Supplemental Information

APPENDIX A: DETAILED MODEL SPECIFICATION AND IMPLEMENTATION

As stated in the main paper, we adopt the following hierarchical model:

$$Y_{ik}|\mu_{ik} = \mathbb{I}_{[\mu_{ik}>0]} \qquad i = 1\ldots, N,\ k = 1, \ldots, K \tag{8}$$

$$\mu_{ik}|\boldsymbol{\lambda}_k, \boldsymbol{\theta}_i, b_k \sim \mathcal{N}(\boldsymbol{\lambda}_k^T\boldsymbol{\theta}_i - b_k, 1) \qquad i = 1\ldots, N,\ k = 1, \ldots, K \tag{9}$$

$$\lambda_{kj}|m_{kjj} \sim \begin{cases} \mathcal{N}_{[0,\infty]}(0, m_{kjj}^2) & \text{if } m_{kjj} > 0 \\ \mathcal{N}_{[-\infty,0]}(0, m_{kjj}^2) & \text{if } m_{kjj} < 0 \\ \delta_{[\lambda_{kj}=0]} & \text{if } m_{kjj} = 0 \\ \mathcal{N}(0, 5) & \text{if } m_{kjj} = \texttt{NA} \end{cases} \qquad k = 1, \ldots, K,\ j = 1, \ldots, d \tag{10}$$

$$b_k \sim \mathcal{N}(0, 1) \qquad k = 1, \ldots, K \tag{11}$$

$$\boldsymbol{\theta}_i|\Sigma \sim \mathcal{N}_d(\mathbf{0}, \boldsymbol{\Sigma}) \qquad i = 1, \ldots, N \tag{12}$$

$$\Sigma|\nu_0, \mathbf{S}_0 \sim \mathcal{IW}_d(\nu_0, \mathbf{S}_0), \tag{13}$$

Note that the d subscript on $\mathcal{N}_d$ and $\mathcal{IW}_d$ refers to their dimension throughout; it is not an index. The outcome model is a classical ogive (probit) IRT (Lord 1953). Specifically, we assume:

$$Pr(Y_{ik} = 1) = \Phi(\boldsymbol{\lambda}_k^T\boldsymbol{\theta}_i - b_k) \tag{14}$$

where Φ is the standard normal CDF, $\boldsymbol{\lambda}_k$ is the usual vector of d loadings associated with item k, $\boldsymbol{\theta}_i$ is the vector of d latent factor scores for unit i, and b_k is the baseline "discrimination" parameter for item k. Note that, as stated in the Extensions subsection, that equation can be replaced with a logistic or multinomial-logistic specification without altering the main core of the model. The (standard) idea behind the model is to decompose the likelihood of an affirmative ("1") answer to item k by unit i into d dimensions, each of which could have either a positive, negative, or zero effect on that likelihood. Since direct estimation of Eq. (14) is often complex due to its nonlinearity, we adopt the classical Gibbs sampling scheme proposed first by Albert (1992), and now widely used. That scheme proposes the creation of the normally-distributed latent-variable μ_{ik} for every outcome (Eq. (9), and then conditional posterior updates for the other parameters in the model can be derived conditional on that variable. Notably, if we condition on μ_{ik}, we see that Y_{ik} becomes:

$$Y_{ik}|\mu_{ik} = \mathbb{I}_{[\mu_{ij}>0]} = \begin{cases} 1 & \text{if } \mu_{ik} > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where we use $\mathbb{I}_{[\cdot]}$ to represent the indicator function.

While so far our modeling choices have been standard, we now introduce a new model for the loadings λ that implements our main theoretical contribution. For each item k, we model λ_k conditionally on a $d \times d$ diagonal matrix specific to that item. The diagonal of such a matrix is allowed to contain either real values, or the special `NA` code to denote missingness. We then assign a prior distribution to $\boldsymbol{\lambda}_k$ conditional on the values of the diagonal of M_k: for each $j = 1, \ldots, d$, we independently draw λ_{kj} from a positive-truncated normal distribution, if $m_{kjj} > 0$, or a negative-truncated normal distribution if $m_{kjj} < 0$. In essence, this constrains item k to only either load positively or negatively on factor j, as specified by the user. Additionally, positive or negative entries of m_{kjj} can also be given an absolute value of the user's response: this should encode any prior on the extent to which the item loads on the factor. If the user believes that item k is strongly determined by factor j, then m_{kjj} can have a large absolute value, whereas if the user believes that factor j only contributes little to item k, then m_{kjj} can have a small absolute value. This is then reflected in the model by using m^2_{kjj} as variance for λ_{kj}. The user can also specify that item k does not load on factor j at all, i.e., a respondent's level of factor j has no impact on whether that respondent will answer positively to item k; the user can do so by setting $m_{kjj} = 0$, in which case λ_{kj} is drawn from a distribution that puts density 1 at 0 and 0 everywhere else, which is represented by Dirac's delta function $\delta_{[\lambda_{kj}=0]}$ in Eq. (4). In this case, $\lambda_{kj} = 0$ w.p. 1. Finally, a user might not know whether item k is expected to load positively, negatively, or not at all on factor j: in this case the user should set $m_{kjj} =$ `NA`, and λ_{kj} will be drawn from a standard normal distribution. The classical "discrimination" parameter is represented by b_k in our modeling framework: we do not constrain sampling of this parameter in any way dependent on the M-matrix. The latent factors $\boldsymbol{\theta}_i$ are sampled independently from a d-dimensional multivariate normal distribution for each respondent, i (Eq. (12)). Notably, the restrictions imposed on the loadings by our framework allow a covariance matrix Σ to be learned for the factors: we accomplish this by placing a d-dimensional Inverse-Wishart conjugate prior on Σ (Eq. (13)). With this full specification, our model has only three hyperparameters: the M-matrix, ν_0, and $\mathbf{S}_0$, the latter being the degrees of freedom and co-variance parameters for the Inverse-Wishart prior on Σ.

Gibbs sampler

Posterior sampling for the model introduced in the previous section is implemented via standard MCMC techniques. Specifically, we derived and implemented a Gibbs sampler for the model based on fully conjugate conditional posterior updates for all the model parameters. We reproduce the steps of the sampler here as pseudocode. Throughout the following we will denote vectors and matrices containing multiple parameters by omitting the relevant indices from the respective symbols, and by displaying those symbols in bold. For example, if μ_{ik} denotes the value of μ for data unit i and item k, then $\boldsymbol{\mu}_i = [\mu_{i1}, \ldots, \mu_{iK}]^T$ denotes a K-dimensional column vector containing all the values of μ associated with data unit i, and $\boldsymbol{\mu}$ a $N \times K$ matrix of values of μ for all N data units and K items in which the i^{th} row is the (transposed) vector $\boldsymbol{\mu}_i$ and the k^{th} column is a N-dimensional vector of all μ values associated with item k.

