t}}$ can be found from the equation:

$$\mathbb{E}_t\left[\beta^{\frac{\alpha}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\alpha-1}\frac{D_{i,t+1}}{D_{i,t}}\cdot\left(\frac{\xi_{t+1}}{\xi_t-1}\right)^{\frac{\alpha}{\rho}-1}\cdot\mathcal{K}(z_{t+1})\cdot(\lambda_{t+1}+1)\right]=\lambda_t.$$

III. Numerical Solution

Following the notation from the paper, aggregate consumption growth is

$$\Delta c_{t+1}=\mu_{s_{t+1}}+\sigma\varepsilon_{t+1},\quad \varepsilon_{t+1}\sim N(0,1).$$

The consumption volatility σ is constant, whereas the mean growth rate $\mu_{s_{t+1}}$ is driven by a two-state Markov-switching process s_{t+1} with a state space:

$$\mathcal{S} = \{1 = \text{expansion},\ 2 = \text{recession}\},$$

a transition matrix

$$\mathcal{P} = \begin{pmatrix} \pi_{11} & 1-\pi_{11} \\ 1-\pi_{22} & \pi_{22} \end{pmatrix}$$

and transition probabilities $\pi_{ii} \in (0,1),\ i = 1, 2$. Let

$$\mathcal{X}(y_1, y_2, y_3) = \frac{1 + \theta \mathbb{I}\left\{\beta e^{\rho y_1}\left(\frac{y_2}{y_3 - 1}\right) \leqslant \delta^\rho\right\}}{1 + \theta\delta^\alpha \mathbb{E}_t\left[\mathbb{I}\left\{\beta e^{\rho y_1}\left(\frac{y_2}{y_3 - 1}\right) \leqslant \delta^\rho\right\}\right]},$$

then, the wealth-consumption ratio $\xi_t = \frac{W_t}{C_t}$ satisfies the equation:

$$\text{(11)} \qquad \mathbb{E}_t\left[\beta^{\frac{\alpha}{\rho}} e^{\alpha \Delta c_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1}\right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}\Big(\Delta c_{t+1}, \xi_{t+1}, \xi_t\Big)\right] = 1,$$

and the price-dividend ratio $\lambda_t = \frac{P_t}{D_t}$ of the asset with a gross return R_{t+1} (I skip the subscript i for convenience) is given by:

$$\text{(12)} \qquad \mathbb{E}_t\left[\beta^{\frac{\alpha}{\rho}} e^{(\alpha-1)\Delta c_{t+1} + \Delta d_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1}\right)^{\frac{\alpha}{\rho} - 1} \cdot \mathcal{X}\Big(\Delta c_{t+1}, \xi_{t+1}, \xi_t\Big) \cdot \frac{\lambda_{t+1} + 1}{\lambda_t}\right] = 1.$$

A. Projection Method

Following Pohl, Schmedders, and Wilms (2018), I apply a projection method of Judd (1992) to solve for the equilibrium pricing functions defined by equations (11)-(12). The model solution consists of two steps. First, I find the wealth-consumption ratio from equation (11). Second, I use the wealth return from the first step and substitute it into equation (12) to find the price-dividend ratio for the equity claim.

The Return on the Aggregate Consumption Claim Asset. I conjecture the wealth-consumption ratio of the form $\xi_t = G(\pi_t)$, in which π_t is the posterior belief. I seek to approximate the functional form of $G(\pi_t)$ by a basis of complete Chebyshev polynomials $\Psi = \{\Psi_k(\pi_t)\}_{k=0}^{n}$ of order n with coefficients $\psi = \{\psi_k\}_{k=0}^{n}$:

$$G(\pi_t) = \sum_{k=0}^{n} \psi_k \Psi_k(\pi_t) \quad \pi_t \in [1-p,\ q]. \tag{13}$$

I further define the function:

$$\Gamma(\pi_t; j) = \mathbb{E}_{t,j}\left[\beta^{\frac{\alpha}{\rho}} e^{\alpha \Delta c_{t+1}} \cdot \left(\frac{\xi_{t+1}}{\xi_t - 1}\right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}\Big(\Delta c_{t+1}, \xi_{t+1}, \xi_t\Big)\right] =$$
$$= \beta^{\frac{\alpha}{\rho}} \int e^{\alpha y} \left(\frac{G(B(y,\pi_t))}{G(\pi_t) - 1}\right)^{\frac{\alpha}{\rho}} \cdot \mathcal{X}\Big(y, G(B(y,\pi_t)), G(\pi_t)\Big) f(y,j) dy, \tag{14}$$

$$B(y,\pi_t) = \frac{(1-q) f(y,1)(1-\pi_t) + p f(y,2)\pi_t}{f(y,1)(1-\pi_t) + f(y,2)\pi_t},$$

$f(y,j)$ is the probability density function of a normal distribution $N(\mu_{s_t}, \sigma^2)$ conditional on $s_t = 1, 2$. I further apply the Gauss-Hermite quadrature to calculate expectations in equation (14).

