Internet Appendix: Central Counterparty Default Waterfalls and Systemic Loss

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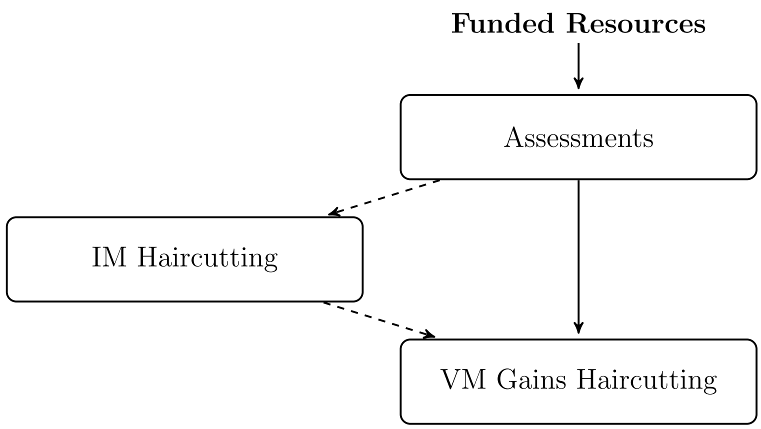
A End-of-Waterfall Mechanisms

The final stages of the default waterfall are based on mechanisms that are not as widely implemented, and their exact rules can vary significantly across CCPs. In addition, their placement at the end of the waterfall means they are used only in rare situations where all previous pre-funded resources have been applied. There is thus limited historical precedent for these recovery mechanisms. Therefore, structurally modeling their effectiveness is crucial to inform expectations in the event they are required.

Figure A.1: Stages of the End-of-Waterfall



Note: The chart depicts the series of end-of-waterfall mechanisms in the waterfall that will be accessed if the funded resources are insufficient to cover total default losses in the event of a clearing member or client default. The solid arrows depict the most common set of waterfall resource contingencies, with the dashed arrows showing alternative contingency paths.
Source: Authors' creation.

The final stages include assessments, initial margin haircutting, and variation margin gains haircutting, depicted in Figure A.1. Assessments are the most commonly employed of these three mechanisms. At the time of the loss, if the CCP has already used up the previous stages of the default waterfall, it will have the ability to assess further funds from its clearing members. There is usually a limit on the amount that can be assessed from each clearing member. This cap varies across CCPs. For instance, ICE Clear Credit imposes a limit of 300 percent of guarantee fund contributions, while CME Group has a limit of 275 percent of guarantee fund contributions for a single clearing member default and 550 percent of guarantee fund contributions for multiple member defaults. Because assessments are not pre-funded, they are less burdensome to clearing members than IM or guarantee fund contributions, which must be given in advance. However, being unfunded also means that assessments may not be available from clearing members that are under stress. Thus, assessments may not provide the CCP sufficient resiliency if many clearing members are under financial stress simultaneously and cannot pay the assessed amounts in a timely fashion.

Initial margin haircutting (IMH) allows the CCP to take the IM of clearing members that did not default. This opens up a very large pool of funds to the CCP, as IM values are usually quite large. However, IMH has the downside of potentially distorting clearing member incentives and causing clearing members to enter contracts that require less IM. It can also conflict with regulatory capital requirements for bank holding companies. Basel regulations place higher risk weights on non-bankruptcy remote capital held with the CCP. Utilizing IMH could thus raise clearing costs for clearing members under the current regulatory system. Finally, in some jurisdictions such as the U.S., IM may also be held at a third party that the CCP does not have access to in a crisis. Because of these issues, IMH tends to be an unpopular mechanism (ISDA (2013)).[1] Though IMH is an option for some CCP jurisdictions, no CCP currently implements it in their default waterfalls (as suggested by the layout of Figure 1).

Variation margin gains haircutting (VMGH) allows the CCP to continue to make VM payments that are owed, but the action reduces the VM payments by some percentage. In theory, VMGH could allow the CCP to withstand an unlimited loss, as the CCP could reduce its VM payments to zero. By reducing the payments that the CCP owes, VMGH spreads losses to other clearing members and thus is a form of risk-sharing. However, as VMGH takes an equal pro rata approach to loss distribution, it can be harmful to firms that are hedged, and therefore generate contagion losses outside of the derivatives market.

[1] Though legally segregated operationally commingled is meant to prevent IMH in event of default of a member in the U.S., under the bankruptcy code even individually segregated client funds can be treated as if IM were commingled in a single account (Ruffini (2015)).

A.1 Member Assessments

Each of the m clearing members is assessed according to the riskiness of its portfolio. This assessment is made via a similar method as in determining guarantee fund contributions. The difference is that assessments are not pre-funded and must be collected at the time of the shock from the remaining capital of the clearing members. The total amount that can be assessed thus depends on the capital that each clearing member has available. We assume that assessments have a lower priority than a firm's VM payment obligations. Firms that are under stress cannot be assessed, and firms that are not under stress can contribute only up to their capital remaining after VM payments.

Given the payments that it receives, a clearing member k will have an amount of capital left over, θ_k, of

$$\text{(A.1)} \quad \theta_k \equiv \Bigg[b_k - \Bigg[\sum_{i \neq k} \bar{p}_{ki} + \sum_{i \in C_k} (\bar{q}_{0ki} + \bar{q}_{ik0}) - \sum_{i \neq k} ((p_{ik} + z_{ik}) \wedge \bar{p}_{ik}) - \sum_{i \in C_k} ((q^c_{ik0} + z_{ik0}) \wedge \bar{q}_{ik0}) + q_{0ki}) \Bigg]^+ \Bigg]^+ .$$

The CCP typically also has an upper cap on the amount it can assess each clearing member. Similar to the rules of existing CCPs, we assume that the assessment amount of a clearing member is capped at β times the guarantee fund contribution. Therefore $\min(\beta\gamma_k, \theta_k)$ is the most that can be raised from clearing member k.

We define the intermediate stress of the CCP, $\dot{s}_0$, as the stress of the CCP after accounting for the guarantee fund and the CCP capital contribution. Formally, $\dot{s}_0$ is given by

$$\text{(A.2)} \quad \dot{s}_0 \equiv \left[\sum_{k \in M} \left(\bar{p}_{0k} + \sum_{i \in C_k} \bar{q}_{0ki} - (p_{k0} + \sum_{i \in C_k} q^m_{ik0} + z_{k0}) \wedge (\bar{p}_{k0} + \sum_{i \in C_k} \bar{q}_{ik0}) \right) - \gamma - b_0 \right]^+ .$$

We note that this is equivalent to the CCP's stress from equation (10) in the main text for the model without the end of waterfall mechanisms.

For a given set of payment vectors, $\alpha_0 \equiv \min(\sum_{k \in M} \min(\beta\gamma_k, \theta_k), \dot{s}_0)$ is the *assessment ability* of the CCP. If this amount covers the remaining payment obligations of the CCP, the CCP will have zero remaining stress after assessments. If this amount is not sufficient, then assessments will not fully cover the CCP's payment obligations, and the CCP will have to implement VMGH, IMH, or move into resolution procedures. We note that α_0 is an endogenous value that depends on the stress of the CCP and its members are given a specified set of payment vectors.

Given an assessment amount α_0, the CCP's remaining stress going on to the next stage is defined as

$$\ddot{s}_0 \equiv \dot{s}_0 - \alpha_0. \tag{A.3}$$

Note that α_0 is endogenous and depends on the equilibrium stress of clearing members. It thus needs to be determined by a fixed point between the value of the stress of each firm s_k. However, this calculation will be affected by VMGH and IMH. We discuss the calculation of the final stress in the following two subsections.

A.2 Initial Margin Haircuts

If a CCP were allowed to implement IMH, it would take unused IM from its contracts with its clearing members and clients to cover its stress. We assume a pro rata haircut of the available IM is applied to all outstanding accounts,

$$z_0^r \equiv \sum_{k \in M} [z_{k0} - (\bar{p}_{k0} - p_{k0})]^+ + \sum_{k \in M} \sum_{i \in C_k} [z_{ik0} - (\bar{q}_{ik0} - q_{ik0}^c)]^+, \tag{A.4}$$

up to a maximum of its stress after assessments $\ddot{s}_0$. Note that p_{i0} and p_{ik0} are determined endogenously, so the remaining pool of IM, z_0^r, is determined endogenously as well.

If IMH can satisfy all of the CCP's stress $\ddot{s}_0$, then the CCP will make all of its payments.

In this case $p_{0k} = \bar{p}_{0k}$ and $q_{0ki} = \bar{q}_{0ki}$. On the other hand, if the pool of IM is not enough to cover the CCP's stress then the CCP will either move onto VMGH if available, or it will default and enter resolution procedures. We do not explicitly model the resolution procedures. If using VMGH, we define the stress as

$$\ddot{s}_0^z \equiv [\ddot{s}_0 - z_0^r]^+. \tag{A.5}$$

A.3 Variation Margin Gains Haircuts

The final stage in the waterfall is VMGH, where the CCP will prorate outgoing VM payments to all firms to cover its shortfall $\ddot{s}_0$, or $\ddot{s}_0^z$ if IMH was used. VMGH occurs through a nearly identical mechanism as is employed by defaulting firms in the model, whereby the CCP prorates its VM payments to others.[2]

We define the combined payment obligations for the CCP to derive how the prorating occurs. We let $\bar{p}_0^g = \sum_{k \in M} \bar{p}_{0k} + \sum_{k \in M} \bar{q}_{0ki}$. We use these combined payment obligations to derive the CCP's relative payment liability to different firms. If IMH is present, then replace $\ddot{s}_0$ by $\ddot{s}_0^z$ in the following equations.

$$a_{0k} = \bar{p}_{0k}/\bar{p}_0^g, \tag{A.6}$$

$$a_{0ki} = \bar{q}_{0ki}/\bar{p}_0^g. \tag{A.7}$$

$$p_{0k} = \bar{p}_{0k} - a_{0k}\ddot{s}_0 \quad \forall k \in M, \tag{A.8}$$

$$q_{0ki} = \bar{q}_{0ki} - a_{0ki}\ddot{s}_0 \quad \forall k \in M. \tag{A.9}$$

We note that under VMGH the model functions similarly as the Eisenberg-Noe model since the CCP prorates its outgoing payments proportionally like any other node.

[2]Note that under VMGH mechanism, the model functions similarly as the Eisenberg-Noe model.

A.4 Calculating Loss at End-of-Waterfall

Clearing members may suffer additional losses from the CCP's use of clearing member contributions to the guarantee fund, $\hat{\gamma}_k$, assessments on the firm by the CCP, $\hat{\alpha}_k$, and IM losses in cases where IMH is used, $\hat{z}_{k0}$. The total default waterfall losses are given by

$$\text{(A.10)} \qquad \text{Default Waterfall Loss} = \hat{\gamma}_k + \hat{\alpha}_k + \hat{z}_{k0}.$$

In calculating these losses, we consider only the usage of funds to cover the obligations of other clearing members. Guarantee funds that are used to cover a clearing member's own default are not counted in losses, but guarantee funds used to cover another clearing member's default are counted as losses.

Recall that the total guarantee fund contributions of clearing member k are denoted γ_k. We denote the amount of guarantee fund used to cover a clearing member's *own obligations* $\dot{\gamma}_k$. This guarantee fund is taken from clearing member k when clearing member k is short payments to the CCP and owes more than its IM z_{k0} can cover. The value of $\dot{\gamma}_k$ is given by

$$\text{(A.11)} \qquad \dot{\gamma}_k \equiv \min\left(\left[\bar{p}_{k0} + \sum_{i\in C_k} \bar{q}_{ik0} - \left(p_{k0} + \sum_{i\in C_k} q^m_{ik0} + z_{k0}\right)\right]^+, \gamma_k\right), \quad \forall k \in M.$$

Suppose clearing member k does not use all of its contributions γ_k and losses to the CCP from other firms go beyond the resources of the CCP's capital contribution b_0. Then clearing member k will have a part, if not all, of its guarantee fund contribution used to cover the obligations of other clearing members. Guarantee fund contributions will be taken pro rata the remaining contributions of each clearing member to cover the CCP's shortfall.

