Multidimensional party polarization in Europe: Cross-cutting divides and effective dimensionality

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– SUPPLEMENTARY MATERIALS –

A Chapel Hill Expert Survey

We rely on the 1999-2019 Chapel Hill Expert Survey trend file (Jolly et al., 2022) throughout our article. This covers all of the EU member states and all parties that were represented in parliament and/or passed a 2-percent threshold in the popular vote. The number of experts per country-year-party ranges from 2 to 27 with an average of 11.6.

Three survey items are relevant to the analyses reported in this paper:

1. **LRGEN:** Position of party in [year] in terms of its overall ideological stance. The response scale runs from 0 ("extreme left") to 10 ("extreme right").

2. **LRECON:** Position of the party in [year] in terms of its ideological stance on economic issues. Parties can be classified in terms of their stance on economic issues such as privatization, taxes, regulation, government spending, and the welfare state. Parties on the economic left want government to play an active role in the economy. Parties on the economic right want a reduced role for government. The response scale runs from 0 ("extreme left") to 10 ("extreme right").

3. **GALTAN:** Position of the party in [year] in terms of their views on social and cultural values. "Libertarian" or "postmaterialist" parties favor

expanded personal freedoms, for example, abortion rights, divorce, and same-sex marriage. "Traditional" or "authoritarian" parties reject these ideas in favor of order, tradition, and stability, believing that the government should be a firm moral authority on social and cultural issues. The response scale runs from 0 ("libertarian/postmaterialist") to 10 ("traditional/authoritarian").

We use LRGEN as a measure of general left-right ideology, whereas LRECON serves as a measure of the economic dimension, and GALTAN as a measure of the cultural dimensions.

B The Mathematics of Effective Dimensionality

This section provides additional information on the computation of the effective number of dimensions.

Consider the correlation matrix $\boldsymbol{R}$. First, we compute the eigenvalues over this matrix. Those eigenvalues add up to D, the number of spatial dimensions under consideration. If the dimensions are uncorrelated, each has an associated eigenvalue of 1. If the dimensions all correlate perfectly, then the first eigenvalue is D and the remaining ones are all 0. For those familiar with principal component analysis, this means that a single component perfectly accounts for the correlations.

Next, the eigenvalues are normalized by dividing them by their sum. Thus, the ith normalized eigenvalue is

$$\lambda_i^* = \frac{\lambda_i}{\sum_{j=1}^{D} \lambda_j} = \frac{\lambda_i}{D}, \tag{4}$$

where λ_i is the original eigenvalue and $i = 1, \cdots, D$.

In a third step, we convert the combined normalized eigenvalues into Shannon's entropy:

$$H(\lambda) = -\sum_{i=1}^{D} \lambda_i^* \ln[\lambda_i^*] \tag{5}$$

Here we treat the normalized eigenvalues as probabilities. The purpose of this step is to assess the information contained in the correlation matrix. If the dimensions are perfectly correlated, then $H(\lambda) \to 0$. This is the lowest conceivable level of entropy; there is no uncertainty. If the dimensions are uncorrelated, then $H = \ln[D]$, which is the highest possible level of entropy.

We see that Shannon's entropy is bounded between 0 and $\ln[D]$. We simply need to exponentiate H to obtain boundaries between 1 and D. Hence,

$$ED = \exp[H] = \prod_{i=1}^{D} (\lambda_i^*)^{-\lambda_i^*} \tag{6}$$

In our article, we focus on the two dimensional case. Let the correlation matrix between the two dimensions be given by

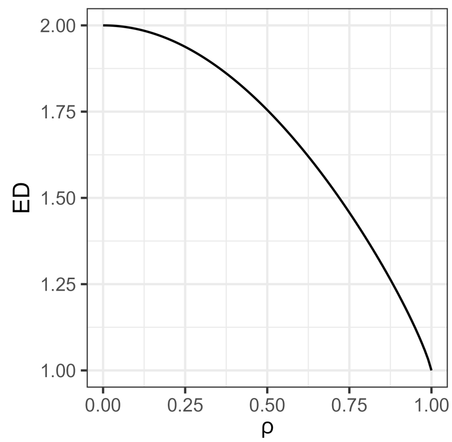
$$\boldsymbol{R} = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