Substituting $G(\pi_t)$ from equation (13) and $\Gamma(\pi_t; j)$ from equation (14) into equation (11), I obtain:

$$R^c(\pi_t; \psi) = (1 - \pi_t)\Gamma(\pi_t, 1) + \pi_t \Gamma(\pi_t, 2) - 1.$$

The objective is to choose the unknown coefficients ψ to make $R^c(\pi_t; \psi)$ close to zero $\forall \pi_t \in [1 - p,\ q]$. I apply the orthogonal collocation method. Formally, I evaluate the residual function in the collocation points $\{r_k\}_{k=1}^{n+1}$ given by the roots of the $n + 1$ order Chebyshev polynomial and then solve the system of $n + 1$ equations:

$$R^c(r_k; \psi) = 0 \quad k = 1, ..., n + 1$$

for $n + 1$ unknowns $\psi = \{\psi_k\}_{k=0}^{n}$. Let $\tilde{\xi}_t = \tilde{G}(\pi_t) = \sum_{k=0}^{n} \tilde{\psi}_k \Psi_k(\pi_t)$ denote an approximation of the wealth-consumption ratio, which will be used in the second step.

The Return on the Aggregate Dividend Asset. I conjecture the price-dividend ratio of the form $\lambda_t = H(\pi_t)$. Now, I seek to approximate the functional form of $H(\pi_t)$, which solves equation (12). I approximate $H(\pi_t)$ by a basis of complete Chebyshev polynomials $\Upsilon = \{\Upsilon_k(\pi_t)\}_{k=0}^{n}$ of order n with coefficients $\upsilon = \{\upsilon_k\}_{k=0}^{n}$:

$$H(\pi_t) = \sum_{k=0}^{n} \upsilon_k \Upsilon_k(\pi_t) \quad \pi_t \in [1 - p,\ q]. \tag{15}$$

I define the function:

$$\Lambda(\pi_t; j) = \mathbb{E}_{t,j}\left[\beta^{\frac{\alpha}{\rho}} e^{(\alpha-1)\Delta c_{t+1}+\Delta d_{t+1}} \left(\frac{\tilde{\xi}_{t+1}}{\tilde{\xi}_t - 1}\right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{X}\Big(\Delta c_{t+1}, \tilde{\xi}_{t+1}, \tilde{\xi}_t\Big) \cdot \frac{\lambda_{t+1}+1}{\lambda_t}\right] =$$

$$(16) \qquad = \beta^{\frac{\alpha}{\rho}} \iint e^{(\alpha+\lambda-1)y+g_d+z} \left(\frac{\tilde{G}(B(y,\pi_t))}{\tilde{G}(\pi_t)-1}\right)^{\frac{\alpha}{\rho}-1} \cdot \mathcal{X}\Big(y, \tilde{G}(B(y,\pi_t), \tilde{G}(\pi_t)\Big) \cdot$$

$$\cdot \frac{H(B(y,\pi_t))}{H(\pi_t)-1} f(y,j) g(z,j) dy dz,$$

in which $f(y,j)$ and $g(z,j)$ are probability density functions of normal distributions $N(\mu_{s_{t+1}}, \sigma)$ and $N(g_d, \sigma_d)$, respectively, conditional on $s_{t+1} = 1, 2$. Substituting $H(\pi_t)$ from equation (15) and $\Lambda(\pi_t; j)$ from equation (16) into equation (12), I obtain:

$$R^d(\pi_t; \upsilon) = (1-\pi_t)\Lambda(\pi_t, 1) + \pi_t \Lambda(\pi_t, 2) - 1.$$

Again, I apply the orthogonal collocation method. Formally, I evaluate $R^d(\pi_t; \psi)$ in the collocation points $\{s_k\}_{k=1}^{n+1}$ given by the roots of the $n+1$ order Chebyshev polynomial and solve the system of $n+1$ equations

$$R^d(s_k; \upsilon) = 0 \quad \forall k = 1, ..., n+1$$

for $n+1$ unknowns $\upsilon = \{\upsilon_k\}_{k=0}^{n}$.

B. Implementation in Matlab

This paper implements a one-dimensional projection method for solving functional equations. I approximate unknown functions using Chebyshev polynomials of the first kind and compute them recursively as:

$$T_0(z) = 1, \quad T_1(z) = z, \quad T_k(z) = 2zT_k(z) - T_{k-1}(z), \quad k = 2, ..., n \ \wedge \ z \in [-1, 1].$$

I adjust the domain of Chebyshev polynomials to the state space of pricing ratios and use modified polynomials in the approximation. Thus, the following equalities hold on the interval $[\pi_{\text{min}}, \pi_{\text{max}}] = [1 - p, \ q]$:

$$\Psi_k(\pi_t) = \Upsilon_k(\pi_t) = T_k\left(2\left[\frac{\pi_t - \pi_{\text{min}}}{\pi_{\text{max}} - \pi_{\text{min}}}\right] - 1\right), \quad k = 0, ..., n.$$

I present the results based on the collocation method. For this purpose, I evaluate residual functions in a set of nodes corresponding to $n + 1$ zeros of the $(n + 1)$-order Chebyshev polynomial, which are formally defined as:

$$z_k = \cos\left(\frac{2k + 1}{2n + 2}\pi\right), \quad k = 0, ..., n.$$

I adjust the nodes $z_k \in [-1, 1]$ to the domain of the state variable π_t :

$$\pi_k = \pi_{\text{min}} + \frac{\pi_{\text{max}} - \pi_{\text{min}}}{2}(1 + z_k), \quad k = 0, ..., n.$$

The numerical algorithm, which requires solving a system of nonlinear equations, is efficiently programmed in Matlab. I experiment with different nonlinear solvers to achieve better performance of the code. Initially, I used the simple solver ”fsolve”. Then I found the solution to the system of nonlinear equations by minimizing a constant subject to the system of nonlinear functions. I apply the nonlinear programming solver ”fmincon” with the SQP algorithm for this purpose. Similar to Pohl et al. (2018), I found that ”fmincon” provides faster running of the code and a more accurate solution compared to ”fsolve”. Thus, I present all results based on the ”fmincon” approach.