All the full conditional posterior updates for each of the parameters in our sampler are derived using the standard formulas for normal-normal, and normal-inverse Wishart conjugate updates given, for example, in Hoff (2009), and as such we omit a full derivation here.

Our Gibbs sampler is as follows:

1. Initialize $\boldsymbol{\mu} \in \mathbb{R}^{N\times K}$, $\mathbf{b} \in \mathbb{R}^K$, $\boldsymbol{\theta} \in \mathbb{R}^{N\times P}$, $\boldsymbol{\lambda} \in \mathbb{R}^{K\times d}$, $\Sigma \in \mathbb{R}^{d\times d}$ by drawing once from their prior distributions as specified in Eqns. (9)-(13).

2. For $s = 1, \ldots S$ sampling iterations:

   1. For $i = 1, \ldots, N$ sample $\boldsymbol{\theta}_i \sim \mathcal{N}_d(B_i, V_i^{-1})$, where $V_i = \boldsymbol{\lambda}^T\boldsymbol{\lambda} + \Sigma^{-1}$, and $B_i = V_i^{-1}\boldsymbol{\lambda}^T(\mathbf{b} + \boldsymbol{\mu}_i)$.

2. For $i = 1, \dots, N$, $k = 1, \dots, K$ sample:

$$\mu_{ik} \sim \begin{cases} \mathcal{N}_{[0,\infty]}(B_{ik}, 1) & \text{if } Y_{ik} = 1 \\ \mathcal{N}_{[-\infty,0]}(B_{ik}, 1) & \text{if } Y_{ik} = 0 \\ \mathcal{N}(B_{ik}, 1) & \text{if } Y_{ik} = \texttt{NA}, \end{cases}$$

   where $B_{ik} = \boldsymbol{\lambda}_k^T \boldsymbol{\theta}_i + b_k$.

3. Sample $\Sigma^* \sim \mathcal{IW}_d(N + \nu_0, \boldsymbol{\theta}^T \boldsymbol{\theta} + S_0)$. Initialize $d \times d$ diagonal matrix Σ, and for $i, j = 1, \dots, d$, $j \neq i$, set $\Sigma_{ij} = \Sigma^*_{ij} / \Sigma^*_{ii}$.

4. For $k = 1, \dots, K$, sample $b_k \sim \mathcal{N}(B_k V_k)$, where $V_k = \frac{1}{N+1}$, and $B_k = V_k \sum_{i=1}^N \boldsymbol{\theta}_i \boldsymbol{\lambda}_k^T - \mu_{ik}$

5. For $k = 1, \dots, K$, let $\mathbf{L}_k$, and $\mathbf{U}_k$ be d-dimensional vectors of lower and upper bounds for item k, where, for $j = 1, \dots, d$:

$$\mathbf{L}_{kj} = \begin{cases} -\infty & \text{if } m_{kjj} < 0 \\ -\infty & \text{if } m_{kjj} = \texttt{NA} \\ 0 & \text{if } m_{kjj} \geq 0 \end{cases}, \quad \mathbf{U}_{kj} = \begin{cases} \infty & \text{if } m_{kjj} > 0 \\ \infty & \text{if } m_{kjj} = \texttt{NA} \\ 0 & \text{if } m_{kjj} \leq 0. \end{cases}$$

   Additionally define the $d \times d$ diagonal matrix Ω_k, such that the j^{th} element of the diagonal is defined as: $\Omega_{kjj} = |m_{kjj}|^2$. Finally, sample: $\boldsymbol{\lambda}_k \sim \mathcal{N}_{d,[\mathbf{L}_k, \mathbf{U}_k]}(B_k, V_k^{-1})$, where: $V_k = \boldsymbol{\theta}^t \boldsymbol{\theta} + \Omega_k^{-1}$, and $B_k = V_k^{-1} \boldsymbol{\theta}^t (b_k + \boldsymbol{\mu}_k)$, and $\mathcal{N}_{[d,\mathbf{L}_k,\mathbf{U}_k]}$ is the truncated multivariate normal distribution of dimension d, where each dimension is truncated between the bounds defined in the respective dimensions of the vectors $\mathbf{L}_k$ and $\mathbf{U}_k$, and such that if $L_{kj} = U_{kj}$, then $\lambda_{kj} = L_{kj}$ when sampled from this distribution.

6. Store the values of $\boldsymbol{\mu}$, $\boldsymbol{\theta}$, $\boldsymbol{\lambda}$, $\mathbf{b}$, Σ sampled at this iteration.

Learning independent factors with correlated loadings

Our model can be modified to allow for learning correlated loadings but independent factors. This is useful in case the analyst is interested in explicitly independent latent dimensions, and maintains the rotation invariance requirements needed for model identification. In order to implement this modification it is sufficient to first replace the prior on $\boldsymbol{\theta}_i$ defined in Eq. (12) with $\boldsymbol{\theta}_i \sim \mathcal{N}_d(\mathbf{0}, \mathbf{I})$, where $\mathbf{I}$ is the $d \times d$ identity matrix. Second, the prior on λ_{kj} in Equation (10) should be modified to $\boldsymbol{\lambda}_k \sim \mathcal{N}_{[d,\mathbf{L}_k,\mathbf{U}_k]}(\mathbf{0}, \Omega_k)$, where $\mathbf{L}_k$ and $\mathbf{U}_k$ are lower and upper bounds on each of the d-dimensions of the normal distribution and they are defined at step (e) of the Gibbs sampler introduced earlier. Finally, the d-dimensional matrix Ω_k should be given the prior induced by sampling $\Omega_k^* \sim \mathcal{IW}_d(\nu_0, \mathbf{S}_0)$, and then setting each diagonal element of Ω_k to:

$$\Omega_{kjj} = \begin{cases} \Omega^*_{kjj} & \text{if } m_{kjj} \neq 0 \\ 0 & \text{if } m_{kjj} = 0 \end{cases},$$

and each off-diagonal element of $\Omega_{kj\ell}$ to:

$$\Omega_{kj\ell} = \begin{cases} \Omega^*_{kj\ell}/\Omega_{kjj} & \text{if } m_{kjj} \neq 0 \\ 0 & \text{if } m_{kjj} = 0 \end{cases}.$$

Sampling from this model is possible by adapting the Gibbs sampling scheme used for the model with correlated factors.