Let the amount that the CCP needs to cover if it uses the previous waterfall layers be given by g_0:

$$\text{(A.12)} \quad g_0 \equiv \left[\sum_{k\in M}\left(\bar{p}_{0k} + \sum_{i\in C_k} \bar{q}_{0ki} - (p_{k0} + \sum_{i\in C_k} q^m_{ik0} + z_{k0}) \wedge (\bar{p}_{k0} + \sum_{i\in C_k} \bar{q}_{ik0}) - \dot{\gamma}_k\right) - b_0\right]^+.$$

The guarantee fund losses for clearing member k, $\hat{\gamma}_k$, are given by

$$\text{(A.13)} \qquad \hat{\gamma}_k \equiv \min\left(\frac{\gamma_k - \dot{\gamma}_k}{\sum_{j\in M} \gamma_j - \dot{\gamma}_j} g_0, \gamma_k - \dot{\gamma}_k\right), \quad \forall k \in M.$$

If this amount is still not enough to cover the CCP's losses, then the end-of-waterfall mechanisms will be used. Each clearing member will be assessed pro rata its total amount available for assessments, $\min(\beta\gamma_k, \theta_k)$. The assessment losses for clearing member k are given by

$$\text{(A.14)} \qquad \hat{\alpha}_k \equiv \frac{\min(\beta\gamma_k, \theta_k)}{\sum_{j\in M} \min(\beta\gamma_j, \theta_j)} \min\Big(\dot{s}_0, \sum_{j\in M} \min(\beta\gamma_j, \theta_j)\Big), \quad \forall k \in M.$$

If IMH is an option for the CCP and is necessary, it will be performed pro rata across the clearing member contributions to the IMH pool. Recall that the total pool of IM that the CCP has available is

$$\text{(A.15)} \qquad z_0^r = \sum_{k\in M} \left[z_{k0} - (\bar{p}_{k0} - p_{k0})\right]^+ + \sum_{k\in M} \sum_{i\in C_k} \left[z_{ik0} - (\bar{q}_{ik0} - q_{ik0}^c)\right]^+.$$

The total IM left for a clearing member k including its client clearing transactions is

$$\text{(A.16)} \qquad \dot{z}_{k0} \equiv \left[z_{k0} - (\bar{p}_{k0} - p_{k0})\right]^+, \quad \dot{z}_{k0}^0 \equiv \sum_{i\in C_k} \left[z_{ik0} - (\bar{q}_{ik0} - q_{ik0}^c)\right]^+, \quad \forall k \in M.$$

The total amount of IMH that the CCP needs depends on the level of its stress $\ddot{s}_0$. The amount of IMH losses for each clearing member k, $\hat{z}_{k0}$, is given by

$$\text{(A.17)} \qquad \hat{z}_{k0} \equiv \frac{\dot{z}_{k0} + \dot{z}_{k0}^0}{z_0^r} \min(\ddot{s}_0, z_0^r), \quad \forall k \in M.$$

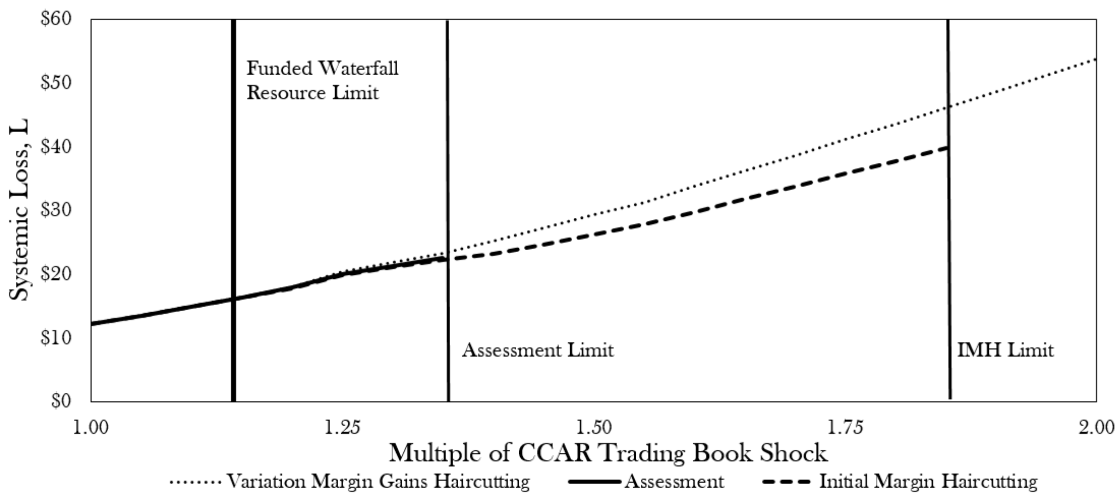
A.5 Empirical Results

If losses are significant enough to exceed the total available funded waterfall resources, the CCP must employ an end-of-waterfall mechanism. As discussed previously, the major end-of-waterfall mechanisms are assessments, VMGH, and IMH. Our survey of global CCPs finds that assessments and VMGH are both common in CCP waterfall structures today. IMH, on the other hand, is not part of any current CCP mechanisms; however, it is a widely discussed mechanism and its use may come into place in the future.

For simplicity, we isolate each mechanism in our evaluations of their impact on resilience and clearing member incentives, and we assume that the CCP cannot combine them. For each mechanism, we vary the size of the CCAR shock by α, and we calculate the total systemic loss L. This allows us to find the level of α at which the CCP runs out of resources to cover payments. Note that under VMGH, the CCP never technically defaults because it can always reduce its outgoing payments to zero. However, once the CCP uses VMGH it will start transmitting its stress, $\ddot{s}$, to other institutions. On the other hand, assessments and IMH will not add stress to the network as long as the CCP has enough unfunded resources to fulfill its payment obligations. With these mechanisms, the CCP transmits stress only when it runs out of clearing member funds to assess or initial margin to haircut.

Figure A.2 shows the systemic losses of the three mechanisms. The vertical lines on the plot highlight important thresholds. Going from left to right, the first line indicates where the CCP's funded waterfall resources hit their limit, at $\alpha = 1.13$. After this threshold, the three mechanisms are activated by the CCP, which results in differences in the systemic losses. The second line, at $\alpha = 1.35$, indicates where the assessment mechanism reaches its limit in providing enhanced resiliency to the CCP, due to clearing members running out of resources to assess. The final line, at $\alpha = 1.85$, indicates where the IMH mechanism reaches its limit. The greater α limit for IMH highlights how much larger the pool of IM funds is relative to assessments for the CCP. Additionally, it is important to note that both of these mechanisms provide lower total systemic losses than VMGH.

Figure A.2: Total Market Losses and CCP Default Threshold by End-of-Waterfall Mechanism



Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario. Each line represents the amount of loss suffered under the three end-of-waterfall mechanisms, assuming each is implemented independently of the other. The vertical lines represent the limit at which a mechanism no longer has the resource to draw upon. The assessment mechanism has limited additional resilience in our example, whereas initial margin haircuts (IMH) allows the CCP to maintain full payments under a 50 percent larger stress. Variation margin gains haircuts provide the most resilience, though they create the largest systemic losses.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

The caveat to both assessments and IMH is that they have limits to the protection they offer. They require additional resources to cover payments, but in times of stress, such resources may be limited. In contrast VMGH, the worst performing of the mechanisms, by the measure of systemic losses, provides the greatest degree of resiliency in allowing the CCP to continue operating even with limited funds. This result suggests why VMGH is typically used by CCPs as a final stage for the waterfall, and why it has been accepted for use in the United States.

B Collateral Illiquidity and Fire Sales

In this appendix, we explore the impact of collateral quality on systemic losses, as collateral illiquidity can create fire sale effects and lead to deadweight losses. Collateral in our theoretical model is composed of initial margin holdings as well as capital buffers. To incorporate the potential consequences of collateral illiquidity, we assume that for all i and j every z_{ij}, z_{ij0} and b_i that exists is held in an asset with a normalized price π. The value of π can range from zero to one, with zero indicating a full reduction in value and one indicating no reduction in value (i.e. cash). For instance, πz_{ij} is the reduced value of the IM from firm i to firm k, and πb_i is the reduced value of the capital buffer of firm i. Following the price-impact formulation used in Cifuentes et al. (2005) and Amini et al. (2016), we posit that π is a strictly decreasing function $G(1, \Delta)$ of the total proportion of collateral asset liquidated Δ, with the first argument of G indicating the initial price of 1. To be concrete, we set

$$\pi = G(1, \Delta) = e^{-\lambda\Delta} \tag{B.1}$$

for some $\lambda > 0$.

Equation B.1 states that, for a given proportion of collateral liquidations Δ, the final normalized collateral price is equal to $\pi = G(1, \Delta) = e^{-\lambda\Delta}$ for some $\lambda > 0$. A larger λ corresponds to a less liquid asset. The value of Δ is then derived from the payment vector and the stress equations. Note that as the value of Δ affects the value of firm payments, calculating the equilibrium entails additional steps to derive Δ along with the payment vector.

We note that in the case of illiquid collateral, uniqueness of equilibrium may not hold and will in general depend on additional restrictions as shown in Amini et al. (2016). We instead consider the greatest fixed point clearing vector in our equilibrium analysis, which is guaranteed to exist due to the monotonicity of π by Tarski's fixed point theorem (1955), as applied in Cifuentes et al. (2005), Elliott et al. (2014), Gofman (2017), and Amini et al. (2016). We provide an algorithm to calculate the fixed point in Internet Appendix Section D.

Given π, Δ and a total initial collateral value of IM and capital buffers, the total deadweight loss in our system, D, can be defined as

$$D = \Delta(1-\pi)\left[\sum_i \sum_j z_{ij} + \sum_{i \in C} \sum_{k \in M_i} z_{ik0} + \sum_i b_i\right] \tag{B.2}$$

This notion of deadweight loss corresponds to the amount of liquidated collateral in the payment equilibrium and can be seen as a net welfare loss for the system. The other forms of losses are transfers between agents and thus not net welfare losses, whereas the deadweight loss represents money that is taken out of the system. Note that we only consider the reduction in the value of assets that are liquidated and not the reductions for collateral that are not liquidated. This is because we assume in the long run the collateral will recover its value. Such an assumption is standard in the literature, see for instance Allen and Gale (2000) and Acemoglu et al. (2015).

Next, we re-estimate losses after incorporating the influence of collateral illiquidity and fire sales. We employ the Federal Reserve's CCAR scenario once again, which contains estimates of the price impact consequences of the scenario on collateral assets by credit quality.[3] Table B.1 presents the price impacts on corporate lending in advanced economies, which we use as a proxy in our collateral liquidity analysis. Using the formulation of illiquidity given in equation (B.1), we can back out the value of λ that produces an equivalent price impact under an $\alpha = 1$ shock. The price of the collateral used for IM and the capital buffer will decrease as more of the collateral is liquidated.

Figure B.1 plots the size of the systemic loss, L, and deadweight loss, D, under different assumptions of collateral quality through the parameter λ. By comparing these different scenarios we can get a sense of the increase in systemic losses from lower collateral liquidity and derive an estimate of the impact of fire sales on losses. We find that as the average collateral quality decreases, the impact on systemic and deadweight losses sharply increases.

[3]Information is from the Federal Reserve 2015 CCAR Severely Adverse Scenario spreadsheet (https://www.federalreserve.gov/supervisionreg/ccar-2015.htm) on advanced country corporate loans.

Table B.1: Advanced Economy CCAR Shock on Corporate Loans By Quality

Loan Quality	**Value Loss**
AAA	-6.2%
AA	-6.7%
A	-13.4%
BBB	-22.6%
BB	-26.9%

Note: The table presents the decrease in value of Corporate Loans based on the Federal Reserve 2015 Severely Adverse CCAR Shock for advanced economies.
Source: Federal Reserve CCAR Severely Adverse Market Shock.

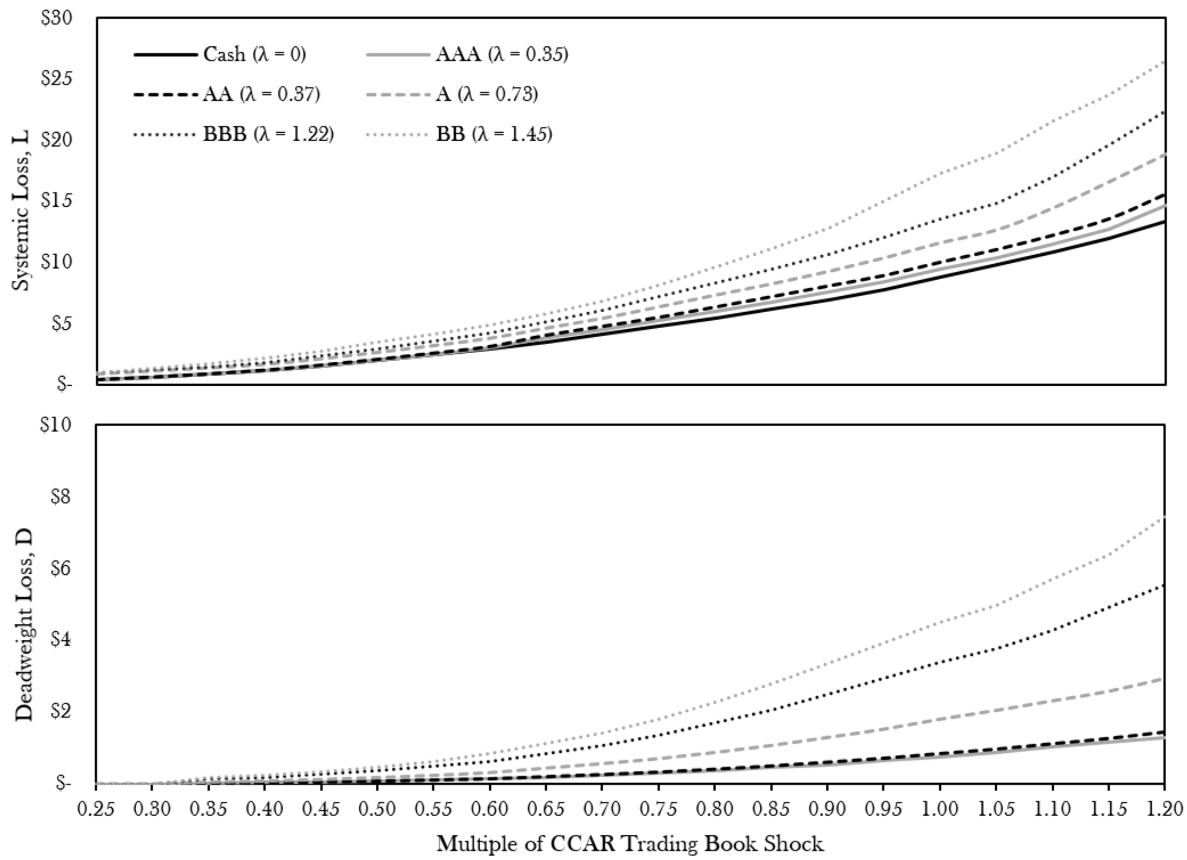
At $\alpha = 1.2$, when collateral quality is set to BBB the systemic losses are 40 percent greater when compared to AAA collateral. The consequences are even more dramatic when comparing deadweight losses, as BBB collateral produces three times the level of deadweight losses. Overall, the comparative results highlight how collateral illiquidity can amplify the losses from stress and speaks to the importance of eligible collateral standards and haircuts.[4] Importantly, CCPs for the most part require highly liquid collateral in practice and high haircuts on lower-grade collateral, which both help to mitigate these losses.