Additional numerical details involve the choices of an order of Chebychev polynomials used in the approximation of unknown functions (n), a number of Gauss-Hermite quadrature points used in the numerical integration of expectations in the residual functions (N_{GH}), and a number of draws used in Monte-Carlo simulations to compute model-based European put prices (N_{MC}). I report the results of all models in the main text based on the numerical solution, in which $n = 400$, $N_{GH} = 150$, and $N_{MC} = 4,000,000$. The next section performs a sensitivity analysis of alternative approximation choices.

C. Accuracy of Numerical Solution

To better assess the numerical accuracy, I first calculate the root mean squared error (RMSE) in the residual function for the wealth-consumption ratio. I evaluate $R^c(\pi_t;\psi)$ on a dense grid of points $\{\pi_i\}_{i=1}^{N_{\text{RMSE}}}$ that are equally spaced on the interval $[\pi_{\text{min}},\ \pi_{\text{max}}]$. I choose

$N_{\text{RMSE}} = 10,000$ of these points. The RMSE is calculated as:

$$\text{RMSE}^c = \sqrt{\frac{1}{N_{\text{RMSE}}} \sum_{k=1}^{N_{\text{RMSE}}} \Big[R^c(\pi_k;\psi)\Big]^2}, \tag{17}$$

$$\pi_k = \pi_{min} + \frac{\pi_{\max} - \pi_{\min}}{N_{\text{RMSE}} - 1}(k-1), \quad k = 1, ..., N_{\text{RMSE}}.$$

I consider four pairs of $(n, N_{GH}) : (200, 100), (200, 150), (400, 100), (400, 150)$. For each pair, I solve different model calibrations of this paper and compute the RMSE.

Table A1 reports the Euler errors implied by various approximation and integration choices. Several observations are noteworthy. First, the numerical solution technique is highly accurate, producing errors consistently below 6e-7 for all cases. Second, the projection method generates smaller RMSE for the models with Epstein-Zin preferences relative to the calibrations with disappointment aversion and generalized disappointment aversion utility functions. This result is expected in light of nonlinearities in the pricing kernel implied by disappointing outcomes in consumption growth. Third, increasing either the degree of Chebyshev polynomials or the number of quadrature points generally leads to a better approximation precision.

[Insert Table A1 here]

Figure A1 conducts further robustness checks. It compares the results of the two solutions of the original GDA calibration. First, the ”GDA” lines correspond to the variance term structures as presented in the main text. Second, the ”GDA2” curves represent the results of the same calibration, which is solved with a twice larger order of Chebyshev polynomials. The panels in Figure A1 show that the results across the two solutions are very similar, confirming the high-precision solution obtained by the projection method.

[Insert Figure A1 here]

IV. Asset Prices

The empirical evidence concerning the variance term structure and higher moment risk premiums is based on the data at the daily frequency and is then expressed in monthly terms. The risk-neutral neutral expectation of return variance can be synthesized using options data in a model-free way or proxied by synthetic variance swap rates (Britten-Jones and Neuberger (2000), Bakshi et al. (2003), Carr and Wu (2009), Dew-Becker, Giglio, Le, and Rodriguez (2017)). The ex-post total return variance is commonly estimated by a sum of squared daily returns. The ex-ante expectation of total return variance under the physical measure requires using the high-frequency data to compute ex-post return variation and then forecasting the future return variance using lagged realized variance or additional predictors. Kozhan et al. (2013) further extend these approaches for computing the risk-neutral and physical expectations of return skewness.

Turning to the model-based asset prices, one needs to calibrate the model at a daily frequency in order to exactly follow the procedure used to obtain empirical estimates. Bonomo, Garcia, Meddahi, and Tédongap (2015) build a discrete-time model with the daily interval. I want to be as close as possible to the existing long-run risk and rare disaster models in discrete time, particularly Drechsler and Yaron (2011) and Gabaix (2012), which fail to replicate the variance term structure, a key focus of my paper, as shown by Dew-Becker et al. (2017). Therefore, I calibrate my framework at the monthly frequency and present the model-based asset prices in this section.