Here, ρ is the correlation between the two dimensions. The eigenvalues of this matrix are $1+\rho$ and $1-\rho$, respectively. In this case, the exponentiated Shannon entropy is

$$ED = \left(\frac{1-\rho}{2}\right)^{\frac{\rho-1}{2}} \left(\frac{1+\rho}{2}\right)^{\frac{-\rho-1}{2}}$$

Here effective dimensionality is a nonlinear function of the correlation, as is illustrated in Figure B1.

Figure B1: The Relationship Between the Effective Number of Dimensions and Dimensional Correlation in Two Dimensions



C More on Measurement Validation

This section contains additional information relevant for the section on measurement validation in the main article.

In Table C1, the dependent variables are the difference between the standardized two- and one-dimensional polarization measures. In column 1, the dependent variable is the raw difference between the two, while it is the absolute difference as a measure of similarity in column 2. The two explanatory variables are effective dimensionality and the (absolute) difference between the mean of the weighted variances along the two dimensions compared to the weighted variance along the general left-right dimension. In general, the two-dimensional polarization measure is larger (column 1) and more dissimilar (column 2) than the one-dimensional measure when effective dimensionality increases and when the mean variance along the two dimensions is larger than along the left-right dimension.

Table C2 is the regression table of Figure 6b in the main article. All variables are standardized to a standard deviation of 1 and a mean of 0 prior to

	Model 1	Model 2
Intercept	-0.00	0.45^{***}
	(0.03)	(0.09)
Eff. Dim.	0.59^{***}	0.27^{***}
	(0.02)	(0.06)
Diff. in Var.	0.89^{***}	
	(0.02)	
Abs. Diff. in Var.		0.58^{***}
		(0.08)
AIC	11.13	235.12
BIC	25.58	249.58
Log Likelihood	-0.57	-112.56
Num. obs.	133	133
Num. groups: country	24	24
Var: country (Intercept)	0.02	0.09
Var: Residual	0.04	0.25

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table C1: Differences in Polarization Measures

estimation, so that coefficient sizes reflect changes in standard deviations. In the first column, the dependent variable is one-dimensional polarization, while it is two-dimensional polarization in the second column. As highlighted in the main article, the estimated coefficients of the two independent variables are very similar. Additionally, aspects such as information criteria are very similar across the two models.

	Model 1	Model 2
Intercept	-0.02	-0.01
	(0.14)	(0.15)
Proportionality	0.24^{*}	0.15
	(0.12)	(0.12)
Eff. No. of Parties	0.18	0.31^{**}
	(0.11)	(0.11)
AIC	365.00	358.61
BIC	379.46	373.06
Log Likelihood	-177.50	-174.31
Num. obs.	133	133
Num. groups: country	24	24
Var: country (Intercept)	0.35	0.47
Var: Residual	0.64	0.58

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table C2: Polarization and System-level Features

D Exploring three-dimensionality

To illustrate the possibility of extending our approach to any multidimensional setting, we explore a three-dimensional space below. Specifically, in addition to an economic and a cultural dimension, we may include the issue of European integration as a third divide (see Bakker, Jolly and Polk, 2012). In line with Figures 2 and 5 in the main text, Figures D1 and D2 show the over time variation in effective dimensionality and multidimensional polarization by country.

It is interesting to note that no space is truly three-dimensional in the same way that we find effective two-dimensionality (see Figure 2). This suggests that, while the European issue may have some cross-cutting potential, the different divides tend to correlate to some degree. Indeed, the correlation between effective dimensionality in a two- vs. three-dimensional space is considerable, at 0.67. This correlation is even higher for the polarization measures ($r = 0.88$). As before, we do see a great degree of variation in polarization scores, both within and between countries. Future research may want to unpack these differences.