We provide below some empirical statistics on how CCP collateral is held globally. As shown above, the liquidity of the collateral resources held by the CCP can have a large impact on systemic losses. Using cash as collateral helps to reduce systemic losses. However, although intra-firm payments are made in cash, holding IM and guarantee fund collateral in cash alone creates significant costs for clearing members. As a result, other forms of collateral that pay higher interest rates are typically held, or the CCP may rely on credit lines in the case of short-term delays in payments. Table B.2 highlights the percent of collateral and credit lines held by 30 CCPs as of the fourth quarter of 2017.

The data in Table B.2 shows that CCPs tend to hold very high-quality collateral in cash or cash equivalents. Though CCPs may take wider forms of collateral if needed, they generally encourage collateral delivered to be of high quality through applying steep haircuts relative to the general market. Such practices help to mitigate the impact of fire sales.

[4] See the CFTC's Margin Rule.

Figure B.1: Collateral and Financial System Losses



Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario. Each line represents a different level of collateral liquidity, λ, which accounts for the fire sale effects that could occur as firms sell collateral to fulfill payments. At low λ values, i.e. high liquidity, and low shock multiples additional loss consequences are minimal. However, the consequence of high λ values, i.e. low liquidity, or large shock multipliers are significant, as they create greater systemic losses.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

However, high collateral standards have still been insufficient to prevent payment disruptions in the past. For instance, in March 2020 despite high collateral standards several European CCPs had difficulty making their payments promptly, which caused overnight liquidity drains (European Systemic Risk Board (2020)).[5]

[5] In Internet Appendix Section B, we use our model and data to analyze the impact of collateral illiquidity on the effectiveness of default waterfalls.

Table B.2: Pre-funded Resource Collateral and Credit Lines

	Mean	Median	Std Dev	Min	Max
Collateral					
Secured Cash Deposits	44.3	47.5	35.0	-	100.0
Unsecured Cash Deposits	14.6	1.8	31.0	-	100.0
Repo Lent Cash/Securities	10.3	-	21.1	-	81.4
Government Securities	28.2	21.0	30.2	-	99.0
Other	2.6	-	13.5	-	74.2
Unsecured Credit Lines	8.1	-	23.4	-	121.8

Note: The table presents the percentage of collateral and liquidity resources held by 30 OTC derivative CCPs as of the fourth quarter of 2017. The majority of CCP collateral holdings are in cash, repo, or government securities. A small percentage of holdings are in other less liquid assets. Additionally, some CCPs have unsecured credit lines, which we present as a percentage of their total collateral holdings, that they may draw on in times of short-term liquidity impairment.
Source: CCPView Clarus Financial Technology; authors' analysis.

C Proof of Theorems

C.1 Proof of Theorem 1

THEOREM 1: *There exists a unique payment equilibrium, $\Phi(p^*, q^*) = (p^*, q^*)$, for the financial clearing system.*

Proof. **Network with Auxiliary IM Nodes**

For the purposes of the proof we transform our model into a format without IM holdings. Suppose that a firm i has a bilateral liability $\bar{p}_{ij}$ to a firm j with IM z_{ij}. Having IM in the model is equivalent to having instead an auxiliary node i_j, whose capital buffer b_{i_j} is equal to z_{ij}, that intermediates the original payment between firm i and j. Thus, firm i has its obligation to firm j sent to auxiliary node i_j. The auxiliary node receives the payment from node i and has a single obligation to firm j of the same amount $\bar{p}_{ij}$. The payment function of node i_j is denoted as $\tilde{p}_{i_j j} = (\tilde{p}_{ii_j} + z_{ij}) \wedge \bar{p}_{ij}$, and the total amount received by node j for this obligation is denoted as $\tilde{p}_{i_j j}$, which substitutes for $(p_{ij} + z_{ij}) \wedge \bar{p}_{ij}$ in the original network. Also, for a client i that clears through a member k, there is an associated auxiliary node for the client's IM. We denote this node as i_{k0}, and it receives the payment q^c_{ik0} and has the payment obligation $\bar{q}_{ik0}$.

In this way, node i's obligation is rerouted through auxiliary node i_j, which uses its capital buffer to pay any shortfalls from i. IM is similarly replaced for all other bilateral, direct cleared, and client cleared transactions through the use of these auxiliary nodes, with a distinct auxiliary node inserted for each holding of IM. This is straightforward for most cases. The one complication is with the IM held by the CCP for a member k. This auxiliary node, which we denote k_0, combines direct and client clearing payments from member k together. It receives payments from member k that in the original network were $p_{k0} + \sum_{i \in C_k} q^m_{ik0}$, and it has a payment obligation to the CCP of $\bar{p}_{k0} + \sum_{i \in C_k} \bar{q}_{ik0}$.

Stress is defined for all firms similarly to before, except with the IM and payments in the stress equations replaced by the payments going to and from auxiliary nodes. Stress is defined analogously for auxiliary firms as the positive of total obligations minus total payments. For instance, stress for the auxiliary node i_j of a purely bilateral firm i to its counterparty firm j is given by $s_{i_j} = [\bar{p}_{ij} - p_{ii_j} - z_{ij}]^+$. The payment function for the alternate system is $\tilde{\Phi}(p, q)$, defined over the set of firms $\tilde{N}$ which includes the original set of firms N plus the new auxiliary nodes.

Any equilibrium of the original system is thus an equilibrium of this new system, and vice versa. To see this, note for instance that for bilateral transactions each firm j still receives the amount $\tilde{p}_{i_j j} = (p_{ij} + z_{ij}) \wedge \bar{p}_{ij}$ for the obligation from firm i, where p_{ij} is the original equilibrium payment made by firm i to j. The same logic holds for all other transaction types as well. In particular, if there is a unique equilibrium in the alternate system, then there is a unique equilibrium in the original system.

Monotonocity of Payment Functions

We next prove the monotonicity of payment functions under the new system.

Lemma 1. *All payment functions in the original system and the alternate system are weakly monotone in the payments received. Further, if a firm's stress is positive then a strict increase in payments received leads to a strict increase in payments made to all counterparties.*

Proof. This statement is simple to see for all auxiliary nodes from their payment functions.

We thus focus on the remaining firm types in the system.

The payment functions for the firms in our original model are given by $p_{ij} = \bar{p}_{ij} - a_{ij}s_i$ for purely bilateral firm payments, equation (4) for client payments, equation (7) for clearing members, and equation (12) for the CCP. These payment functions remain the same in the network with auxiliary IM nodes, except that the recipients of payments are switched to auxiliary IM nodes as appropriate. Thus, showing that the Lemma holds for these payment functions implies that the Lemma holds for the respective payment functions in the network with auxiliary nodes as well.

Note that the previously defined relative liabilities $a_{ij}, a^c_{ik0}, a_{0ki}$ were constant in payments received for bilateral firms, client firms, and the CCP.[6] For these firms, payment functions are thus weakly monotone in payments received. Further, if stress is positive then a strict increase in payments received strictly decreases stress, which by the payment functions strictly increases payments made to all counterparties.

Finally, with respect to members, we need to consider two cases. The first, identical to those just discussed, is when the higher payment is a bilateral or direct clearing payment. In this case a_{ki}, a^m_{0ki}, and a^m_{ik0} in equation (6) remain unchanged. Thus the payment functions in equation (7) are strictly increasing as stress strictly decreases.

In the second case, the higher payment is from a client clearing payment. We assume that the auxiliary node of client i increases its payment to k for the obligation $\bar{q}_{ik0}$, and the case where the CCP increases its payment to k for the obligation $\bar{q}_{0ki}$ is symmetric. Then equation (6) shows that a_{ki} and a^m_{0ki} strictly increase and a^m_{ik0} strictly decreases. Since s_k strictly decreases, equation (7) implies that q^m_{ik0} strictly increases as payments received increase. It thus remains to check that p_{ki} and q^m_{0ki} also strictly increase from a higher client clearing payment.

Suppose that the increase in the auxiliary node of client i's payment to member k, $\tilde{q}^c_{i_{k0}k0}$, is equal to x. We assume without loss of generality that $x + \tilde{q}^c_{i_{k0}k0} \leq \bar{q}_{ik0}$. Suppose first that

[6] When working with the network with auxiliary nodes, although each relative liability is now to an auxiliary node, the amount of the liability remains the same, i.e. $a_{ii_j} = a_{ij}$.

$s_k > x$. Then the decrease in member k's stress s_k and remaining liability $\bar{p}_k^r$ must also be equal to x. Since both s_k and $\bar{p}_k^r$ are reduced by x, and $s_k < \bar{p}_k^r$, the product $a_{ki}s_k = \bar{p}_{ki}s_k/\bar{p}_k^r$ must be strictly decreasing. Then equation (7) shows p_{ki} is strictly increasing. The other case is if $s_k \leq x$. In this case, s_k is reduced to zero and thus $p_{ki} = \bar{p}_{ki}$, so the payment again increases. A similar argument shows that q_{0ki}^m is also strictly increasing if payments received strictly increase and stress is positive.

□

Since all payment functions are monotonic, the Tarski fixed point theorem can be applied to ensure the existence of a maximal and a minimal fixed point payment vector.

Equity Conservation

We next define the *equity* of each firm, as in papers like Eisenberg and Noe (2001) and Banerjee and Feinstein (2019), as the positive of each firm's total resources minus total payment obligations. For instance, for a purely bilateral firm i, its equity V_i is given by:[7]

$$V_i \equiv [b_i + \sum_{j \in \tilde{N}} \tilde{p}_{ji} - \sum_{j \in \tilde{N}} \bar{p}_{ij}]^+ \tag{C.1}$$

Equity is closely related to the definition of stress. While equity is the positive of total resources minus total obligations, stress is the positive of total obligations minus total resources. Thus, if equity is greater than or equal to zero, then stress is equal to zero and vice versa. Note also that equity is weakly increasing in the payments received by each firm. As such, the equity in the maximum payment equilibrium must be weakly greater than the equity in any other equilibrium.

We can show in the following Lemma that in equilibrium the value of equity at each node in the system is the same in every fixed point payment vector. Furthermore, the sum of all firm equity is equal to the sum of the capital in the network, i.e. the total capital buffer plus total IM plus guarantee fund in the original network, $\sum_{i \in N} \sum_{j \in N} z_{ij} + \sum_{i \in C} \sum_{k \in M_i} z_{ik0} +$

[7]Note that $j \in \tilde{N}$ indicates selection over all nodes in $\tilde{N}$, including auxiliary nodes.

$\sum_{i\in N} b_i + \gamma$.

Lemma 2. *The value of equity at each node in the system is the same in every fixed point payment vector. Furthermore, the sum of all firm equity is equal to the sum of the capital in the network.*

Proof. Let $\tilde{p}$ represent an arbitrary clearing vector and V the associated equity vector. Let $\tilde{p}_i$ denote the sum of payments made by a firm i to all its counterparties in this clearing vector, including bilateral, direct clearing, and client clearing payments. Also, let $\alpha_{ji}(\tilde{p})$ be defined as the total relative payment obligation of firm j to i given $\tilde{p}$. This equals the equilibrium payment made by j to its counterparty i divided by the total equilibrium payment made by j to all its counterparties, and it differs from the previous relative liability definitions as it combines all payment types together. For instance, for bilateral firms this is simply $\alpha_{ji}(\tilde{p}) = a_{ij}$, while for a member k's obligation to the CCP $\alpha_{kk_0}(\tilde{p}) = (\tilde{p}_{kk_0} + \sum_{i\in C_j} \tilde{q}^m_{ikk_0})/\tilde{p}_i$. Then we have

$$\sum_{i\in\tilde{N}} V_i = \gamma + \sum_{i\in\tilde{N}} [b_i + \sum_{j\in\tilde{N}} \alpha_{ij}\tilde{p}_i - \bar{p}_i]^+ \tag{C.2}$$

$$= \gamma + \sum_{i\in\tilde{N}} \left(b_i + \sum_{j\in\tilde{N}} \alpha_{ij}(p)[\bar{p}_j - s_j]^+ - [\bar{p}_i - s_i]^+ \right) \tag{C.3}$$

$$= \gamma + \sum_{i\in\tilde{N}} b_i + \sum_{j\in\tilde{N}} [\bar{p}_j - s_j]^+ \sum_{i\in\tilde{N}} \alpha_{ij}(p) - \sum_{j\in\tilde{N}} [\bar{p}_i - s_i]^+ \tag{C.4}$$

$$= \gamma + \sum_{i\in\tilde{N}} b_i = \gamma + \sum_{i\in N}\sum_{k\in N} z_{ik} + \sum_{i\in C}\sum_{k\in M_i} z_{ik0} + \sum_{i\in N} b_i \tag{C.5}$$

The sum of total equity is thus the same in any equilibrium and equal to the sum of capital. In addition, the equity of each firm in the maximal payment equilibrium is weakly greater than the equity of that firm in any other equilibrium. Thus, the equity of each firm must be the same in every equilibrium, or otherwise, the maximal payment equilibrium would have a greater sum of total equity.