Over-Identification

Our approach provides a method for researchers to link their theoretical expectations about latent constructs to models that estimate those constructs from data. That implies that, in some settings, researchers' theoretical expectations might result in more than the $d(d-1)$ model constraints required for model identification. In this case it is said that the model is *over-identified*: an identified (but not over-identified) model will learn the unique value of the latent dimensions that best fits the data according to some numerical measure of fit (this can be shown to approximately equal a penalized L2 distance between observed data and factor-loading combinations in the case of our model). Conversely, an over-identified model may not return latent dimensions that best maximize model fit, since the additional constraints imposed on the model may be ruling out precisely those values of the latent parameters that maximize fit.

Depending on the researcher's needs, over-identification may or may not be desirable: a researcher that has no strong theoretical priors over the *meaning* of their dimensions may be fine with latent quantities that are learned to maximize some numerical measure of fit, while researchers who specifically want to target the estimation of latent dimensions that conform to some theoretical expectation may still want to impose constraints on such latent quantities, even if this means that their model will not be the one that best fits the data. If the latent dimensions found by a model that is not over-identified are very different from those found by an over-identified model, then that should be a sign for the researcher that the data may not support the theoretical priors they hold about their latent dimensions. In that case, either the theory needs revision or the data are a poor match to the theory. Because of this, a practical suggestion for researchers is to first fit a model with the bare minimum number of constraints required for identification, and then fit a model with all the constraints that they want to impose. Comparing the dimensions returned by each model will be informative: if the latent quantities are very different, then either the data are a poor fit for the theory or the theory behind the over-identified model may need to be revised.

Finally, we remark that our Simulation depicted in Figure 4 shows that even in the event that a model is over-identified with constraints that are misspecified, according to some population model, latent dimensions output by IRT-M are still largely similar to their target values as long as not too many of the constraints are misspecified.

APPENDIX B: ADDITIONAL SIMULATION INFORMATION

General Simulation Setup

All simulations were run for each N, K, d-triple a total of 50 times. At each iteration data was generated from the model detailed in the paper as follows:

$$
\begin{aligned}
\rho_{j,\ell} &\sim \text{Uniform}(-1,1) && j,\ell = 1,\dots,d \\
\Sigma &= \begin{bmatrix} 1, & \dots, & \rho_{1,d} \\ \rho_{1,2}, & \dots, & \rho_{2,d} \\ & \ddots & \\ \rho_{1,d}, & \dots, & 1 \end{bmatrix} && \\
\boldsymbol{\theta}_i &\sim \mathcal{N}_d(\mathbf{0}, \Sigma) && i = 1,\dots,N \\
u_{kj} &\sim \text{Uniform}(0,1), && k = 1,\dots,K,\ j = 1,\dots,d \\
\lambda_{k,j} &\sim \begin{cases} \mathcal{N}(0,1) & \text{if } u_{ij} > 0.25 \\ 0 & \text{otherwise.} \end{cases} && k = 1,\dots,K,\ j = 1,\dots,d \\
b_k &\sim \mathcal{N}(0,1) && k = 1,\dots,K, \\
\mu_{ik} &\sim \mathcal{N}(\boldsymbol{\lambda}_k^T \boldsymbol{\theta}_i - b_k, 1) && i = 1,\dots,N,\ k = 1,\dots,K \\
Y_{ik} &= \mathbb{I}_{[\mu_{ik} > 0]}. && i = 1,\dots,N,\ k = 1,\dots,K
\end{aligned}
$$

We keep the proportion of loadings that are 0 fixed at 25% in all our simulations. M-matrices for our models are then generated according to the sign of the generated $\lambda_{k,j}$; i.e., we generate, for $k = 1, \dots K$:

$$
\mathbf{M}_k = \begin{bmatrix} \text{sign}(\lambda_{kj}), & \dots, & 0 \\ & \ddots & \\ 0, & \dots, & \text{sign}(\lambda_{kj}) \end{bmatrix}, \quad \text{with} \quad \text{sign}(\lambda_{kj}) = \begin{cases} 1 & \text{if } \lambda_{kj} > 0 \\ -1 & \text{if } \lambda_{kj} < 0 \\ 0 & \text{if } \lambda_{kj} = 0 \end{cases}.
$$

Additional Simulation Results

We report additional results from the simulations conducted and introduced in the main paper.

Tables 4 and 5 respectively show MSE and 95% credible interval coverage for learning the loadings, $\boldsymbol{\lambda}$. We see that IRT-M generally performs well in learning loadings, and largely outperforms PCA and traditional IRT. Adding factor correlation does seem to give the model a small boost in performance for this task, especially as the amount of respondents (N) grows. In terms of coverage all models seem to undercover the true parameter values: this is somewhat expected as learning individualized parameters with good uncertainty estimation is a generally hard problem. Nonetheless, IRT-M presents a substantial improvement in terms of coverage for both $\boldsymbol{\lambda}$ and $\boldsymbol{\theta}$ over the traditional methodologies.

Table 6 displays 95% credible interval coverage for the factors, $\boldsymbol{\theta}$. Here we see that IRT-M performs well above standard IRT in terms of coverage. This gain is likely due to the fact that IRT-M can estimate more precise posteriors, thanks to the additional information provided to it by the M-matrices. Together with gains in estimation error as measured by MSE (Table 2) these gains in coverage are substantial enough to justify use of IRT-M instead of traditional IRT.

Tables 7 and 8 display convergence of the compared models as measured by the Geweke (Geweke 1992) and adjusted $\hat{R}$ (Vehtari et al. 2021) statistics. Both tables show that IRT-M generally converges faster than classical IRT. Table 8 shows that all IRT-M models display $\hat{R}$ coefficients below 1.1, the recommended threshold of Vehtari et al. (2021), and in most cases these coefficients are very close to 1.0, indicating almost optimal convergence.

APPENDIX C: ROLL CALL CODING

We specified our coding rules in stages. The first set of rules was written prior to examining bills. We specified five latent dimensions, chosen to be theoretically distinguishable and encompass many different policy areas. The dimensions are Defense/Security, Economic Development, Civil Rights/Social equality, Entitlements/Redistribution/Welfare, and Socio-cultural. The second set of latent dimensions was created after having coded bills for the first set. It comprises six latent dimensions: Economic Policy, Foreign Policy, Public Distribution, Redistribution, Power, and Civil Rights. One guiding

TABLE 4. MSE for λ.