A. Prices and Returns of Variance Claims

Consider an n-month variance swap, a claim to realized variance over months $t+1$ to $t+n$. Given the discrete nature of the model, the total variance of the return is equal to the sum of conditional variances RV_{t+i} in each subperiod. Following Dew-Becker et al. (2017), the price of an n-month variance swap is

$$VS_t^n = \mathbb{E}_t^Q \left[\sum_{i=1}^{n} RV_{t+i} \right].$$

In turn, the price of a zero coupon forward claim on realized variance is

$$F_t^n = \mathbb{E}_t^Q \left[RV_{t+n} \right].$$

Thus, F_t^n is equal to the risk-neutral expectation of return variance during the n-th month from the current period. F_t^0 is naturally defined as the realized variance in the current period. Next, I define the return on the n-month variance forward as a return on the trading strategy in which investors buy the n-month forward at the time t and sell it in the next period as a forward claim with maturity $n-1$. The proceeds from selling the forward are then used to purchase a new n-month variance at price F_{t+1}^n. Formally, the excess return of an n-period variance forward is

$$R_{t+1}^n = \frac{F_{t+1}^{n-1} - F_t^n}{F_t^n}.$$

Using the law of iterated expectations and the Radon-Nikodym derivative defined as $\frac{d\mathbb{Q}}{d\mathbb{P}} = \frac{M_{t+1}}{\mathbb{E}_t(M_{t+1})}$, I recursively compute the prices and returns of variance forwards for different maturities.

B. Variance and Skew Risk Premiums

The focus of this paper is on the monthly variance and skew risk premiums associated with equity returns. Since I calibrate the economy at the monthly frequency, the t-time monthly variance premium vp_t is defined as the difference between risk-neutral and physical expectations of the total return variance between t and $t+1$. The monthly decision horizon of a discrete-time model considered in this paper implies that the variance premium simply equals

$$vp_t = var_t^{\mathbb{Q}}(r_{e,t+1}) - var_t^{\mathbb{P}}(r_{e,t+1}), \tag{18}$$

in which $var_t^{\mathbb{Q}}(r_{e,t+1})$ and $var_t^{\mathbb{P}}(r_{e,t+1})$ are t-period conditional variances of the log return $r_{e,t+1} = \ln(R_{e,t+1})$ under the risk-neutral $\mathbb{Q}$ and physical $\mathbb{P}$ probability measures, respectively. Drechsler and Yaron (2011) call the definition (18) as the level difference. Furthermore, they argue that calibrating the model at a higher frequency would imply

$$vp_t = \mathbb{E}_t^{\mathbb{Q}}\left[\sum_{i=1}^{n-1} var^{\mathbb{Q}}_{t+\frac{i-1}{n}}\left(r_{e,t+\frac{i-1}{n},t+\frac{i}{n}}\right)\right] - \mathbb{E}_t^{\mathbb{P}}\left[\sum_{i=1}^{n-1} var^{\mathbb{P}}_{t+\frac{i-1}{n}}\left(r_{e,t+\frac{i-1}{n},t+\frac{i}{n}}\right)\right], \tag{19}$$

in which $var_{t+\frac{i-1}{n}}\left(r_{e,t+\frac{i-1}{n},t+\frac{i}{n}}\right)$ denotes the conditional variance of the market return between $t+\frac{i-1}{n}$ and $t+\frac{i}{n}$. Following equation (19) for calibrations at the higher frequency, Drechsler and Yaron (2011) define the variance premium as

$$vp_t = \mathbb{E}_t^{\mathbb{Q}}(var_{t+1}^{\mathbb{Q}}(r_{e,t+2})) - \mathbb{E}_t^{\mathbb{P}}(var_{t+1}^{\mathbb{P}}(r_{e,t+2})), \tag{20}$$

in which vp_t is the sum of the level difference and the drift difference defined as:

(21)

$$\text{drift difference} = \left[\mathbb{E}_t^{\mathbb{Q}}(var_{t+1}^{\mathbb{Q}}(r_{e,t+2})) - var_t^{\mathbb{Q}}(r_{e,t+1})\right] - \left[(\mathbb{E}_t^{\mathbb{P}}(var_{t+1}^{\mathbb{P}}(r_{e,t+2})) - var_t^{\mathbb{P}}(r_{e,t+1}))\right].$$

As I compare the predictions of our model with those implied by Drechsler and Yaron (2011), I similarly define the variance premium by equation (20). However, in the unreported results, I confirm that the main results related to the variance premium are robust to the alternative formulation in equation (21) because the drift difference strongly dominates vp_t, the finding also reported by Drechsler and Yaron (2011) and Lorenz, Schmedders, and Schumacher (2020).

The t-time monthly skew premium is defined as a return on a skew swap, a contract paying the realized skew of the return between time t and $t+1$. Following Kozhan et al. (2013), I define the skew premium as

$$sk_t = \frac{\mathbb{E}_t^{\mathbb{P}}(skew_{t+1}^{\mathbb{P}}(r_{e,t+2}))}{\mathbb{E}_t^{\mathbb{Q}}(skew_{t+1}^{\mathbb{Q}}(r_{e,t+2}))} - 1,$$

in which $skew_{t+1}^{\mathbb{Q}}(r_{e,t+2})$ and $skew_{t+1}^{\mathbb{P}}(r_{e,t+2})$ are $(t+1)$-period conditional skewness of the log return $r_{e,t+2} = \ln(R_{e,t+2})$ under the risk-neutral $\mathbb{Q}$ and physical $\mathbb{P}$ probability measures, respectively. Note that Kozhan et al. (2013) define the skew premium as the ratio between the risk-neutral and physical expectation of return skewness, unlike the difference between the expectations in the case of the variance premium. This leads to an economic interpretation that is different from the variance premium: the skew premium measures the average return on a skew swap, a synthetic instrument with a price equal to the risk-neutral expectation of return skewness that pays off the realized return skewness. I want to be consistent with Kozhan et al. (2013) and their estimates, therefore, I follow their definition of the skew premium.