□

Uniqueness

The proof of uniqueness for the financial system's equilibrium payment vector follows closely from Theorem 2 of Eisenberg and Noe (2001). Consider the risk orbit for a firm i as the set of firms reachable from i along a directed path of payment obligations. These payment obligations can be of any type, including bilateral, direct clearing, and client clearing. Since each firm has a positive capital buffer by assumption, there must be a firm with positive equity in every firm's risk orbit in the system. The proof of this follows along identically as in Lemma 1 and Lemma 2 of Eisenberg and Noe (2001).

We next proceed with proof by contradiction. We suppose that there is a firm i that has two different equilibrium payments in the maximal and minimal payment equilibria, $\{p^+_{ij}\}_{j\in\tilde{N}}$ and $\{p^-_{ij}\}_{j\in\tilde{N}}$, with $p^+_i > p^-_i$. This firm must have nonpositive equity in equilibrium or else it would make the full payment in both equilibria. There must also be a firm x in i's risk orbit such that the equilibrium equity of firm x is positive. Let the directed path from firm i to firm x be given by $i = i^0, i^1, i^2, ..., i^n = x$, and suppose without loss of generality that each firm in this path is distinct and each firm except x has nonpositive equity in both equilibria.

Next, proceed by induction along this path. By assumption firm i^0 is making more payments to its counterparty i^1. Assume that firm i^y, where $y \in \{0, 1, ..., n\}$, is making strictly more payments in the maximal equilibrium than in the minimal equilibrium to firm i^{y+1}, $p^+_{i^y i^{y+1}} > p^-_{i^y i^{y+1}}$. We show that firm i^{y+1} must also be making strictly more payments to firm i^{y+2} in the maximal equilibrium than in the minimal equilibrium.

Note that by monotonicity, i^{y+1} must be receiving weakly more payments from every other firm in the maximal payment equilibrium. Thus firm i^{y+1} is receiving weakly more payments from firms other than i^y, and receiving strictly more payments from i^y. This implies that the stress of firm i^{y+1} must be positive in the minimal equilibrium or else i^{y+1} would have positive equity in the maximal equilibrium, a contradiction. Thus firm i^{y+1} must also make strictly more payments to all its counterparties by Lemma 1, and in particular, it makes strictly more payments to firm i^{y+2}. This establishes the induction argument.

This implies that the final firm x must also be receiving more payments in the maximal equilibrium, and so its equity is strictly higher. This is a contradiction since every firm with positive equity must have the same equity in all equilibrium by Lemma 1. This argument has shown that a contradiction must arise if the equilibrium clearing equity vector is not unique. Therefore the equilibrium payments are unique. □

C.2 Proof of Theorem 2

THEOREM 2: *Given a shock and a fixed level of CCP pre-funded resources, CCP stress and systemic losses weakly decrease as any member's contribution of mutualized funds relative to their contribution of segregated funds is increased.*

Proof. Here we will prove that raising the mutualized contribution (i.e. guarantee fund) for any member i and lowering its segregated contribution (i.e. IM) by an equal amount weakly decreases CCP stress and systemic losses in the payment equilibrium (assuming all other default waterfall contributions of other members are held fixed). Consider the payment equilibrium under the original default waterfall. There are two cases: first, if the CCP's stress as defined by equation (10) is equal to zero, and second if the CCP's stress is positive.

In the first case, note that the change from IM to GF of a member i will cause the CCP's stress in equation (10) to remain at zero. As a result the change in waterfall structure has no impact on the CCP's solvency and thus no impact on the payments made by any firms in the maximal payment equilibrium. Thus all payment-related losses are unchanged. The only change in losses must be directly associated with the waterfall. Hence there are two possibilities. In the first case member i defaults and its IM and guarantee fund contributions are insufficient to cover its obligations to the CCP. The result is all IM and guarantee fund contributed by the member is used in the early stages of the waterfall, and so the transfer of IM to guarantee fund for this member has no influence on the resources used by member i, nor on the waterfall resources used by any other member. Thus waterfall losses are unchanged.

Now suppose member i either does not default or that if it does default, its IM and guarantee fund are sufficient to cover its obligations. If the CCP could cover all its losses with earlier waterfall resources without using any of the pooled guarantee fund in the original equilibrium, changing member i's contribution from IM to the guarantee fund has no impact. However, if the CCP could not cover all its losses without using the mutualized guarantee fund contributions, then changing member i's contribution will increase the waterfall losses of member i. Simultaneously, this would reduce the waterfall losses of other non-defaulting members who have their guarantee fund used. Thus the aggregate waterfall losses are unchanged.

Suppose the second case where the CCP stress is positive in the payment equilibrium. Payments in the equilibrium may be affected by the shift from IM to the guarantee fund. Assume first that member i does not default in the original payment equilibrium. Then the CCP would have the additional guarantee fund contribution from member i to pay its obligations, and its stress according to equation (10) would be strictly reduced. In this case, the payments made by the CCP strictly increase, and the payments made in the rest of the system weakly increase in the new equilibrium due to monotonicity as shown in Lemma 1. Thus the losses in the new equilibrium are strictly lower. On the other hand if member i defaults and uses all of its IM and guarantee fund, then the change in IM to guarantee fund has no impact on the equilibrium. As a result, the CCP stress according to equation (10) and the losses in the financial system will be the same as in the prior case. □

C.3 Proof of Theorem 3

THEOREM 3: *Given a fixed market shock, if the client clearing obligation of any client i of a member k is converted to a direct clearing obligation, then*

a. *the CCP requires weakly more funded resources to maintain solvency,*

b. *though if the CCP is solvent in both settings, systemic losses are weakly lower in the direct clearing network.*

Proof. Let us consider the auxiliary IM node model defined in Theorem 1's proof, which is isomorphic to the original model and whose systemic losses are identical when the losses of the auxiliary nodes are not included. Let $\eta = 0$ denote the original network where i is still a client for member k and $\eta = 1$ denote the network where i directly clears with the CCP. Let the amount of funded resources held by the CCP be denoted as R. For simplicity, we will assume that R is composed entirely of b_0 and that $\gamma = 0$. The exact breakdown of R between b_0 and γ is irrelevant for this theorem as only the sum affects the CCP's solvency and changing the proportion of b_0 and γ only results in a transfer of losses between the CCP and its members without affecting aggregate systemic losses.

First note that if a fixed market shock does not cause the CCP to default given a funded resource level R', then increasing its funded resources does not change the amount of payments it makes in equilibrium. Thus, the equilibrium payments of the CCP are the same at all funded resource levels $R > R'$. This observation means that for each η we can find the equilibrium payments $p(\eta), q(\eta)$ given an R large enough that the CCP does not default. We can then determine the minimum level of funded resources the CCP needs to maintain solvency by analyzing the CCP's payments received in equilibrium. We denote the minimum level of funded resources as

$$R_0(\eta) \equiv \left[\sum_{k\in M}\left(\bar{p}_{0k} + \sum_{i\in C_k}\bar{q}_{0ki} - (p_{kk_0} + \sum_{i\in C_k} q^m_{ikk_0} + z_{k0}) \wedge (\bar{p}_{k0} + \sum_{i\in C_k}\bar{q}_{ik0})\right)\right]^+ \tag{C.6}$$

If we assume that the CCP does not default, then if the CCP owes VM to the client i of member k the payment is made in full, and so the presence of the member's guarantee makes no difference to the equilibrium outcome. Thus the CCP receives the same equilibrium payments in $\eta = 0$ and $\eta = 1$, and therefore the minimum level of resources the CCP requires to not default is the same in both. We thus analyze the other case in which client i owes the VM payment to the CCP.

Next, consider the obligation structure of $\eta = 1$, and the sequence of payment vectors

$\{(p^n, q^n)\}_{n=0}^{\infty}$. The initial payment vector (p^0, q^0) is equal to the equilibrium payments in $\eta = 0$ for all firms other than member k, while k makes its payments after re-balancing its obligations by removing the client clearing obligation of i to the CCP. Thus, the payments made by member k in (p^0, q^0) are weakly larger than its payments in the equilibrium of $\eta = 0$ to all its counterparties (other than k_0, its auxiliary IM node to the CCP).

Let the equity vector given (p^0, q^0) be denoted V^0. Note that V^0 is greater than the V in the $\eta = 0$ equilibrium for every firm other than the CCP and k_0 since total payments made to all such firms are weakly greater in (p^0, q^0). For auxiliary node k_0, equity in V^0 is also higher since the client clearing payment obligation for client i is no longer present, and it receives weakly more payments from member k for every other obligation to the CCP.

Then if we define the sequence of payment vectors $\{(p^n, q^n)\}_{n=1}^{\infty}$ inductively as $(p^n, q^n) = \Phi(p^{n-1}, q^{n-1})$, and the sequence of equity vectors $\{V^n\}_{n=0}^{\infty}$ as the corresponding equity sequence, the CCP will have sufficient R at every step of the sequence to meet its full payment obligations. $\{(p^n, q^n)\}_{n=0}^{\infty}$ is a weakly increasing sequence due to the monotonicity of the payment functions shown in Lemma 1. $\{V^n\}_{n=0}^{\infty}$ is thus also a weakly increasing sequence, and it converges to the equilibrium equity vector of $\eta = 1$. This argument shows that equilibrium equity in $\eta = 1$ is weakly greater than the equilibrium equity in $\eta = 0$ for every firm other than the CCP.

Now suppose that CCP's equity is strictly greater in the equilibrium of $\eta = 1$. The sum of equity overall firms in the network is then strictly greater in $\eta = 1$ than in $\eta = 0$, which violates the equity conservation that was shown to hold in Lemma 2 in the proof of Theorem 1. Note that changing client clearing obligations to direct clearing obligations does not impact the initial equity in the system, so it does not affect the sum of equity in equilibrium either.

Thus, the CCP's equity must be weakly less in the equilibrium of $\eta = 1$. If the equity of the CCP is weakly lower in the equilibrium of $\eta = 1$, then it is receiving weakly fewer payments in equilibrium. Thus the minimum CCP resources required for solvency in $\eta = 1$

are weakly greater, i.e. $R_0(1) \geq R_0(0)$, which proves the statement in part (a).

Finally, let us consider systemic losses if the CCP does not default in either system, i.e. CCP resources are greater than $R_0(1)$. As all firms other than the CCP receive weakly more payments in the $\eta = 1$ equilibrium, their losses must be weakly lower than in $\eta = 0$. Meanwhile, the CCP receives weakly less payments, and thus it has weakly more losses. If the CCP has the same losses, then the result holds. If instead we assume that the CCP's losses increase by an amount $w > 0$, then member k should have a loss that is w greater in $\eta = 0$ than in $\eta = 1$. This is because the payment made to member k from the auxiliary node of client i in $\eta = 0$ is weakly lower than the payment made to the CCP by the auxiliary node of i in $\eta = 1$. Thus, systemic loss in $\eta = 1$ is weakly lower than in $\eta = 0$, which proves the statement in part (b).

□

D Fictitious Stress Algorithm

Here we describe the fictitious stress algorithm used to find the clearing vectors for p and q. The essence of the algorithm is simple. First, determine each market participant's payout, assuming that all other market participants satisfy their obligations. The iterative algorithm starts with the assumption that no market participants are under stress. If this is a feasible outcome, then it is the outcome of the clearing equilibrium. If, however, some market participants are stressed, then we update the payment vector given the stress and check for additional stress. The algorithm terminates when the level of additional stress added to the financial system is below a given tolerance threshold.

I At step m of the algorithm, let Λ_m be the set of stressed market participants. Denote p_h and q_h as the payment vector for firm h. Initialize $\Lambda_0 = \{\}$ and $p_h = \bar{p}_h, q_h = \bar{q}_h$.

II Compute the stress s_h at each firm h given the payment vectors. Recalculate Λ_m as the set of all market participants such that entry h of the stress vector is positive, i.e.

$s_h > 0$.

III Terminate if there are no stressed firms, or if the increase in total stress $S_m \equiv \sum_h s_h$ is lower than a minimum threshold, $S_m - S_{m-1} < \epsilon$.

IV Otherwise determine the clearing payments by recalculating the maximal values of $p_h, q_h \ \forall h \in \Lambda_m$. Iterate $m \to m+1$ and repeat starting at II.