N	K	d=2				d=3				d=5				d=8			
		IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M				
Correlated $\boldsymbol{\theta}$?		No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes				
10	10	2.077	3.811	**0.418**	0.421	2.446	3.147	**0.468**	0.481	2.429	2.947	**0.592**	0.592	–	–	–	–
10	50	1.788	3.341	0.301	**0.296**	1.985	2.904	0.339	**0.338**	2.079	2.799	**0.359**	0.362	–	–	–	–
10	100	2.146	6.458	**0.295**	0.299	1.970	4.284	**0.302**	0.302	1.976	3.087	0.334	**0.333**	**–**	**–**	**–**	**–**
10	250	2.036	7.229	0.283	**0.282**	1.931	4.621	0.301	**0.3**	2.036	3.247	0.324	**0.322**	–	–	–	–
10	500	1.925	9.382	**0.285**	0.285	1.933	6.991	0.287	**0.286**	1.955	3.099	**0.312**	0.312	**–**	**–**	**–**	**–**
50	10	2.245	3.477	0.31	**0.299**	2.204	3.320	0.393	**0.391**	2.369	3.365	0.521	**0.515**	2.507	2.981	**0.575**	0.575
50	50	2.017	3.266	**0.158**	0.158	1.990	3.246	**0.178**	0.179	2.040	2.666	0.207	**0.206**	2.067	2.511	0.257	**0.253**
50	100	2.394	6.855	0.151	**0.149**	1.917	4.472	**0.157**	0.157	2.062	2.917	0.175	**0.174**	2.027	2.495	0.219	**0.216**
50	250	2.002	15.945	**0.139**	0.139	2.074	9.536	**0.14**	0.141	2.083	4.385	0.16	**0.158**	1.949	2.847	**0.186**	0.186
50	500	2.052	16.590	**0.122**	0.123	2.049	17.645	**0.132**	0.132	2.028	4.888	**0.15**	0.15	2.009	3.168	0.185	**0.184**
100	10	2.319	3.616	0.285	**0.273**	2.286	3.635	0.402	**0.386**	2.327	3.250	0.478	**0.463**	2.403	3.031	0.609	**0.582**
100	50	2.080	3.419	0.139	**0.138**	2.145	2.877	0.139	**0.137**	2.094	2.821	0.17	**0.165**	2.060	2.473	0.21	**0.203**
100	100	1.763	6.281	**0.104**	0.104	1.984	3.265	0.109	**0.108**	2.104	2.781	0.134	**0.133**	2.014	2.454	0.155	**0.153**
100	250	1.967	7.106	**0.082**	0.082	1.870	5.350	0.098	**0.097**	2.049	3.724	**0.11**	0.111	2.007	2.982	0.127	**0.126**
100	500	2.013	27.811	**0.074**	0.074	1.878	11.669	0.095	**0.094**	2.054	4.519	**0.101**	0.101	2.019	3.136	**0.115**	0.115
250	10	2.443	4.047	0.289	**0.254**	2.433	3.888	0.437	**0.411**	2.463	3.603	0.556	**0.545**	2.494	3.304	0.629	**0.603**
250	50	2.191	4.766	0.118	**0.103**	2.040	3.027	0.132	**0.116**	2.060	2.898	0.141	**0.126**	2.084	2.545	0.195	**0.172**
250	100	1.753	4.496	0.065	**0.06**	2.018	3.728	0.093	**0.087**	2.084	3.079	0.118	**0.11**	2.002	2.376	0.127	**0.117**
250	250	1.982	5.132	0.04	**0.039**	1.998	3.818	**0.068**	0.069	2.066	3.386	**0.073**	0.073	1.971	2.648	**0.079**	0.079
250	500	1.992	15.136	0.06	**0.059**	2.069	4.849	**0.053**	0.053	1.994	4.257	**0.064**	0.065	2.027	3.005	0.072	**0.071**
500	10	2.357	5.684	0.185	**0.161**	2.156	3.039	0.398	**0.354**	2.387	3.006	0.66	**0.562**	2.468	3.139	0.724	**0.688**
500	50	2.066	4.473	0.106	**0.074**	2.180	2.962	0.127	**0.094**	2.145	2.564	0.18	**0.133**	2.046	2.491	0.204	**0.161**
500	100	2.039	5.515	0.093	**0.063**	1.948	3.081	0.093	**0.074**	2.041	2.630	0.114	**0.088**	1.983	2.430	0.128	**0.102**
500	250	1.830	7.330	0.067	**0.05**	1.857	5.287	0.063	**0.054**	1.965	2.499	0.079	**0.067**	1.965	2.405	0.09	**0.08**
500	500	1.785	4.906	0.042	**0.035**	1.997	6.309	0.063	**0.059**	2.088	3.419	0.056	**0.055**	2.043	2.687	0.057	**0.054**
1000	10	2.379	3.173	0.177	**0.162**	2.255	3.427	0.251	**0.222**	2.478	3.250	0.691	**0.586**	2.406	3.255	0.85	**0.763**
1000	50	1.990	5.189	0.064	**0.04**	2.089	4.736	0.228	**0.122**	2.029	2.706	0.19	**0.128**	1.998	2.734	0.196	**0.143**
1000	100	1.966	3.733	0.129	**0.077**	2.145	3.403	0.164	**0.096**	2.102	2.691	0.142	**0.099**	2.013	2.410	0.147	**0.09**
1000	250	1.796	3.445	0.064	**0.028**	2.073	3.943	0.126	**0.09**	2.010	2.647	0.118	**0.091**	2.082	2.397	0.098	**0.068**
1000	500	2.054	3.820	0.072	**0.027**	1.950	2.967	0.074	**0.057**	2.025	2.635	0.087	**0.069**	1.989	2.367	0.081	**0.067**
2500	10	2.389	6.342	0.34	**0.281**	2.377	5.862	0.339	**0.315**	2.421	3.248	0.838	**0.635**	2.420	3.033	0.8	**0.688**
2500	50	2.093	3.569	0.141	**0.042**	2.018	3.427	0.206	**0.154**	2.040	3.081	0.238	**0.112**	2.086	2.622	0.243	**0.161**
2500	100	2.013	5.508	0.15	**0.063**	1.962	3.364	**0.146**	0.154	2.023	2.830	0.193	**0.095**	2.024	2.536	0.183	**0.103**
2500	250	1.852	9.136	0.135	**0.098**	2.067	4.997	**0.24**	0.252	2.066	3.071	0.214	**0.134**	2.046	2.535	0.173	**0.129**
2500	500	2.023	8.175	0.154	**0.031**	2.021	5.870	0.205	**0.177**	1.952	2.682	0.165	**0.119**	2.063	2.494	0.164	**0.144**

Note: Lower is better; best method for each N,K,d, in bold. Values are Root Mean Square Error for estimated vs true loadings, averaged over d dimensions, N units, and 50 simulations. For Bayesian models estimates are posterior means computed by averaging over 10000 posterior samples. All results from Bayesian models are computed from 4000 posterior samples obtained from 4 parallel MCMC chains after 2000 burn-in iterations.