C. Option Prices and Implied Volatilities

I now describe how I compute model-based option prices and solve for their Black-Scholes implied volatilities. Consider a European put option written on the price of the equity that is traded in the economy. Note that the equity price should not include dividend payments; that is, options are written on the ex-dividend stock price index. Using the Euler equation, the relative price $\mathcal{O}_t(\pi_t, \tau, K) = \frac{P_t^o(\pi_t,\tau,K)}{P_t^e(\pi_t)}$ of the τ-period European put option with the strike price K, expressed as a ratio to the initial price of the equity P_t^e, should satisfy

$$\mathcal{O}_t(\pi_t, \tau, K) = \mathbb{E}_t\left[\prod_{k=1}^{\tau} M_{t+k} \cdot \max\left(K - \frac{P_{t+\tau}^e}{P_t^e}, 0\right)\right]. \tag{22}$$

Note that a put price P_t^o depends on the equity price P_t^e, whereas the normalized price $\mathcal{O}_t$ does not. One can express the ratio $\frac{P_{t+\tau}^e}{P_t^e}$ in terms of dividend growth rates and price-dividend ratios on the equity and hence the state belief π_t provides sufficient information for the calculation of the option prices. Specifically, I compute model-based European put prices $\mathcal{O}_t = \mathcal{O}_t(\pi_t, \tau, K)$ via Monte Carlo simulations. I convert them into Black-Scholes implied volatilities with a properly annualized continuous interest rate $r_t = r_t(\pi_t)$ and dividend yield $q_t = q_t(\pi_t)$. Thus, given the maturity τ, the strike price K, the risk-free rate r_t, and dividend yield q_t, the implied volatility $\sigma_t = \sigma_t^{\text{BS}}(\pi_t, \tau, K)$ solves the equation:

$$\mathcal{O}_t = e^{-r_t\tau} \cdot K \cdot N(-d_2) - e^{-q_t\tau} \cdot N(-d_1), \tag{23}$$

$$d_{1,2} = \left[-\ln\left(K\right) + \tau\left(r_t - q_t \pm \sigma_t^2/2\right)\right] / \left[\sigma_t\sqrt{\tau}\right].$$

V. Sensitivity Analysis

This appendix presents additional results of alternative calibrations of GDA, DA, and EZ specifications.

A. Equity Returns and Moment Risk Premiums

Figure A2 provides sensitivity results for the risk-free rate, the equity premium, the price-dividend ratio, and the moment risk premiums for a broad range of parameter choices in the three models. In particular, I change a key parameter in each of the three preference specifications, while holding the remaining parameters at the values in the original calibration. In the GDA model, I vary the disappointment threshold between 0.915 and 0.945. In the DA model, I change the disappointment aversion parameter between 0.45 and 0.75. In the EZ model, the results are provided for the coefficient of relative risk aversion ranging from 4.5 to 7.5. The panels in Figure A2 present the model-based average statistics implied by the GDA, DA, and EZ frameworks. The asset pricing moments are expressed as a function of a varying parameter, which is indicated on the corresponding axis.

Figure A2 shows that the risk-free rate decreases with the disappointment threshold, disappointment aversion, and relative risk aversion in the GDA, DA, and EZ models, respectively. Further, the equity premium increases and equity prices decline in δ, θ, and $1-\alpha$. Intuitively, when the agent faces more disappointing outcomes or becomes more averse to low consumption growth rates, he demands larger premiums in expected returns for bearing the additional risk in consumption growth. The impact of δ and $1 - \alpha$ on the volatility of asset prices is similar across the GDA and EZ models: a higher disappointment threshold or a higher risk aversion leads to a more volatile risk-free rate, while the volatility of equity returns and the price-dividend ratio exhibits a hump-shaped pattern with a maximum approximately in the middle of the parameter intervals considered. In the DA model, raising disappointment aversion slightly increases the volatility of the risk-free rate, equity returns, and prices. Overall, the magnitude of changes in the risk-free rate, equity returns, and price-dividend ratio is quite comparable across the three models, especially when looking at the GDA and EZ frameworks. These findings suggest that all three preference specifications can reasonably explain the first and second moments of equity returns by adjusting a key preference parameter. In contrast, the four bottom panels in Figure A2 indicate the crucial importance of generalized disappointment aversion for generating significant risk premiums in higher moments of equity returns.

[Insert Figure A2 here]

Figure A2 shows that, in the DA setting, changing the disappointment aversion for a wide range of values does not improve the model's performance, as the variance and skew risk premium moments are not very sensitive to changes in θ. Moreover, no value of the disappointment aversion parameter can support the negative skew premium. Figure A2 also shows that Epstein-Zin preferences provide a better fit of the model with the data. In particular, when the risk aversion increases from 4.5 to 6, the average variance premium increases from less than 2 to around 5, while the skew premium declines from around -10% to -20%. However, the mean and volatility of the variance premium actually start to decline at some point, and thus the higher risk aversion will move the model away from the data. Finally, the comparative analysis with respect to the disappointment threshold in the GDA model generates patterns in the variance and skew risk premiums similar to those predicted by different risk aversion parameters in the EZ economy. However, with generalized disappointment aversion, the magnitude and time-variation of variance and skew risk premiums are significantly amplified.