Following Theorem 3.1 from Rogers and Veraart (2013), the stress algorithm above produces a well-defined sequence of payment vectors p_h, q_h which reaches the clearing vector of Φ. Similar algorithms have been used to find the clearing vector(s) in Blume et al. (2011) and Elliott et al. (2014).

The algorithm can be modified to incorporate collateral illiquidity. A few conditions are added to the algorithm to update collateral prices based on how much collateral is liquidated in the prior iteration of the algorithm. The updated algorithm with illiquidity, which we apply in Internet Appendix Section B, is:

I At step m of the algorithm, let Λ_m be the set of stressed market participants. Denote p_h and q_h as the payment vector for firm h. Let Δ_m be the total value of collateral liquidated. Initialize $\Lambda_0 = \{\}$, $p_0 = \bar{p}, q_0 = \bar{q}$, and $\Delta_m = 0$.

II Compute the stress s_h at each firm h given the payment vectors and the collateral liquidated. Recalculate Λ_m as the set of all h market participants such that entry h of the stress vector is positive, i.e. $s_h > 0$.

III Terminate if there are no stressed firms, or if the increase in total stress $S_m \equiv \sum_h s_h$ is lower than a minimum threshold, $S_m - S_{m-1} < \epsilon$.

IV Otherwise, determine the remaining clearing payments and the collateral liquidated by recalculating the maximal values of $p_h, q_h \ \forall h \in \Lambda_m$ given a value of collateral liquidated Δ_{m-1}. Then compute the corresponding total liquidations Δ_m from these new payments. Iterate $m \to m+1$ and repeat starting at II.

E Types of Losses

This appendix provides a detailed description of the different types of losses in the model. Losses in our model represent shortfalls suffered by an institution as a result of a variation margin shock. For firms other than the CCP, three types of loss can occur in the model: bilateral losses, client clearing losses, and default waterfall losses. A bilateral loss arises when a firm does not receive all the payments owed by another firm in a bilateral transaction. A client clearing loss arises when a client or clearing member does not receive the resources owed in a client clearing transaction. A default waterfall loss arises when a clearing member has its guarantee fund contribution used to cover another clearing member's shortfall to the CCP. We define each type of loss precisely below.

The first type of loss comes from bilateral transactions. For a firm i, these losses are given by

$$\text{Bilateral Loss} = \sum_{j \neq i} \left[\bar{p}_{ji} - (p_{ji} + z_{ji})\right]^{+}. \tag{E.1}$$

Bilateral losses can accrue to clearing members, clients, and bilateral firms. For CCP members, we also include direct clearing losses from the CCP in this category.

The second type of loss comes from client clearing transactions. Such losses are experienced by clients that have shortfalls in funds owed to them by the CCP. Clearing members also experience client clearing losses when they do not receive the entirety of the funds that they are liable to pass through for the obligation. Client clearing losses are given by the following set of equations

$$\text{Client Clearing Losses} = \begin{cases} \sum_{k \in M_i} \left(\bar{q}_{0ki} - q^{m}_{0ki}\right) & \forall i \in C, \\ \sum_{i \in C_k} \left(\left[\bar{q}_{ik0} - (q^{c}_{ik0} + z_{ik0})\right]^{+} + \bar{q}_{0ki} - q_{0ki}\right) & \forall k \in M, \end{cases} \tag{E.2}$$

where the first equation is the losses for clients and the second equation is the losses for clearing members.

Finally, the third type of loss comes from non-defaulting clearing members that have their guarantee fund contributions used by the CCP to cover another clearing member's default. We denote the default waterfall loss for a firm k as $\hat{\gamma}_k$.[8] To calculate $\hat{\gamma}_k$, recall that we consider only funds used by a clearing member to cover obligations of *other* clearing members to be a loss. Guarantee funds that are used to cover a clearing member's default are thus not counted in losses, but guarantee funds used to cover another clearing member's default are counted in losses.[9]

Recall that the total guarantee fund contributions of a clearing member k are denoted γ_k. We denote the amount of guarantee fund used to cover a member's *own obligations* as $\dot{\gamma}_k$. This guarantee fund is taken from clearing member k when clearing member k is short payments to the CCP and owes more than its IM z_{k0} can cover. The value of $\dot{\gamma}_k$ is given by

$$\text{(E.3)} \qquad \dot{\gamma}_k \equiv \min\left(\left[\bar{p}_{k0} + \sum_{i \in C_k} \bar{q}_{ik0} - \left(p_{k0} + \sum_{i \in C_k} q^m_{ik0} + z_{k0} \right) \right]^+, \gamma_k \right), \quad \forall k \in M.$$

Suppose clearing member k does not use all of its contributions γ_k, and losses to the CCP from other firms go beyond the resources of the CCP's capital contribution b_0. Clearing member k would have a part, if not all, of its guarantee fund contribution used to cover the obligations of other clearing members. Guarantee fund contributions will be taken pro rata the remaining contributions of each clearing member to cover the CCP's shortfall.

Let the additional amount that the CCP needs to cover if it uses the preceding waterfall layers be denoted as g_0:

$$\text{(E.4)} \quad g_0 \equiv \left[\sum_{k \in M} \left(\bar{p}_{0k} + \sum_{i \in C_k} \bar{q}_{0ki} - (p_{k0} + \sum_{i \in C_k} q^m_{ik0} + z_{k0}) \wedge (\bar{p}_{k0} + \sum_{i \in C_k} \bar{q}_{ik0}) - \dot{\gamma}_k \right) - b_0 \right]^+ .$$

[8]In Appendix A.4 we consider default waterfall losses due to the use of assessments and IMH at the end of the waterfall.

[9]Note that our notion of losses is not directly equivalent to our previous notion of stresses. While the CCP does not transmit stress to its clearing members until the default waterfall is fully depleted, clearing members will suffer default waterfall losses as soon as their guarantee fund is used to cover other clearing member obligations.

As a function of g_0, the guarantee fund losses for clearing member k, $\hat{\gamma}_k$, is given by

$$\text{(E.5)} \qquad \text{Default Waterfall Loss } \hat{\gamma}_k = \min\left(\frac{\gamma_k - \dot{\gamma}_k}{\sum_{j \in M} \gamma_j - \dot{\gamma}_j} g_0, \gamma_k - \dot{\gamma}_k\right), \quad \forall k \in M.$$

F Estimating the CDS Margins and Buffers

This appendix explains how the estimation of variation margin, initial margin, and capital buffers from the supervisory data is performed. The DTCC data reports the positions on all standardized and confirmed CDS involving U.S. entities. Positions represent swap transactions with comparable risk characteristics between counterparties. They include detailed information about underlying reference entities, notional amount bought and sold, inception and termination dates, and other terms of contracts.

F.1 Variation Margin

Variation margin (VM) payments are cash transfers made by a firm to its counterparties to account for changes in the value of the CDS contracts. These variation margin payments are made daily. From the protection seller's perspective, a CDS derives positive value from premia received until the contract terminates or the underlying reference entity defaults (whichever comes first); in the latter case, the seller's contract value is reduced by the expected protection payment. The sources of value are switched from the standpoint of the protection purchaser: at contract inception, the present value of premia paid is balanced by the expected value of default payments. The value of the contract varies with market credit spreads through their concurrent impact on the present value of premia receipts and the expected value of default payments. These changes in value trigger VM payments that make the future expected payments equal to zero for both parties.

Consider a contract x that is established between counterparties i and j at time t, on a set of reference entity characteristics r_x and a notional amount of protection N_x. Through the use of a bootstrapping procedure to value CDS contracts using the term structure of credit spreads at t, we are able to estimate the net present value of the contract (Luo (2005)). The change in value of contract k between successive periods t and $t+1$ determines the variation margin $\mathrm{VM}_{ij}(N_x, r_x, t, t+1)$ payable on the xth contract. The sum of changes across all contracts between i and j is the bilateral variation margin

$$\mathrm{VM}_{ij}(t, t+1) = \sum_x \mathrm{VM}_{ij}(N_x, r_x, t, t+1). \tag{F.1}$$

The term of each CDS contract come from DTCC, data on credit spreads (which accounts for documentation clause, seniority, maturity, and currencey) come from Markit, and the discount rates come from Bloomberg. The details of these calculations are described in Appendix A of Paddrik et al. (2020).

F.2 Initial Margin

The initial margin (IM) collected from counterparties are held in segregated accounts and can only be used to cover losses induced by a given counterparty's failure to pay. IM is typically held in cash, or cash equivalents, and assets that can be liquidated on short notice but not necessarily at full value. We address initial margin liquidity concerns in Appendix B.

To determine the amount of IM posted we adopt a historical portfolio-at-risk measure approach with respect to the margin period of risk (MPOR) presented in Duffie et al. (2015) (see equation (F.2)) and validated in Capponi et al. (2022). For each pair of firms i and j, the DTCC data report the portfolio of CDS contracts for which i and j were the counterparties on the date of the shock. Using Markit data we can infer the price changes, and hence the VM that would have been exchanged between i and j if they had held this same portfolio over the prior 1,000 days. We then find the amount c_{ij} with the maximum shortfall (MS) over 1,000 (net amount of VM that i owed j was less than c_{ij}) with respect to the MPOR of the portfolio. The MPOR horizon is 10-days for bilateral positions and 5-days for cleared positions.

$$\text{IM} = \text{MS}(\text{VM}_{ij}(t, t + \text{MPOR}) + 0.02\text{AS}(N_x, t) \tag{F.2}$$

Additionally, as IM for CDS is asymmetric due to jump-to-default risk, the Duffie et al. (2015) approach applies an additional add-on of two percent of gross outstanding short notional (AS) held in CDS portfolios. We calculate this level for the CCP, and we additionally scale up the estimates by a common factor so that the total IM collected corresponds to the CCP's total reported IM at the end of 2014 (ICE (2016)).

The difference between the Duffie et al. (2015) IM method and VaR is the increase in margin held due to the aggregate short position requirements. VaR similarly is a maximum shortfall calculation, which by itself is symmetric between counterparties, however, the addition of the aggregate short position introduces asymmetry. This aggregate short requirement is thus well-suited for the CDS market, as it captures the jump-to-default risk associated with the contract.

F.3 Capital Buffers

The IM collected by firm i from its counterparties is dedicated to covering shortfalls in payments to i *from* its counterparties; it cannot be accessed to meet i's obligations *to* others. To cover its own obligations the firm maintains a capital buffer b_i, which includes cash or cash-equivalents and short-term lines of credit. These buffers are not part of the DTCC data, nor are they available from public data sources. Instead, we estimate their magnitude by considering how much cash a prudently managed firm would need to manage its *net* VM obligations. These numbers can be estimated from the weekly inflows and outflows of VM at the firm level, which is derived from DTCC data as described above.

Fix a firm i and let $X_i(t)$ denote the total net VM payment that i owes to all of its counterparties on a given day t. If $X_i(t) > 0$ then i owes more than it is owed; the reverse holds if $X_i(t) < 0$. As we noted earlier, prior VM payments make future expected net payments equal to zero for both counterparties, hence $\mathrm{E}[X_i(t)] = 0$. Let $N_i(t)$ be the gross notional value of i's CDS contracts at time t and let $\tilde{X}_i(t) = X_i(t)/N_i(t)$. The *volatility* of i's CDS portfolio over a given period $[0, T]$ is

$$\sigma_i = \sqrt{(1/T)\sum_{t=1}^{T}(\tilde{X}_i(t))^2} \tag{F.3}$$

G Evaluating Initial Margin

Initial margin (IM) provides the first line of defense against the default of a counterparty, and it plays a critical role in keeping a counterparty's default losses minimized. In this appendix, we perform a series of robustness tests on the total losses suffered by firms and the CCP's default waterfall resilience level under differing assumptions. The primary purpose is to understand how consequential these assumptions are on the results presented in the paper.

G.1 Comparing Initial Margin Models

While the MS Plus Short margin framework is used through out the main body of paper, given its closest resemblance to CDS margin model practices, there are other margin models that could considered. In this subsection, we extend our examination of IM models from Section VII.B. We compare the two models considered in the text, MS Plus Short and CoMargin, against the traditional Value-at-Risk (VaR) model. Unlike the other two methods, VaR is a symmetric method in which both counterparties contribute the same amount of IM for the contract. As such, it lacks the jump-to-default asymmetry that characterizes MS Plus Short. Though VaR is not typically used for CDS markets, the method provides us a benchmark to assess the importance of the jump-to-default asymmetry for our results.

We estimate VaR by applying a similar method as described in Internet Appendix Section F.2 for MS Plus Short. For each pair of firms i and j, the DTCC data reports the portfolio of CDS contracts for which i and j were the counterparties on the date of the stress test. Using Markit data we infer the price changes, and hence the VM that would have been exchanged between i and j if they had held this same portfolio over the last 1,000 days, holding all contract variables constant. We then find the amount c_{ij} at a 99.5 percent level (i.e., the net amount of VM that i owed j was less than c_{ij} on the fifth-largest VM date) with respect to the 10-day MPOR of the portfolio.