TABLE 5. 95% Credible interval coverage for λ.

N	K	d=2			d=3			d=5			d=8		
		IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M
Correlated θ?		No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes
10	10	**0.676**	0.596	0.598	**0.685**	0.581	0.585	**0.684**	0.587	0.589	–	–	–
10	50	0.56	**0.602**	0.6	0.586	0.585	**0.588**	**0.62**	0.587	0.584	–	–	–
10	100	0.516	**0.602**	0.599	0.526	0.591	**0.593**	0.548	0.584	**0.586**	–	–	–
10	250	0.478	**0.596**	0.595	0.488	**0.593**	0.593	0.497	0.586	**0.588**	–	–	–
10	500	0.479	**0.594**	0.594	0.469	0.596	**0.598**	0.48	0.587	**0.588**	–	–	–
50	10	0.54	**0.598**	0.589	0.57	0.587	**0.589**	**0.626**	0.568	0.575	**0.66**	0.585	0.583
50	50	0.368	0.609	**0.611**	0.408	**0.614**	0.612	0.472	0.596	**0.598**	0.518	0.592	**0.596**
50	100	0.369	0.613	**0.615**	0.377	**0.621**	0.621	0.412	0.608	**0.611**	0.453	0.59	**0.592**
50	250	0.38	0.626	**0.627**	0.368	0.623	**0.624**	0.369	0.617	**0.619**	0.399	**0.601**	0.601
50	500	0.365	**0.635**	0.634	0.361	**0.63**	0.63	0.373	**0.623**	0.623	0.39	0.602	**0.603**
100	10	0.396	0.568	**0.573**	0.457	0.529	**0.553**	0.527	0.546	**0.558**	**0.579**	0.552	0.558
100	50	0.298	0.598	**0.601**	0.327	0.602	**0.605**	0.368	0.581	**0.585**	0.419	0.574	**0.579**
100	100	0.322	**0.623**	0.616	0.303	**0.617**	0.616	0.336	**0.602**	0.601	0.366	0.595	**0.599**
100	250	0.308	**0.637**	0.637	0.291	**0.624**	0.623	0.308	0.619	**0.621**	0.327	0.61	**0.611**
100	500	0.304	**0.644**	0.644	0.3	**0.631**	0.63	0.306	**0.628**	0.628	0.306	0.617	**0.618**
250	10	0.318	0.581	**0.586**	0.373	0.504	**0.512**	0.379	**0.52**	0.519	0.449	0.505	**0.512**
250	50	0.226	**0.592**	0.592	0.222	0.545	**0.559**	0.251	0.537	**0.557**	0.287	0.515	**0.534**
250	100	0.208	**0.618**	0.615	0.21	0.583	**0.584**	0.228	0.548	**0.554**	0.243	0.546	**0.557**
250	250	0.223	**0.635**	0.632	0.216	**0.612**	0.604	0.203	**0.602**	0.594	0.223	**0.595**	0.593
250	500	0.267	**0.629**	0.625	0.207	**0.633**	0.629	0.218	**0.622**	0.614	0.228	**0.613**	0.609
500	10	0.225	0.567	**0.576**	0.252	0.496	**0.509**	0.317	0.454	**0.477**	0.343	0.473	**0.491**
500	50	0.193	0.542	**0.556**	0.183	0.499	**0.531**	0.182	0.475	**0.51**	0.209	0.461	**0.494**
500	100	0.173	0.566	**0.574**	0.152	0.525	**0.544**	0.157	0.492	**0.519**	0.173	0.477	**0.511**
500	250	0.177	**0.576**	0.565	0.148	**0.567**	0.565	0.134	0.541	**0.544**	0.151	0.525	**0.533**
500	500	0.183	**0.614**	0.607	0.159	**0.585**	0.575	0.154	**0.584**	0.574	0.152	**0.576**	0.57
1000	10	0.194	**0.534**	0.534	0.2	0.445	**0.486**	0.224	0.392	**0.437**	0.257	0.416	**0.436**
1000	50	0.15	0.547	**0.582**	0.136	0.443	**0.502**	0.129	0.411	**0.471**	0.151	0.41	**0.461**
1000	100	0.144	0.51	**0.545**	0.12	0.466	**0.512**	0.111	0.443	**0.502**	0.122	0.411	**0.477**
1000	250	0.136	0.564	**0.575**	0.11	0.494	**0.507**	0.094	0.461	**0.492**	0.099	0.455	**0.495**
1000	500	0.14	**0.549**	0.547	0.106	0.545	**0.557**	0.093	0.496	**0.504**	0.093	0.491	**0.504**
2500	10	0.154	0.473	**0.525**	0.13	0.382	**0.432**	0.152	0.37	**0.395**	0.189	0.367	**0.398**
2500	50	0.11	0.524	**0.579**	0.091	0.386	**0.468**	0.088	0.354	**0.436**	0.103	0.346	**0.417**
2500	100	0.12	0.464	**0.539**	0.078	0.421	**0.498**	0.071	0.365	**0.453**	0.078	0.348	**0.438**
2500	250	0.122	0.513	**0.555**	0.074	0.425	**0.486**	0.061	0.385	**0.456**	0.062	0.354	**0.435**
2500	500	0.11	0.506	**0.535**	0.076	0.453	**0.503**	0.058	0.414	**0.471**	0.057	0.383	**0.449**

Note: Higher is better. Best method for each N,K,d, in bold. Values are proportion of times that the true value of $lambda_{kj}$ falls within the 95% Credible Interval generated by the posterior draws of each estimated latent loading. Proportions are computed over K items separately for each dimension of λ, resulting values are averaged across dimensions and 50 simulations.
All results from Bayesian models are computed from 4000 posterior samples obtained from 4 parallel MCMC chains after 2000 burn-in iterations.

TABLE 6. 95% Credible interval coverage for θ.