B. Implied Volatilities

Figures A3 and A4 further provide comparative statics of the implied volatility curves in the three preference specifications. Several observations are noteworthy. First, in all economies, the implied volatility curve for one-month options is not very sensitive to a further increase in effective risk aversion. In all cases, an incremental increase is less than 1% for any particular maturity and moneyness. Second, in the model with Epstein-Zin preferences, the slope of the ATM and OTM volatilities stays the same for higher risk aversion. In the DA economy, even though ATM volatilities for longer maturities seem to increase more in response to raising disappointment aversion, the levels are significantly below the empirical curves. In the GDA economy, changes in θ and δ have a larger impact on the term structure of ATM and OTM volatilities. Specifically, Figure A3 suggests that a higher disappointment threshold increases the prices of options with longer maturities, helping to explain a slightly upward-sloping shape in ATM volatilities. Meanwhile, a higher disappointment aversion parameter seems to increase prices of short-term OTM options more than those with longer maturities, helping to explain a slightly downward-sloping pattern in OTM volatilities. Therefore, in the setting of my model, simultaneously increasing θ and decreasing δ could allow one to keep the one-month implied volatilities close to the empirical curves while even better matching the salient statistics of ATM and OTM volatilities. Finally, a lower degree of effective risk aversion implies that the implied volatility curves become flatter and shift down in all models, especially in the economies with GDA and Epstein-Zin preferences.

[Insert Figure A3 here]

[Insert Figure A4 here]

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TABLE A1

Euler Errors: GDA, DA, and EZ

The table reports the RMSE for different models. For each specification, it shows the results for two different degrees of Chebyshev polynomials n and two different numbers of Gauss-Hermite quadrature points N_{GH}. The Euler errors are computed using Equation (17) with 10,000 points equally spaced on the interval $[\pi_{\min}, \pi_{\max}]$.

Model	$n = 200$ $N_{GH} = 100$	$n = 200$ $N_{GH} = 150$	$n = 400$ $N_{GH} = 100$	$n = 400$ $N_{GH} = 150$
GDA	4.71e-07	4.18e-07	1.83e-07	1.47e-07
GDA_{δ_l}	3.40e-07	2.83e-07	1.25e-07	1.16e-07
GDA_{δ_h}	5.01e-07	4.40e-07	1.83e-07	1.75e-07
GDA_{θ_l}	4.28e-07	4.12e-07	1.61e-07	1.53e-07
GDA_{θ_h}	5.42e-07	4.76e-07	1.86e-07	1.84e-07
DA	1.28e-08	9.48e-09	3.97e-09	3.47e-09
DA_{θ_l}	8.82e-09	7.95e-09	3.16e-09	2.52e-09
DA_{θ_h}	1.30e-08	1.17e-08	4.63e-09	3.53e-09
EZ	7.59e-14	7.37e-14	9.15e-14	9.32e-14
$\text{EZ}_{(1-\alpha)_l}$	5.57e-14	4.95e-14	6.85e-14	6.58e-14
$\text{EZ}_{(1-\alpha)_h}$	9.94e-14	9.86e-14	1.34e-13	1.26e-13

FIGURE A1

Accuracy of the Projection Method

The figure plots annualized Sharpe ratios and average prices for variance forwards for the original GDA calibration, which is solved and simulated with different precisions. "GDA" denotes the results of the original solution. "GDA2" shows the results of the original calibration, which is solved with a twice larger order of Chebyshev polynomials.

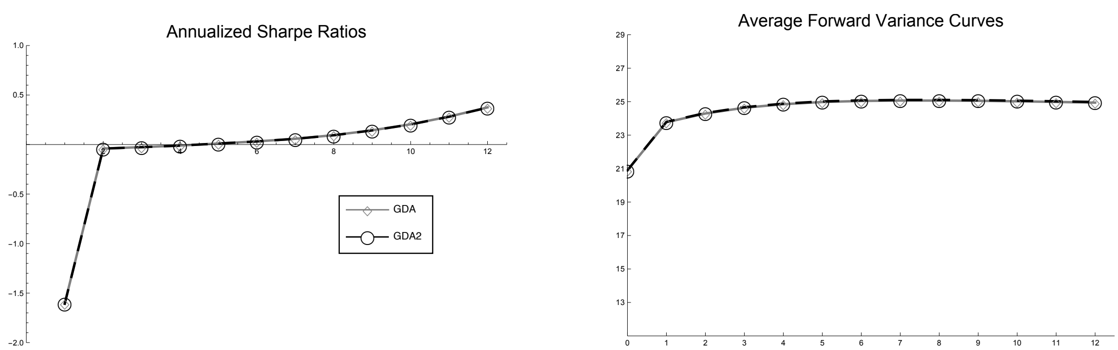

FIGURE A2

Sensitivity of Asset Prices: GDA, DA, and EZ

The figure plots asset pricing moments in the GDA, DA, and EZ models, in which a single parameter is changed while others are fixed at the original values. Specifically, I change the disappointment threshold, the disappointment aversion parameter, and the coefficient of risk aversion in the original GDA, DA and EZ models, respectively, over a range of values. For each calibration, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart. The entries of the figure are medians of sample statistics (annualized for the risk-free rate, the equity premium and the price-dividend ratio; monthly for the variance and skew risk premiums). The model-implied results for equity returns (moment risk premiums) are based on the simulations with (without) consumption disasters, consistent with the historical data. I use common notations for mean E and standard deviation σ.