As in the main body of the paper, we continue to use CoMargin as a point of comparison. We provide more details about the estimation of CoMargin here. CoMargin is introduced in Cruz Lopez et al. (2017) and accounts for the interdependencies in tail risks between firms. It is defined as the variation margin VaR of a clearing member or client's portfolio conditional on one or several members being in financial distress. We empirically estimate CoMargin by running a Conditional VaR for each member and client on the two largest members' simulated IM portfolios. To apply this methodology, we find the two largest CCP members by estimating the VaR margins of all members and selecting the two largest. Then, an estimation of the variation margin payments of each account portfolio is conditioned on the sample of dates in which the two largest members suffered their collective largest losses (at a 1 percent level). The CoMargin quantity for each account is determined by taking the greatest VM payment from the conditioned sample dates or the unconditional VaR (α=0.01) of the account, whichever is larger. We note that this empirical method is not identical to the simulation approach proposed in Cruz Lopez et al. (2017), though this empirical method is consistent with how margins estimates are calculated at CCPs.

For both VaR and CoMargin, we scale IM contributions to equalize the total amount of IM collected across all members and clients in the system. We present in Table G.1 the distributional statistics for each IM model held by the CCP, assuming the total margin is held constant but redistributed under the margin models. Additionally, we test the implication of the margin models on payments in the cleared and bilateral market, and we evaluate systemic losses and Guarantee Fund Usage given different shock sizes. In Table G.1 we present our results under three stresses (α's): 0.5x, 1x, and 2x the 2015 CCAR Global Market stress test.

Table G.1: Initial Margin Model Distribution and Loss Effects

	Initial Margin Models								
	VaR			MS Plus Short			CoMargin		
Mean ($M):	38.96			38.96			38.96		
Median ($M):	5.00			4.19			4.25		
Std Dev ($M):	152.44			173.24			194.36		
CCAR Stress (α)	0.5	1	2	0.5	1	2	0.5	1	2
Systemic Loss ($B):	0.92	10.11	47.40	2.34	12.04	47.81	0.95	9.30	43.97
Guarantee Fund Usage (%):	0.39	59.92	100	0.48	39.84	100	1.36	37.44	100
Guarantee Fund Exhausted (α)	1.09			1.13			1.18		

Note: The table presents three initial margin setting models: standard Value-at-Risk (VaR), a CDS margin model which accounts for jump-to-default risk (MS Plus SHort), and CoMargin. For each model, we apply a normalized allocation percentage across an equal quantity of historical initial margin. The tables present the mean, median, and standard deviation of the initial margin held by each cleared account. For each margin model, we provide the estimated systemic loss (in $ billions) under three CCAR stress levels (α = 0.5, 1, and 2). Additionally, the percentage of guarantee fund resources used is provided to reflect on the capacity of the CCP to cover payments in full using collected resources. Finally, the α level at which guarantee fund resources are completely exhausted.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

When we examine the IM distributional characteristics of the three models, we find that MS Plus Short and CoMargin models have a larger positive skew (heavy tail), highlighted by the shift down of the median and greater standard deviation relative to the VaR distribution. When we compare the IM account similarity for the methods, we find the correlation between the MS Plus Short and VaR IM distribution is very high at 98.5 percent. This is likely due to the MS Plus Short method being a simple add-on to VaR. In contrast, the correlation between the CoMargin and VaR accounts is only 94 percent.

Next, examining the stress test results, we find the standard VaR method results in lower systemic losses than MS Plus Short. However, the amount of guarantee fund usage is significantly higher under VaR. This suggests that VaR may reduce losses across firms relative to MS Plus Short, but it also places higher stress on the CCP. When compared against the stress test results of the CoMargin model, we find that CoMargin dominates VaR and MS Plus Short in CCP resilience and systemic loss. This result suggests that CoMargin is generally more effective than both VaR and MS Plus Short at stabilizing the CCP and reducing systemic losses.[10]

G.2 Excess Client Margin Requirements

While client cleared portfolio margins are both held and determined by the CCP, clearing members are ultimately responsible for covering the client's default losses and assuming the client's portfolio. As a result, members may choose to collect additional IM above the CCP's requirements, particularly as members can be asked by the CCP to make variation margin payments within an hour while clients typically use t+1 payment cycles. In this subsection, we test the implication of excess client clearing margins held by members. We evaluate how these margins influence member losses and systemic losses. Finally, we compare how guarantee fund usage is affected by different excess margin levels.

We consider two scenarios beyond the base scenario of no excess margin, one where the excess margin is 10 percent and a second with 20 percent, which we compare to the base scenario. In Table G.2 we present our results under three stresses (α's): 0.5x, 1x, and 2x the 2015 CCAR Global Market stress test.

As expected, higher excess margin levels reduce member losses, systemic losses, and guarantee fund usage. However, the decreases in each case are not linear to the amount of additional margin collected, and the impact of the margin varies with the size of the shock. Under an $\alpha = 1$, a 10 percent increase (+\$1.09 billion) in margin results in lower member losses of \$0.4 billion, systemic losses of \$0.5 billion, and guarantee fund usage of 15 percent. However, a 20 percent increase (+\$2.17 billion) has almost no marginal impact on member

[10] Note that these analyses do not take into account any endogenous actions by firms that may occur if there were a switch to traditional VaR or CoMargin methods. As described in Cruz Lopez et al. (2017), a switch to CoMargin could result in incentives to merge positions among members or to move contracts off the CCP. Such actions could alter the effectiveness of CoMargin relative to the other methods.

Table G.2: Excess Client Margin Impact

	Excess Margin								
	0% (+$0)			10% (+$1.09B)			20% (+$2.17B)		
CCAR Stress (α)	0.5	1	2	0.5	1	2	0.5	1	2
Member Loss ($B):	1.96	8.77	30.49	1.96	8.49	30.13	1.96	8.48	29.76
Systemic Loss ($B):	2.34	12.04	47.81	2.34	11.77	47.02	2.34	11.76	46.23
Guarantee Fund Usage (%):	0.48	39.84	100	0.28	24.86	100	0.12	24.22	100

Note: The table presents three scenarios based on the level of additional margining members place on clients outside of CCP's required initial margin: a base of no excess initial margin is collected (0%), and two where 10 or 20 percent additional margin is collected. Given the additional client initial margin level, the table provides the estimated member and systemic losses (in $ billions) under three CCAR stress levels ($\alpha = 0.5$, 1, and 2). Additionally, the percentage of guarantee fund resources used is provided to reflect on the capacity of the CCP to cover payments in full using collected resources.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

losses or systemic losses compared to the 10 percent increase. In contrast, under extreme stress ($\alpha = 2$), there is a far greater benefit to a 20 percent excess margin as systemic losses reduce by almost the same amount as the increase in margin collected. This highlights the nonlinear benefits that excess margin provides as a function of the shock size, and raises an important (but difficult) question of how to estimate the likelihood of extreme events, as discussed in Paddrik and Young (2021).

G.3 Margin Rehypothecation Restrictions

IM is generally made of liquid assets, such as sovereign debt and currencies, to assure that in the event of default the margin can assuredly cover the payments within the day. In the case of securities used as collateral, there are likely some limitations to how quickly this can occur due to rehypothecation restrictions placed on how the margin is delivered to the CCP. As a result, some percentage of the margin may be inaccessible within the payment default day, though the holder of the margin (i.e. the CCP or a settlement bank) will eventually be able to liquidate the collateral or provide it to the new owner to liquidate themselves.

In this subsection, we provide further testing on the impact of the rehypothecation restrictions discussed in Section V.D. We evaluate how limitations on IM use affect systemic losses. We consider two scenarios, one in which 10 percent of margin is inaccessible and a second with 20 percent inaccessible, and we compare to the base scenario with no restrictions. In Table G.3 we present our results under three stresses (α's): 0.5x, 1x, and 2x the 2015 CCAR Global Market stress test.

Table G.3: Rehypothecation Restrictions Impact on Systemic Losses

	Percentage of Margin Inaccessible								
	0% ($0)			10% ($2.03B)			20% ($4.06B)		
CCAR Stress (α)	0.5	1	2	0.5	1	2	0.5	1	2
Systemic Loss ($B):	2.34	12.04	47.81	2.45	12.72	49.45	2.57	13.48	51.06
Guarantee Fund Usage (%):	0.48	39.84	100	0.68	59.30	100	0.87	82.73	100

Note: The table presents three scenarios based on the potential limits of initial margin rehypothecation: a base of full rehypothecation (0%), and two where 10 or 20 percent of margin cannot be used to cover an outgoing payment. Given the rehypothecation level, the table provides the estimated systemic loss (in $ billions) under three CCAR stress levels ($\alpha = 0.5$, 1, and 2). Additionally, the percentage of guarantee fund resources used is provided to reflect on the capacity of the CCP to cover payments in full using collected resources.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

The results show that a greater percentage of margin inaccessible will increase systemic losses and guarantee fund usage. However, at lower shock sizes of $\alpha = 0.5$ or 1, the increase in inaccessible margin does not translate into a one-to-one increase in systemic losses. For instance, at a 10 percent margin inaccessible level under $\alpha = 1$, the systemic losses increase by $1 billion. The increase in systemic losses is limited because many firms are not defaulting at the lower shock levels. The passthrough is much greater at the 2x CCAR stress. Under $\alpha = 2$, systemic losses increase by almost $2 billion. Since many more firms are defaulting, not being able to access IM translates directly into losses for the firms' counterparties. These results highlight the increasing risks posed by rehypothecation restrictions at high shock levels.

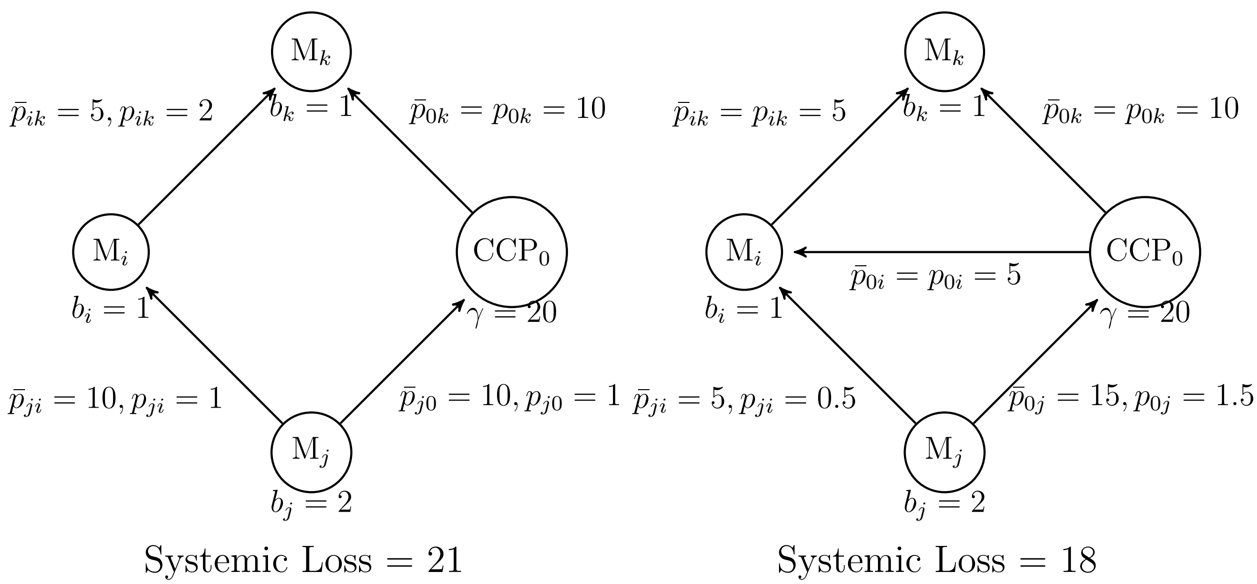
H Central Clearing Participation and Systemic Losses

The social planner must consider the trade-off between greater default waterfall resources and lower central clearing rates. Requiring more waterfall resources from members places additional costs on them, which can lead them to reduce their central clearing participation.

This decrease in central clearing participation will in turn impact systemic losses. In this appendix, we use the theoretical model to investigate the potential impacts of central clearing participation on systemic losses. We do so by providing two sets of examples to illustrate results.

First, consider the two payment networks in Figure H.1. The two networks differ in the degree of central clearing and the value of systemic losses in equilibrium. In examples (a) and (b) each members' total exposure remains fixed while a portion of member i's bilateral exposure in (a) is converted into a central clearing exposure in (b). The increase in central clearing in (b) results in a decrease in systemic losses, as the CCP has enough guarantee fund to cover M_j's payment shortfall, allowing it to fulfill the central clearing obligations to member M_i and reduce the payment shortfalls and losses in the rest of the network.