N	K	d=2			d=3			d=5			d=8		
		IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M
Correlated θ?		No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes
10	10	0.577	**0.68**	0.668	0.583	**0.718**	0.715	0.622	**0.775**	0.769	–	–	–
10	50	0.434	**0.66**	0.658	0.450	0.669	**0.67**	0.501	**0.711**	0.704	–	–	–
10	100	0.296	**0.629**	0.613	0.351	**0.66**	0.656	0.391	0.685	**0.687**	–	–	–
10	250	0.219	**0.576**	0.574	0.226	0.597	**0.6**	0.268	0.649	**0.653**	–	–	–
10	500	0.149	**0.534**	0.534	0.173	**0.58**	0.573	0.198	0.613	**0.618**	–	–	–
50	10	0.460	**0.607**	0.601	0.497	**0.616**	0.615	0.538	0.633	**0.635**	0.566	**0.667**	0.665
50	50	0.254	0.599	**0.604**	0.304	0.591	**0.6**	0.355	0.604	**0.611**	0.406	0.623	**0.626**
50	100	0.215	0.556	**0.568**	0.217	0.565	**0.576**	0.257	0.561	**0.566**	0.312	0.574	**0.579**
50	250	0.139	0.516	**0.519**	0.141	0.53	**0.531**	0.159	0.514	**0.519**	0.199	0.49	**0.495**
50	500	0.097	**0.484**	0.484	0.103	0.489	**0.503**	0.121	0.471	**0.481**	0.142	0.438	**0.444**
100	10	0.440	**0.599**	0.597	0.480	0.604	**0.607**	0.492	0.604	**0.607**	0.540	0.63	**0.631**
100	50	0.263	0.589	**0.6**	0.294	0.587	**0.6**	0.325	0.588	**0.602**	0.369	0.594	**0.605**
100	100	0.194	0.572	**0.585**	0.211	0.564	**0.58**	0.237	0.561	**0.579**	0.276	0.571	**0.587**
100	250	0.114	**0.546**	0.545	0.121	0.507	**0.525**	0.137	0.518	**0.535**	0.170	0.494	**0.516**
100	500	0.081	0.473	**0.487**	0.086	0.479	**0.489**	0.100	0.47	**0.487**	0.118	0.444	**0.463**
250	10	0.426	0.597	**0.609**	0.462	0.592	**0.601**	0.472	0.585	**0.591**	0.494	0.581	**0.588**
250	50	0.256	0.594	**0.607**	0.284	0.586	**0.607**	0.309	0.585	**0.609**	0.341	0.586	**0.609**
250	100	0.173	0.574	**0.583**	0.201	0.578	**0.589**	0.246	0.569	**0.59**	0.271	0.569	**0.598**
250	250	0.106	**0.562**	0.558	0.130	0.544	**0.555**	0.140	0.536	**0.546**	0.161	0.529	**0.558**
250	500	0.081	0.499	**0.507**	0.079	0.517	**0.531**	0.092	0.5	**0.524**	0.105	0.469	**0.508**
500	10	0.395	0.597	**0.606**	0.436	0.593	**0.604**	0.461	0.574	**0.583**	0.482	0.57	**0.575**
500	50	0.261	0.592	**0.61**	0.279	0.589	**0.613**	0.314	0.585	**0.616**	0.330	0.581	**0.609**
500	100	0.236	0.581	**0.582**	0.222	0.579	**0.589**	0.242	0.573	**0.593**	0.264	0.571	**0.598**
500	250	0.133	**0.544**	0.522	0.137	**0.542**	0.532	0.157	**0.537**	0.533	0.183	0.537	**0.543**
500	500	0.078	**0.519**	0.499	0.087	**0.494**	0.481	0.100	**0.502**	0.495	0.108	0.506	**0.515**
1000	10	0.413	0.593	**0.605**	0.430	0.59	**0.602**	0.440	0.571	**0.583**	0.469	0.559	**0.565**
1000	50	0.234	0.589	**0.604**	0.297	0.594	**0.624**	0.309	0.586	**0.612**	0.326	0.584	**0.608**
1000	100	0.204	0.576	**0.587**	0.229	0.576	**0.581**	0.243	0.578	**0.596**	0.269	0.575	**0.598**
1000	250	0.135	**0.523**	0.515	0.167	**0.505**	0.497	0.185	0.504	**0.509**	0.187	0.526	**0.536**
1000	500	0.094	**0.458**	0.452	0.100	**0.481**	0.478	0.125	**0.441**	0.44	0.134	0.461	**0.467**
2500	10	0.412	0.591	**0.606**	0.431	0.581	**0.595**	0.444	0.568	**0.575**	0.461	0.546	**0.557**
2500	50	0.228	0.594	**0.604**	0.294	0.595	**0.616**	0.304	0.59	**0.61**	0.329	0.584	**0.596**
2500	100	0.228	**0.572**	0.566	0.229	0.574	**0.582**	0.252	0.576	**0.59**	0.266	0.574	**0.587**
2500	250	0.134	0.506	**0.516**	0.188	0.468	**0.478**	0.191	0.475	**0.503**	0.206	0.486	**0.518**
2500	500	0.105	0.45	**0.459**	0.125	0.415	**0.45**	0.135	0.396	**0.437**	0.150	0.397	**0.444**

Note: Higher is better. Best method for each N,K,d, in bold. Values are proportion of times that the true value of $theta_{ij}$ falls within the 95% Credible Interval generated by the posterior draws of each estimated latent factor. Proportions are computed over N units separately for each dimension of $\boldsymbol{\theta}$, resulting values are averaged across dimensions and 50 simulations. For All results from Bayesian models are computed from 4000 posterior samples obtained from 4 parallel MCMC chains after 2000 burn-in iterations.

TABLE 7. Geweke convergence for θ.