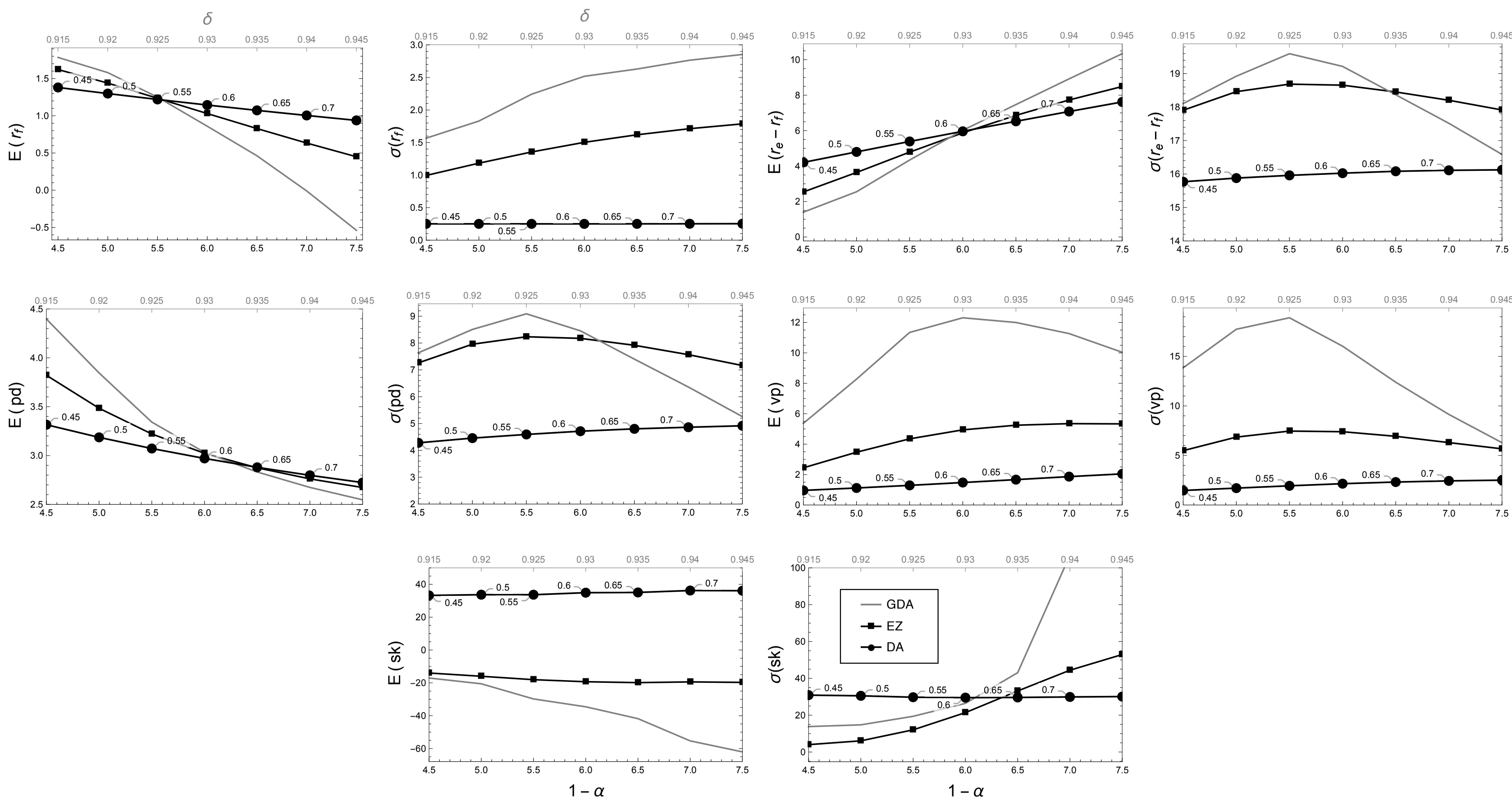

FIGURE A3

Sensitivity of Implied Volatilities: GDA

The figure plots the 1-month implied volatility curve (top) as a function of moneyness, and implied volatility curves for ATM (middle) and OTM (bottom) options as functions of the time to maturity (in months) for different model calibrations with generalized disappointment aversion preferences. GDA corresponds to the original GDA model. In GDA$_{\theta_l}$ and GDA$_{\theta_h}$, the disappointment aversion parameters are $\theta_l = 6.41$ and $\theta_h = 10.41$, respectively. In GDA$_{\delta_l}$ and GDA$_{\delta_h}$, the disappointment threshold parameters are $\delta_l = 0.920$ and $\delta_h = 0.940$, respectively. If not stated otherwise, the remaining parameters in all specifications are set at the original values in the GDA model. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report medians of sample statistics based on these series. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.