Figure H.1: Lower Losses with More Central Clearing



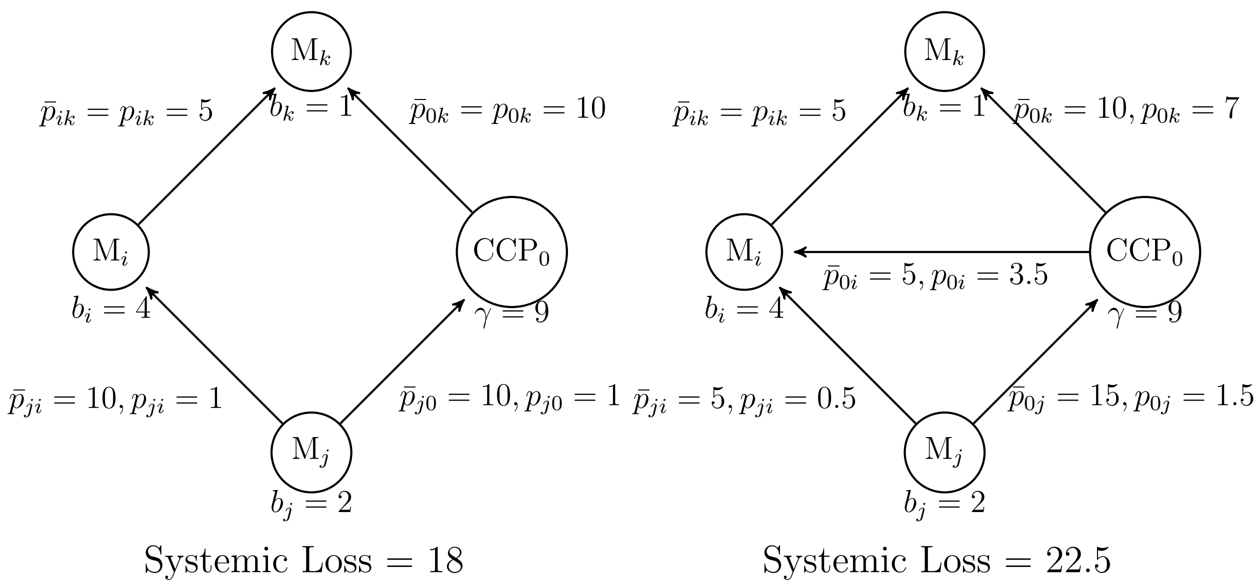
(a) Less Clearing **(b)** More Clearing

Note: The figure depicts two example payment networks with their equilibrium payments and resulting systemic loss. The difference between (a) and (b) lies in the proportion of member j's cleared payments. The increase in central clearing leads to a decrease in systemic loss, as the CCP's guarantee fund allows it to cover member j's payment default, which limits the payment shortfalls and losses in the rest of the network. *Source*: Authors' creation.

In the second example, seen in Figure H.2, we modify the first set of payment networks slightly by changing member i's capital buffer, b_i, and the CCP guarantee fund, γ. The

CCP's guarantee fund is now insufficient and it is thus unable to cover the default of M_j. The CCP fails to make full payments to members M_i and M_k, resulting in greater losses in the network. This example shows that increases in the rate of central clearing can increase systemic losses if the CCP's default waterfall resources are insufficient.

Figure H.2: Higher Losses with More Central Clearing



(a) Less Clearing **(b)** More Clearing

Note: The figure depicts two example payment networks with their equilibrium payments and resulting systemic loss. The difference between (a) and (b) lies in the proportion of member j's cleared payments. The increase in central clearing leads to an increase in systemic loss, as the CCP's guarantee fund is insufficient to cover member j's payment default. This leads to greater payment shortfalls and losses throughout the rest of the network.
Source: Authors' creation.

These two sets of examples highlight the role that central clearing structure and waterfall resources play in systemic losses. In general, greater central clearing participation means that members owed VM payments by the CCP will receive greater payments in equilibrium if the CCP is solvent. If members that are owed VM payments by the CCP represent a significant contagion risk to the network, then increased central clearing participation is helpful. However, a weakly funded default waterfall can reverse the benefits of increased central clearing and intensify payment shortfalls throughout the network.

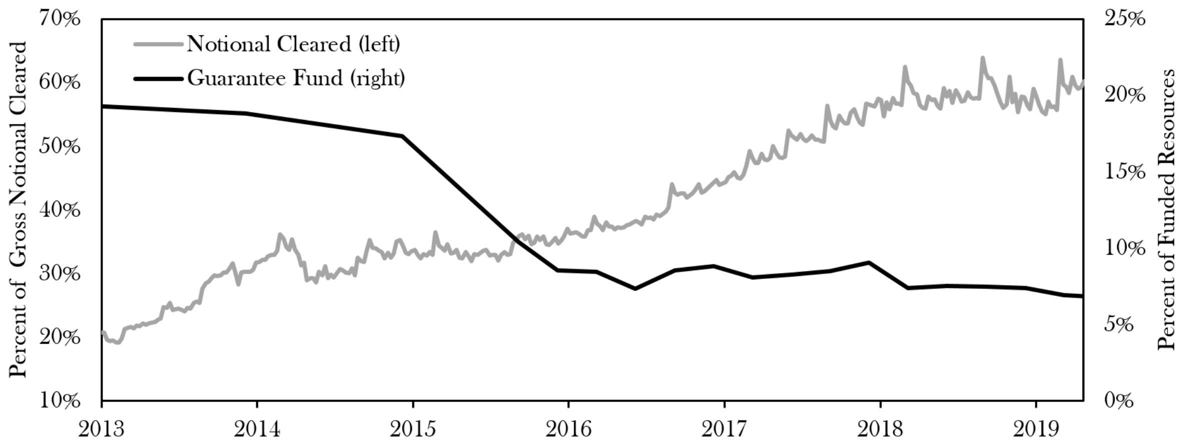
I Impact of Central Clearing Participation

While our analysis shows that systemic losses are lower with greater levels of overall waterfall resources and a greater ratio of mutualized waterfall resources, such requirements also impose additional capital costs on clearing members. If central clearing becomes too costly, then clearing members may pursue bilateral positions, or potentially worse, not hedge their risk exposures. In Internet Appendix Section H we show that the impact of central clearing participation on systemic losses is indeterminate. In this appendix section, we tackle these issues empirically and assess the impact of changes in central clearing participation on systemic losses. We perform a series of robustness tests whereby we shift the central clearing rates of members up or down for a fixed network structure within our empirical setting and then calculate the impact on the system.

As Figure I.3 highlights, the U.S. CDS market has witnessed substantial changes in waterfall resources and central clearing rates since the central clearing mandate took effect in early 2013. As the percentage of cleared positions increased, the percentage of the CCP's mutualized resources (i.e. guarantee funds) simultaneously declined. While potentially coincidental, the data suggest that greater acceptance of central clearing may have required lowering the proportion of guarantee fund resources to incentivize participation. The large magnitudes of these shifts make it difficult to determine whether potential systemic losses have increased or decreased over time.

Before comparing the impact of variations in central clearing rates, let us first consider the two extremes of the default waterfall structure to bound the systemic loss estimates. In the first scenario, assume that there are no resources available in the guarantee fund, whereas in the second scenario assume that there are infinite resources available in the guarantee fund. Comparing these two scenarios in Figure I.4 gives us a measure of the maximum reduction in systemic losses that guarantee fund resources can provide. As the figure highlights, once IM held by the CCP is depleted under scenario one, the difference in losses (gray wedge) with scenario two grows rapidly. The difference in losses at the original CCAR shock level

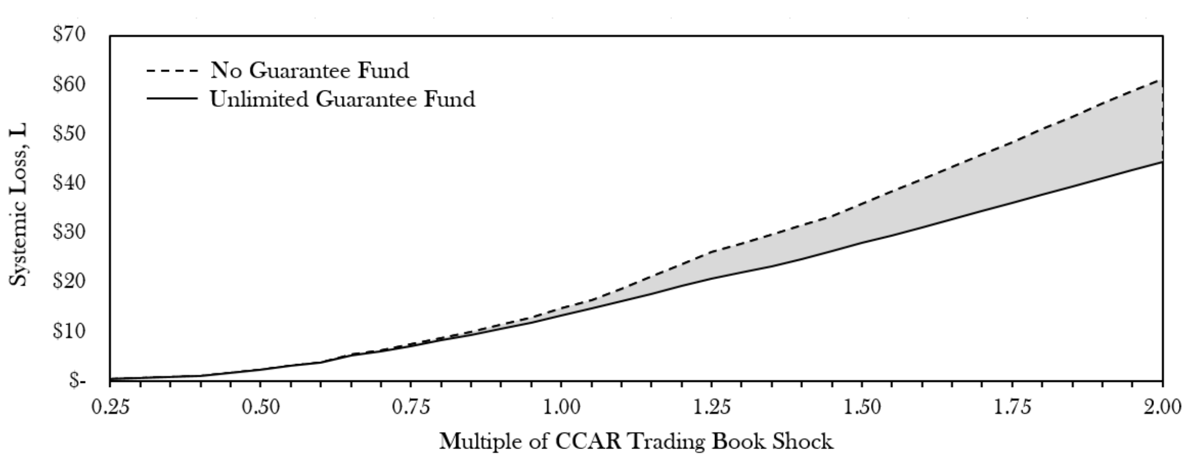
Figure I.3: Central Clearing Rates and Guarantee Fund over Time



Note: The figure plots the percent of gross notional positions cleared and the amount of waterfall resources that can be mutualized.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation; CCPView Clarus Financial Technology; SEC EDGAR 10-K Filing.

is 11 percent, but the difference grows to over 50 percent if the shock level α is doubled.

Figure I.4: CCP Resilience and Systemic Losses



Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario. The solid line represents the systemic loss under the scenario where guarantee funds are unlimited. The dotted line represents the systemic loss under the scenario where guarantee fund is set to zero. The gray area represents the difference in systemic losses suffered under different degrees of risk sharing (through the sizing of the guarantee fund). The additional losses show how consequential the CCP's full payment continuity is to the entire financial system of payments.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

Now let us weigh the tradeoff in setting default waterfall resources versus concerns of central clearing participation. As our framework does not endogenize member responses to changes in the default waterfall structure[11], we consider the upper and lower bounds of the responses by exogenously varying the rates of central clearing participation and measuring the impact on systemic losses at the extremes of default waterfall structure. Two perturbations on the rate of central clearing are performed, one increasing (blue) and one decreasing (red) clearing by 50 percent for members, as shown in Figure I.5. We increase (decrease) the relative size of payments and initial margins between members, and we shift the corresponding payments and margin from (to) the CCP.[12]

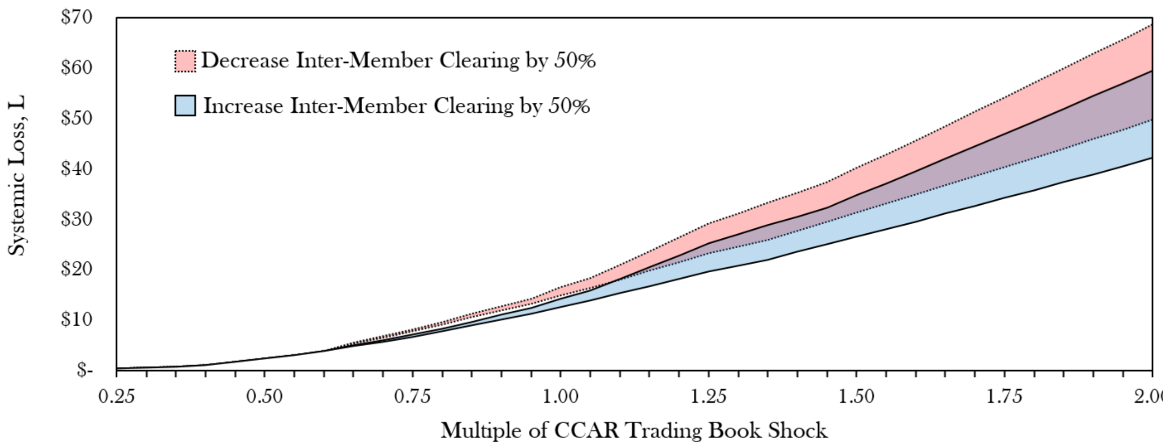
The values of these perturbations are chosen for two reasons. First, given the date of the empirical exercise in late 2014, the adjusted notional amount centrally cleared becomes 20 and 51 percent, respectively. These levels correspond to those in 2013, when central clearing was first mandated for the CDS market in the U.S., and to the higher levels seen more recently, as depicted in Figure I.3. These scenarios thus provide some guidance as to what we could expect if the same stress scenario were run in either period. Second, the choice of CDS clearing is quite limited in practice. As of 2020, just under 20 percent of outstanding notional CDS is eligible to clear at a CCP but not mandated by regulation to be cleared. Thus, while it may be difficult to predict what members will choose to do in response to waterfall adjustments, the bounds of the responses likely fall within the two scenarios.

Figure I.5 shows that at low α levels the increased central clearing wedge is below the reduced central clearing wedge irrespective of the default waterfall structure. When $\alpha < 1.05$, the benefits of the higher clearing rate thus dominate the impact of the lower waterfall resources due to its netting benefits (Duffie and Zhu (2011); Cont and Kokholm (2014)). However, under more extreme conditions where $\alpha \geq 1.05$, the presence of default waterfall resources more intensely impacts losses. A 50 percent lower central clearing rate with a strong default waterfall results in fewer losses than a 50 percent greater central clearing rate

[11]See for instance Wang et al. (2022).

[12]For more details on how this shifting of payments and margins is performed, see Internet Appendix Section I. In the theoretical model, changes in the participation rate can increase or decrease systemic losses. Internet Appendix I provides examples of both cases.

Figure I.5: Central Clearing Rate and Systemic Losses



Note: The figure plots the aggregate amount of systemic losses (in $ billions) under multiples of the 2015 CCAR severely adverse global shock scenario, shaded by two regions representing the difference in systemic losses suffered under differing degrees of risk sharing (through the sizing of the guarantee fund). The two regions represent what happens if there was a hypothetical increase (blue) or decrease (red) in the rate of central clearing positions. The figure highlights that the rate of central clearing is an important determinant in the size of the systemic loss the financial system suffers irrespective of the default waterfall. However, the strength of the waterfall plays a more significant role the higher the rate of central clearing is, as depicted by the difference in the width of the blue region versus the red region.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

with a weak default waterfall.