Figure D1: Effective dimensionality by country (incl. EU); parties weighted by vote share

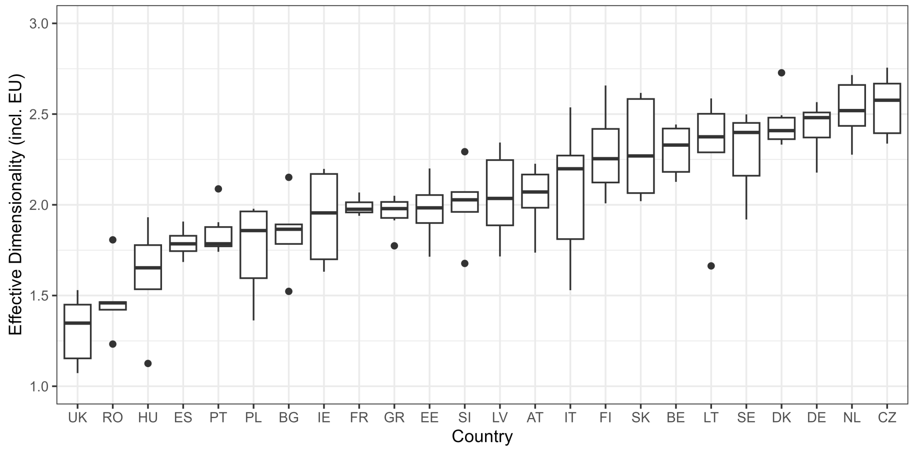
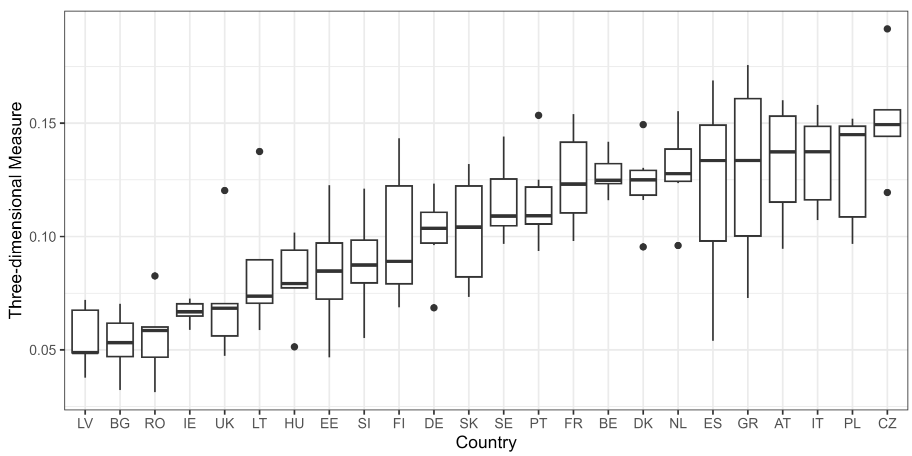


Figure D2: Three-dimensional party polarization by country (parties weighted by vote share)



E More on Application

This section contains additional information relevant for the section on the empirical application in the main article.