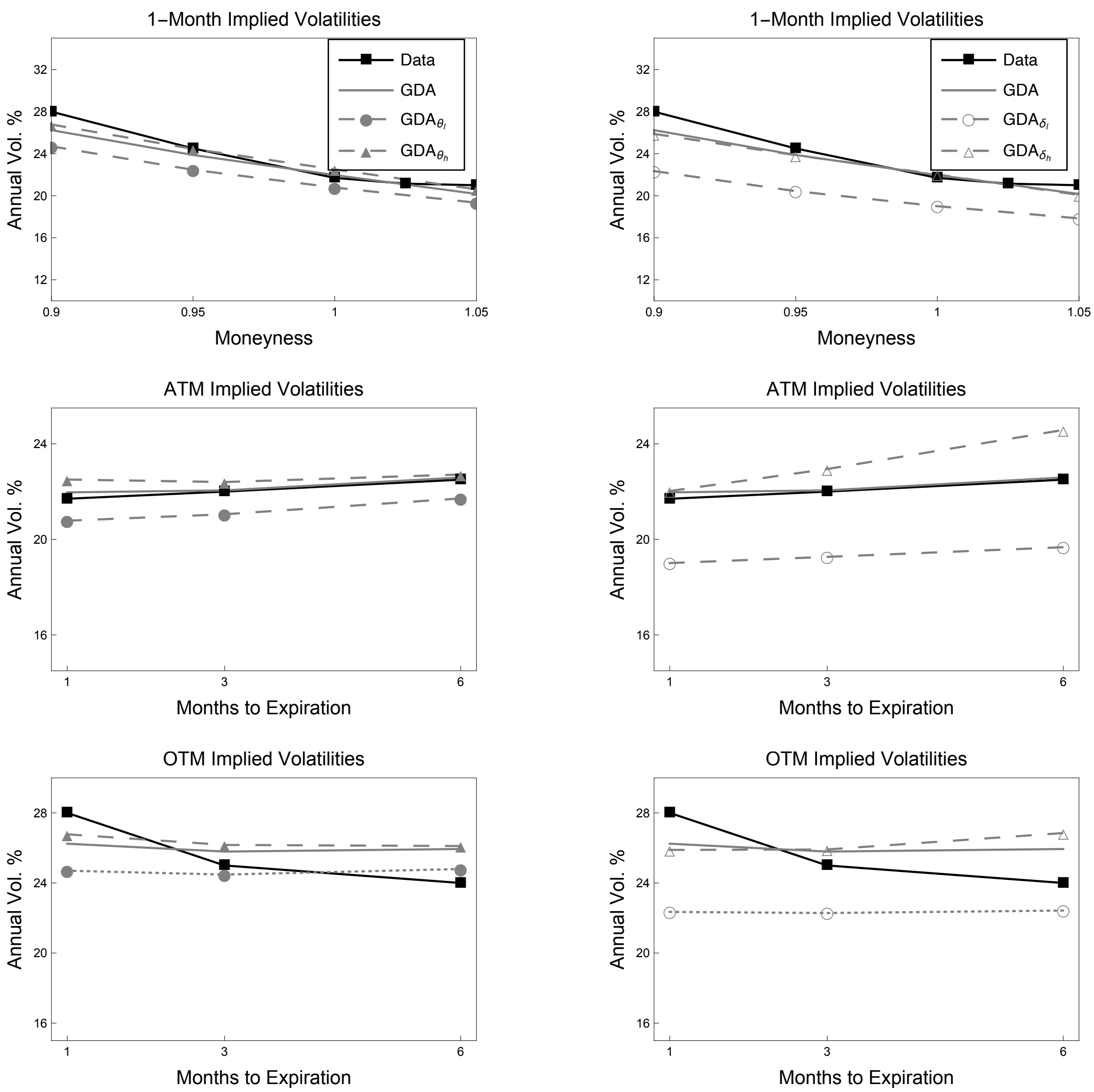

FIGURE A4

Sensitivity of Implied Volatilities: DA and EZ

The figure plots the 1-month implied volatility curve (top) as a function of moneyness, implied volatility curves for ATM (middle) and OTM (bottom) options as functions of the time to maturity (in months) for different model calibrations with disappointment aversion and Epstein-Zin preferences. DA and EZ correspond to the original DA and EZ models. In DA_{θ_l} and DA_{θ_h}, the disappointment aversion parameters are $\theta_l = 0.5$ and $\theta_h = 0.7$, respectively. In $\text{EZ}_{(1-\alpha)_l}$ and $\text{EZ}_{(1-\alpha)_h}$, the risk aversion parameters are $(1-\alpha)_l = 5$ and $(1-\alpha)_h = 7$, respectively. If not stated otherwise, the remaining parameters in all specifications are set at the original values in the DA and EZ models. For each model, I simulate 10,000 economies at a monthly frequency with a sample size equal to its empirical counterpart and report medians of sample statistics based on these series. The model-implied results are based on the simulations without consumption disasters, consistent with the historical data.