N	K	d=2			d=3			d=5			d=8		
		IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M
Correlated θ?		No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes
10	10	0.413	**0.292**	0.3	0.355	0.307	**0.306**	**0.287**	0.355	0.342	–	–	–
10	50	0.512	0.384	**0.375**	0.526	**0.373**	0.373	0.519	**0.418**	0.424	–	–	–
10	100	0.474	0.412	**0.39**	0.506	0.417	**0.41**	0.53	0.44	**0.438**	–	–	–
10	250	0.474	0.471	**0.427**	0.527	0.475	**0.444**	0.549	0.468	**0.461**	–	–	–
10	500	0.508	**0.452**	0.469	0.528	0.484	**0.475**	0.558	0.476	**0.466**	–	–	–
50	10	0.525	**0.249**	0.252	0.546	**0.261**	0.263	0.525	**0.298**	0.306	0.468	**0.327**	0.331
50	50	0.524	**0.289**	0.298	0.594	**0.293**	0.293	0.607	**0.302**	0.302	0.593	**0.33**	0.338
50	100	0.486	**0.353**	0.377	0.566	**0.341**	0.342	0.603	0.353	**0.347**	0.602	0.368	**0.361**
50	250	0.584	**0.433**	0.436	0.586	0.429	**0.419**	0.593	**0.398**	0.403	0.598	**0.4**	0.405
50	500	0.631	0.501	**0.482**	0.615	**0.444**	0.455	0.616	0.433	**0.426**	0.609	**0.434**	0.436
100	10	0.512	**0.205**	0.211	0.577	**0.228**	0.232	0.585	**0.259**	0.268	0.536	**0.3**	0.307
100	50	0.495	0.236	**0.235**	0.561	**0.241**	0.243	0.616	**0.247**	0.25	0.619	**0.273**	0.279
100	100	0.491	0.319	**0.315**	0.559	**0.286**	0.296	0.611	**0.291**	0.292	0.621	0.305	**0.302**
100	250	0.558	**0.434**	0.443	0.587	0.384	**0.375**	0.596	**0.362**	0.367	0.603	0.356	**0.353**
100	500	0.611	**0.487**	0.509	0.615	0.45	**0.445**	0.641	0.401	**0.398**	0.625	0.386	**0.382**
250	10	0.446	0.171	**0.16**	0.531	0.192	**0.191**	0.582	**0.233**	0.235	0.563	**0.272**	0.29
250	50	0.400	**0.17**	0.173	0.509	**0.17**	0.181	0.569	**0.183**	0.194	0.592	**0.207**	0.221
250	100	0.433	**0.228**	0.235	0.500	0.224	**0.222**	0.562	**0.212**	0.221	0.596	**0.222**	0.228
250	250	0.506	0.381	**0.359**	0.536	0.347	**0.341**	0.59	0.297	**0.28**	0.609	0.276	**0.273**
250	500	0.559	0.473	**0.441**	0.613	0.421	**0.402**	0.621	0.359	**0.336**	0.622	0.318	**0.314**
500	10	0.363	0.142	**0.14**	0.488	**0.16**	0.169	0.552	**0.203**	0.218	0.542	**0.25**	0.271
500	50	0.343	**0.134**	0.14	0.434	**0.139**	0.148	0.505	**0.148**	0.164	0.556	**0.166**	0.19
500	100	0.358	0.186	**0.18**	0.427	**0.172**	0.181	0.503	**0.167**	0.184	0.535	**0.177**	0.199
500	250	0.411	0.292	**0.272**	0.463	0.271	**0.244**	0.509	0.235	**0.233**	0.536	**0.226**	0.227
500	500	0.517	0.407	**0.395**	0.530	0.354	**0.331**	0.57	0.318	**0.285**	0.596	0.275	**0.259**
1000	10	0.316	**0.125**	0.125	0.419	**0.14**	0.147	0.479	**0.175**	0.198	0.502	**0.222**	0.254
1000	50	0.249	**0.115**	0.124	0.363	**0.118**	0.129	0.431	**0.125**	0.145	0.492	**0.142**	0.174
1000	100	0.282	**0.136**	0.149	0.345	**0.135**	0.157	0.425	**0.138**	0.159	0.467	**0.147**	0.174
1000	250	0.316	0.23	**0.228**	0.345	**0.202**	0.208	0.417	**0.187**	0.192	0.460	**0.18**	0.195
1000	500	0.368	0.334	**0.304**	0.421	0.283	**0.275**	0.443	0.255	**0.248**	0.471	0.229	**0.224**
2500	10	0.240	**0.107**	0.115	0.332	**0.122**	0.135	0.393	**0.143**	0.18	0.417	**0.178**	0.231
2500	50	0.193	**0.1**	0.108	0.257	**0.104**	0.121	0.345	**0.112**	0.136	0.395	**0.125**	0.163
2500	100	0.210	**0.112**	0.128	0.252	**0.112**	0.13	0.322	**0.117**	0.142	0.375	**0.127**	0.161
2500	250	0.218	**0.153**	0.169	0.270	**0.14**	0.172	0.312	**0.138**	0.175	0.354	**0.141**	0.182
2500	500	0.247	**0.201**	0.215	0.278	**0.189**	0.22	0.323	**0.175**	0.207	0.352	**0.17**	0.201

Note: **Lower is better**. Values are proportion of times that a Geweke test of convergence resulted in a p-value less than 0.5.
Proportions are taken over 100 simulations times d parameters at each N and K value.
Convergence statistics are based on 1000 MCMC samples after 2000 burnin iterations with no thinning.
All results from Bayesian models are computed from 4000 posterior samples obtained from 4 parallel MCMC chains after 2000 burn-in iterations.

TABLE 8. Rhat convergence for θ.