Table E1 contains the main regression results of this section. In models 1–2, system-level independent variables are correlated with individuals' propensity to indicate being partisans. We use one-dimensional polarization in the first column and two-dimensional polarization in the second column. Additional individual-level covariates–such as age, gender, education, income, trade union membership, and left-right self-placement–are included in models 3–4. The main coefficients of interesting relating polarization to partisanship are robust to these additions. In models 5–6, we only include observations for which the time between the election referenced by CSES and the survey wave of CHES is less than two years. Then, we include polarization^2 in models 7–8 to account for a potential non-linear association between these variables. Finally, the conditional association of the electoral strength of second dimension parties on the relationship between two-dimensional polarization and partisanship is shown in the interaction coefficient in model 9. This coefficient is also positive if the average weighted salience of the second dimension in a party system (rather than the electoral strength of these parties) is used as a moderating variable (models 10–11). While the number of available cases drops substantially because this data is only available for 16 of the 73 country-year observations in the CHES data, the positive association holds both if we use the raw salience measure of the second dimension (model 10) or if we use a relative salience measure of the second dimension (subtracting the salience of the economic dimension from that of the GALTAN dimension; model 11). Fundamentally, the documented association between the spatial polarization of parties and mass-level partisanship is similar across all specifications: more spatial polarization of parties is associated with an increase in the probability of individuals' partisanship.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11
1D Polar.	0.09***		0.05**		0.14***		0.07***				
	(0.01)		(0.02)		(0.02)		(0.02)				
2D Polar.		0.15***		0.16***		0.10***		0.15***	0.15***	−0.01	0.10***
		(0.01)		(0.02)		(0.02)		(0.02)	(0.02)	(0.03)	(0.03)
1D Polar.^{2}							−0.07***				
							(0.01)				
2D Polar.^{2}								0.01			
								(0.01)			
2D Polar.*2nd Dim. Parties									0.06***		
									(0.01)		
2D Polar.*2nd Dim. Salience										0.33***	
										(0.05)	
2D Polar.*2nd Dim. Salience (rel.)											0.69***
											(0.06)
2nd Dim. Parties									−0.04***		
									(0.01)		
2nd Dim. Salience										0.33***	
										(0.03)	
2nd Dim. Salience (rel.)											0.39***
											(0.04)
Eff. Nr. of Parties	−0.10***	−0.16***	−0.11***	−0.19***	0.02	−0.03	−0.09***	−0.19***	−0.19***	0.44***	0.55***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.04)	(0.04)
Proportionality	−0.21***	−0.23***	−0.18***	−0.19***	0.13**	0.09*	−0.13***	−0.20***	−0.19***	0.08**	0.32***
	(0.02)	(0.02)	(0.02)	(0.02)	(0.04)	(0.04)	(0.02)	(0.02)	(0.02)	(0.03)	(0.04)
LR Self			0.01*	0.01*	0.01	0.01	0.01*	0.01*	0.01*	−0.00	−0.00
			(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
LR Self^2			0.21***	0.21***	0.20***	0.20***	0.21***	0.21***	0.21***	0.25***	0.25***
			(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)
Union			0.07***	0.07***	0.07***	0.07***	0.07***	0.07***	0.08***	0.08**	0.05*
			(0.01)	(0.01)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.03)	(0.03)
Age			0.20***	0.20***	0.20***	0.20***	0.20***	0.20***	0.20***	0.17***	0.17***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Age^2			0.02***	0.02***	0.02**	0.02**	0.02**	0.02***	0.02***	0.05***	0.05***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Female			−0.09***	−0.09***	−0.11***	−0.11***	−0.09***	−0.09***	−0.09***	−0.08***	−0.08***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)
Education			0.07***	0.07***	0.06***	0.06***	0.07***	0.07***	0.07***	0.06***	0.06***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Income			0.06***	0.07***	0.07***	0.07***	0.06***	0.07***	0.06***	0.06***	0.06***
			(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Intercept	−0.00	0.02	−0.11	−0.06	−0.08	−0.08	−0.05	−0.08	−0.08	−0.69***	−0.95***
	(0.13)	(0.14)	(0.14)	(0.16)	(0.19)	(0.17)	(0.13)	(0.16)	(0.16)	(0.16)	(0.17)
AIC	148408.36	148291.12	93684.93	93588.05	62107.68	62138.19	93622.09	93587.24	93566.95	20162.00	20163.88
BIC	148466.20	148348.95	93813.99	93717.11	62230.99	62261.50	93760.37	93725.52	93714.45	20276.99	20278.87
Log Likelihood	−74198.18	−74139.56	−46828.46	−46780.03	−31039.84	−31055.09	−46796.04	−46778.62	−46767.47	−10066.00	−10066.94
Num. obs.	113485	113485	74515	74515	49429	49429	74515	74515	74515	15773	15773
Num. groups: country	24	24	24	24	19	19	24	24	24		
Num. groups: electionyear	23	23	23	23	12	12	23	23	23	7	7
Var: country (Intercept)	0.24	0.29	0.19	0.23	0.14	0.09	0.19	0.23	0.24		
Var: electionyear (Intercept)	0.23	0.30	0.26	0.33	0.35	0.30	0.19	0.33	0.34	0.17	0.19

$^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Table E1: Polarization and Mass Partisanship