To protect against severe shocks it is critical for the CCP to enforce a sufficient guarantee fund, even if it lowers the rates of central clearing. As Figure I.5 suggests, the recent trends towards higher central clearing rates but lower guarantee fund levels in the CDS market are likely to decrease systemic losses against small shocks, but may ultimately worsen the consequences under more extreme shocks.

J Netting and Allocating Payments

In this appendix, we cover the payment netting and allocation frictions discussed in Section V.D in greater detail. Specifically, we examine how firm netting and payment allocation rules throughout the course of a day can affect whether a firm will have sufficient assets on hand in order to fulfill its obligations. The main model of the paper uses a classical set of payment assumptions, in line with Eisenberg and Noe (2001), where it is assumed firms make payments (a) simultaneously, (b) using a single form of currency/asset ($ dollars), and (c) using a *pro rata* like rule in cases where a full payment cannot be made. In this appendix, we will test how these assumptions affect systemic losses and the ability of the CCP to cover its obligations.

First, let us consider the implications of assumptions (a) and (b). Together they imply that payments can be netted completely, as payment delays between firms or differences in payment forms do not exist. While payments are typically netted through clearing banks and central depositories to reduce frictions, e.g. State Street or the Depository Trust Company, full netting is nearly impossible for large portfolios. For example, if we look at currency variation among notional CDS contracts included in this study, 70-80 percent of notional outstanding is made in dollars for both the cleared and bilateral markets. The remaining currency is predominately made in Euros (19-28%) or Yen (1%).

To explore the consequences of netting inefficiency, let us introduce a netting factor, ξ, into the system of payments as follows,

$$s_i = \left[\sum_{k \neq i,0} \bar{p}_{ik} - \left(\sum_{k \neq i,0} \left((p_{ki} + z_{ki}) \wedge \bar{p}_{ki} \right) \xi + b_i \right) \right]^+ , \tag{J.1}$$

where ξ impacts the efficiency of incoming payments to net against outgoing payments. The result is 100%-ξ of payments *drag* in the financial system, such that capital buffers are necessary for firms to fulfill their outgoing payments obligations in a timely fashion. In Table J.1 we examine the implication of varying ξ, using the variation we observe in payment currencies as a test of the payment drag friction. We present its impact under three stresses (α's): 0.5x, 1x, and 2x the 2015 CCAR Global Market stress test.

Table J.1 highlights how impactful payment netting frictions can be on systemic losses, particularly on the part of the CCP who manages a large number of payments. Comparing

Table J.1: Payment Netting: Losses by Type

	Degree of Netting								
	ξ=100%			ξ=80%			ξ=70%		
CCAR Stress (α)	0.5	1	2	0.5	1	2	0.5	1	2
Bilateral Loss ($B):	0.12	1.06	4.26	0.20	1.43	4.93	0.24	1.66	5.32
Client Loss ($B):	0.26	2.21	13.05	0.42	3.56	16.04	0.51	4.68	19.49
Member Loss ($B):	1.96	8.77	30.49	2.78	10.82	33.18	3.33	11.85	36.07
Guarantee Fund Usage (%):	0.48	39.84	100	48.98	100	100	73.23	100	100

Note: The table presents three payment netting scenarios based on the level $\xi = 100$, 80, and 70 percent. Given the netting level, the table provides the estimated group level losses for bilateral, client, and member firms (in $ billions) under three CCAR stress levels ($\alpha = 0.5$, 1, and 2). Additionally, the percentage of guarantee fund resources used is provided to reflect on the capacity of the CCP to cover payments in full using collected resources.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

the no friction scenario ($\xi = 100\%$) to the one in which 30 percent of payments cannot be netted ($\xi = 70\%$), we find that under the minor stress of $\alpha = 0.5$ losses nearly double and the CCP's usage of the default waterfall goes from near zero to 73 percent. Under the original CCAR stress level of $\alpha = 1$, member losses increase by over $3 billion, and client losses double, while CCP default waterfall usage increases from 40 to 100 percent. The results help explain why CCPs encourage payments be made using a uniform currency or collateral type, and why the timeliness of payments has such a strong repercussion.

Next, we examine assumption (c) on how payments are allocated concerning default. The main model uses a payment rule wherein defaulting firms pro rata their payments to their counterparties, and the counterparties then use IM to fulfill any shortfalls. We denote this rule as *Pro Rata before IM.* While this *pro rata* assumption is typical for assessing the implications of bankruptcy, other allocation methods are also possible. For example, given that IM held can be used to cover obligations, IM could be deducted from the VM payments before the pro rata allocation is applied in the event of default. We denote this rule as *Pro Rata after IM.* While this is not entirely realistic, as IM is meant to cover additional expenses, e.g. contract replacement costs, this method does help assess how non-expended IM may be redistributed. Additionally, this rule could reduce systemic losses by further diversifying the loss across a wider array of firms.

We define the payments made by bilateral firms under this new payment allocation rule. For a bilateral firm, recall that the stress is given by

$$s_i = \left[\sum_{k\neq i,0} \bar{p}_{ik} - \left(\sum_{k\neq i,0} ((p_{ki} + z_{ki}) \wedge \bar{p}_{ki}) + b_i\right)\right]^+ . \tag{J.2}$$

We define the *remaining payment obligation* from firm i to j as $\tilde{p}_{ij} = [\bar{p}_{ij} - z_{ij}]^+$. The total remaining payment obligation across all counterparties of i is given by $\tilde{p}_i \equiv \sum_{k\neq i,0} \tilde{p}_{ik}$. Then, the relative liability of node i to node j is given by $a_{ij} = \tilde{p}_{ij}/\tilde{p}_i$. The final payment made by firm i to firm j is then given by $p_{ij} = \tilde{p}_{ij} - a_{ij}s_i$. Payment obligations for other types of firms in our system can be defined analogously.

In section V.D, we also discussed a payment allocation rule in which no payments are made by the firm in the event of any degree of default. We denote this rule as *Binary Payment*, as either the full payment is made or no payment is made. Such a payment rule is reasonable in practice, as bankruptcy would put a defaulting firm into receivership. This allocation rule is likely to create additional short-term systemic losses that under a longer-term horizon may be reduced after the receivership process is completed. Depending on the horizon of loss considerations, this may be more or less important.

This allocation rule can be simply stated. Let the stress for each firm i, s_i, be defined as in the main text. Then the payment of firm i to a firm j is equal to

$$p_{ij} = \begin{cases} \overline{p}_{ij} & if \ \mathrm{s}_i \leq 0, \\ 0 & if \ \mathrm{s}_i > 0. \end{cases} \tag{J.3}$$

In Table J.2 we examine the implication of varying the allocation rule and testing their implications on loss and default waterfall resources. We present its impact under three stresses (α's): 0.5x, 1x, and 2x the 2015 CCAR Global Market stress test.

Table J.2 highlights the impact of payment allocations on systemic losses. Comparing the allocation scenario Pro Rata before IM to Pro Rata after IM, we find that the losses among

Table J.2: Payment Allocation Rule: Losses by Type

	Payment Rule								
	Pro Rata before IM			Pro Rata after IM			Binary Payment		
CCAR Stress (α)	0.5	1	2	0.5	1	2	0.5	1	2
Bilateral Loss ($B):	0.12	1.06	4.26	0.01	0.07	3.72	0.60	3.06	6.28
Client Loss ($B):	0.26	2.21	13.05	0.07	0.19	15.04	1.02	5.93	16.09
Member Loss ($B):	1.96	8.77	30.49	1.10	3.12	29.01	4.14	14.57	36.79
Guarantee Fund Usage (%):	0.48	39.84	100	0.00	5.69	100	2.18	51.36	100

Note: The table presents three payment allocation scenarios based on default: pro rata payments before initial margin (Pro Rata before IM), pro rata payments discounting for initial margin (Pro Rata after IM), and no payment by defaulting firms (Binary Payment). Given the allocation rule, the table provides the estimated group level losses for bilateral, client, and member firms (in $ billions) under three CCAR stress levels ($\alpha = 0.5$, 1, and 2). Additionally, the percentage of guarantee fund resources used is provided to reflect on the capacity of the CCP to cover payments in full using collected resources.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

the firm types and the guarantee fund usage decline as the initial margins are more efficiently reallocating available cash. While this is particularly effective at reducing losses broadly at lower α's (0.5, 1), once the stress is extremely large (2) its effects are less impactful and lead to a redistribution of losses from bilateral and members to clients when compared to the Pro Rata before IM rule.

When comparing to the Binary Payment rule, not surprisingly we find that the losses and guarantee fund usage increases across all firm types and stress levels than both of the other rules. Similarly, we find that the consequences are more noticeable at the lower levels of stress, leading to a doubling of overall losses at α's of 0.5 and 1 when compared to the Pro Rata before IM rule.

K Payment Seniority

During periods of financial distress, firms may need to prioritize their payments. The decision to prioritize certain payment obligations comes in two general forms: (a) legal-based, where the payment seniority decisions are defined contractually or through regulation; and (b) market power-based, where the payment seniority decisions are based on preferences in the pecuniary benefits or services that counterparties offer. In this appendix, we focus on legal-based seniority, as the interpretation is relatively straightforward and bears strong similarities to a CCP's default waterfall. In contrast, market power-based seniority involves complex game-theoretic considerations that make it a topic deserving of its own paper.

The most topical concern surrounds the prioritization of central clearing obligations. In many jurisdictions, central counterparty obligations are legal stay remote, and they are thus *de facto* senior to bilateral obligations (Acharya and Bisin (2014)). The consequences of seniority on systemic losses are not completely clear or straightforward given the financial system of payments. Here we address the empirical consequences of the legal seniority of cleared vs. bilateral obligations on systemic loss and CCP resilience under our setting. We follow along with the previous theoretical work in examining this issue (see Amini and Minca (2020)).

While the previously stated seniority structure is generally true across several jurisdictions, the circumstances surrounding a firm's default and the implementation of default payments are not likely to be so straightforward in practice. However, for the tractability of our analysis, we assume that all firms follow this ordinal payment rule and prioritize CCP payments over other types of payments.

We present the new payment functions for a member firm i to its counterparties in equation (K.1). We assume for simplicity of exposition that this member is not engaged in any client clearing transactions.[13]

$$p_{ij} = \begin{cases} \left[\bar{p}_{ij} - (\bar{p}_{ij}/(\bar{p}_i - \bar{p}_{i0}))s_i\right]^+ & \forall j \neq 0, \\ \bar{p}_{i0} \wedge [\bar{p}_i - s_i]^+ & j = 0. \end{cases} \tag{K.1}$$

[13] For a member that engages in client clearing transactions, the payment equations are defined analogously with the additional client clearing terms. Importantly, we assume that the member never takes any of the payments that the CCP makes to a client. Thus, the member's pass-through obligations defined for client clearing transactions take precedence even over the member's obligation to the CCP. However, other obligations that the member owes to the client, such as bilateral obligations, will be secondary to any obligations that the member owes to the CCP.

In Table K.1 we examine the implication of the new seniority rule on systemic loss and default waterfall utilization. We present its impact under four stresses (α's): 0.5x, 1x, 1.25x, and 1.5x the 2015 CCAR Global Market stress test.

Table K.1: Seniority in Centrality Cleared and Bilateral Payments

	No Seniority				Seniority			
CCAR Stress (α)	0.5	1	1.25	1.5	0.5	1	1.25	1.5
Bilateral Loss ($B):	0.12	1.06	1.73	2.47	0.10	1.08	1.71	2.52
Client Loss ($B):	0.26	2.21	4.11	6.39	0.18	2.23	3.38	5.67
Member Loss ($B):	1.96	8.77	13.90	18.66	1.92	8.33	13.15	17.50
Guarantee Fund Usage (%):	0.48	39.84	100	100	2.66	27.33	96.28	100

Note: The table presents a comparison of how central clearing payment seniority impacts systemic losses. Given whether or not payment seniority exists, the table provides the estimated group level losses for bilateral, client, and member firms (in $ billions) under four CCAR stress levels (α = 0.5, 1, 1.25, and 1.5). Additionally, the percentage of guarantee fund resources used is provided to reflect the capacity of the CCP to cover its payments using waterfall resources.
Source: Authors' calculations using data provided to the OFR by the Depository Trust & Clearing Corporation and Markit Group Ltd.

Comparing the seniority effect under the four stress scenarios, we find some variation in losses among the firm types and guarantee fund usage. The total size and distribution of losses remain generally similar, but we note three differences across the two modeled payment settings. First, as expected, payment seniority is beneficial for the CCP's resilience. Comparing the guarantee fund usage of the two settings, we find that the CCP can withstand a greater stress event (~20% larger) before completely exhausting its pre-funded guarantee funds.

Second, market participants who centrally clear, i.e. members and clients, suffer reduced total losses as a group. This outcome leads broadly to reduced overall systemic losses in the financial system. Finally, the impact on bilateral firms as a group is mixed at low α stresses. However, at larger values of α, bilateral firms do suffer greater losses than previously. Additionally, compared to the other firm types, bilateral firms suffer proportionally more losses.

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