N	K	d=2			d=3			d=5			d=8		
		IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M	IRT	IRT-M	IRT-M
Correlated θ?		No	No	Yes	No	No	Yes	No	No	Yes	No	No	Yes
10	10	1.110	1.031	**1.03**	1.074	1.034	**1.032**	1.037	1.038	**1.034**	–	–	–
10	50	1.521	1.069	**1.066**	1.513	**1.068**	1.07	1.431	1.09	**1.088**	–	–	–
10	100	1.612	1.096	**1.088**	1.657	1.1	**1.095**	1.742	1.111	**1.106**	–	–	–
10	250	1.795	1.131	**1.124**	1.893	**1.125**	1.131	2.096	1.145	**1.144**	–	–	–
10	500	1.842	**1.155**	1.163	2.066	1.154	**1.151**	2.336	1.157	**1.156**	–	–	–
50	10	1.254	1.019	**1.018**	1.216	1.021	**1.02**	1.147	1.027	**1.026**	1.087	1.03	**1.028**
50	50	1.627	**1.028**	1.028	1.639	**1.029**	1.03	1.589	**1.036**	1.037	1.507	**1.051**	1.051
50	100	1.671	**1.045**	1.046	1.760	1.043	**1.041**	1.800	**1.047**	1.047	1.750	1.064	**1.062**
50	250	1.799	1.089	**1.083**	1.920	**1.071**	1.072	2.014	1.074	**1.069**	2.021	**1.085**	1.092
50	500	1.994	**1.123**	1.123	2.102	1.087	**1.084**	2.175	1.083	**1.082**	2.195	**1.103**	1.112
100	10	1.313	**1.013**	1.013	1.275	**1.015**	1.015	1.208	1.022	**1.021**	1.137	**1.024**	1.024
100	50	1.635	**1.018**	1.019	1.672	**1.019**	1.019	1.618	**1.022**	1.023	1.567	**1.031**	1.033
100	100	1.742	1.032	**1.03**	1.816	**1.028**	1.029	1.849	**1.031**	1.031	1.802	**1.039**	1.039
100	250	2.031	**1.064**	1.064	2.000	1.053	**1.052**	2.057	1.049	**1.048**	2.079	**1.054**	1.054
100	500	2.079	1.109	**1.103**	2.158	1.085	**1.079**	2.250	**1.062**	1.062	2.253	**1.066**	1.067
250	10	1.329	**1.009**	1.009	1.294	**1.013**	1.013	1.260	**1.015**	1.017	1.194	**1.021**	1.024
250	50	1.649	**1.011**	1.011	1.660	**1.011**	1.012	1.643	**1.013**	1.015	1.580	**1.018**	1.02
250	100	1.853	**1.017**	1.017	1.866	**1.016**	1.018	1.806	**1.017**	1.019	1.797	**1.021**	1.023
250	250	2.065	1.044	**1.043**	2.074	1.037	**1.035**	2.147	1.031	**1.029**	2.130	1.032	**1.031**
250	500	2.006	1.085	**1.079**	2.269	1.064	**1.06**	2.292	1.048	**1.043**	2.310	1.044	**1.042**
500	10	1.375	**1.006**	1.007	1.318	**1.009**	1.009	1.283	**1.014**	1.016	1.225	**1.02**	1.023
500	50	1.652	**1.008**	1.008	1.679	**1.008**	1.009	1.617	**1.01**	1.011	1.569	**1.012**	1.015
500	100	1.772	**1.012**	1.012	1.844	**1.011**	1.013	1.806	**1.012**	1.013	1.765	**1.015**	1.018
500	250	1.975	1.029	**1.027**	2.087	1.025	**1.024**	2.078	1.022	**1.021**	2.030	**1.022**	1.023
500	500	2.181	1.056	**1.053**	2.241	1.047	**1.041**	2.319	1.038	**1.033**	2.326	1.033	**1.03**
1000	10	1.368	**1.006**	1.006	1.312	**1.007**	1.01	1.287	**1.012**	1.016	1.235	**1.02**	1.024
1000	50	1.733	**1.007**	1.007	1.629	**1.007**	1.008	1.617	**1.008**	1.01	1.566	**1.01**	1.013
1000	100	1.796	**1.009**	1.011	1.791	**1.009**	1.012	1.815	**1.01**	1.012	1.729	**1.011**	1.014
1000	250	2.038	**1.019**	1.019	1.997	**1.016**	1.019	1.973	**1.015**	1.02	1.992	**1.016**	1.02
1000	500	2.180	**1.035**	1.036	2.280	**1.03**	1.032	2.210	**1.026**	1.03	2.187	**1.024**	1.026
2500	10	1.330	**1.005**	1.006	1.299	**1.008**	1.008	1.289	**1.015**	1.022	1.245	**1.02**	1.03
2500	50	1.676	**1.006**	1.008	1.619	**1.007**	1.011	1.564	**1.008**	1.009	1.541	**1.009**	1.012
2500	100	1.770	**1.008**	1.009	1.784	**1.009**	1.012	1.740	**1.008**	1.011	1.714	**1.009**	1.015
2500	250	1.977	**1.012**	1.018	1.944	**1.011**	1.021	1.947	**1.011**	1.023	1.901	**1.011**	1.02
2500	500	2.090	**1.019**	1.023	2.169	**1.018**	1.053	2.180	**1.016**	1.032	2.110	**1.017**	1.026

Note: Lower is better. Best method for each N,K,d, in bold. Values are Rhat averaged over N units, d dimensions, and 50 simulations. Rhat is a statistic that outputs an adjusted autocorrelation between MCMC posterior samples. Here Rhat is computed over 10000 posterior samples.
All results from Bayesian models are computed from 4000 posterior samples obtained from 4 parallel MCMC chains after 2000 burn-in iterations.

principle of the second set was to have Civil Rights as its own latent dimension, to enable clearer comparisons to DW-Nominate. For each set, in addition to analyzing the five- or six-dimensional latent space, we also combined theoretical concepts in different ways to create and analyze latent spaces with fewer dimensions. The three coding rules discussed in the paper's analysis were derived from three different such combinations. Coding rule A came from the first set of latent dimensions, while coding rules B and C came from the second set. Further discussion and justification of each set of latent dimensions, particularly the second set, can be found in the next two subsections.

Once we had specified latent dimensions, we then determined coding rules for how one would assign a value of 1, -1, or 0 for each voting opportunity-latent dimension pair. Recall that a 1 (-1) is assigned if larger (smaller) values of that latent dimension would theoretically predict a more likely yea (nay) vote, and a 0 is assigned if that dimension's value would not theoretically help to predict a vote. The rules we used can be found in the next two subsections.

Finally, we coded all bills from both Congresses according to those coding rules. For the first set of dimensions we had two coders, who had both contributed to specifying the set of dimensions, independently code bills and then coordinate the final coding between them. For the second set, one coder specified the set of dimensions, while the other coded according to it. We note that our coding method leads to many bills not being used at all to compute latent dimensions. That occurs when none of our theoretically-derived dimensions are deemed to be relevant for predicting votes on that bill, implying that we effectively use fewer bills in determining latent positions than does a method such as DW-Nominate.

First Set

-Defense/Security: votes that are intended to support initiatives that improve or increase the power of U.S. defense, further national security aims, improve counterterrorism capacity or improve the lives of veterans are coded as (+1). Actions that negatively affect any of these aims, weaken U.S. defense or national security, offer leniency to terrorism suspects or limit engagement for counterterrorism are coded with (-1). Votes on bills that commend, decry, or congratulate other nations' actions (holding elections, celebrating a deceased leader, or expressing opposition to curbing of freedoms) are coded as

(+1) as they presume positive engagement in the international community. Examples: - clerk session vote number 608 of the 109th House: HRES571 Express the Sense of the House of Representatives that the deployment of United States forces in Iraq be terminated immediately. Coded as (-1) because it negatively affects U.S. national security and defense. -Clerk session vote number 611- HRES479: Recognizing the 50th Anniversary of the Hungarian Revolution that began on October 23, 1956 and reaffirming the friendship between the people and governments of the United States and Hungary. Coded as (+1).

-Economic Development: votes on bills/questions that advance overall economic development by bringing in more activity, increased budget, support for small business initiatives or international trade initiatives/trade deals are coded as (+1). Actions that decrease government spending and/or budget, decrease support for small businesses, or increase taxation on or eliminate subsidies for big business are coded as (-1). An example of this is clerk session vote number 232 of the 109th Senate: HR 2862 To prohibit weakening any law that provides safeguards from unfair foreign trade practices. Coded as (+1).

-Civil Rights/Social Equality: This category captures actions to protect historically oppressed or racial and ethnic minority groups, undocumented immigrants and other vulnerable groups. It also supports initiatives that improve access to basic needs. Actions that support their protections are coded as (+1) and actions that would strip them of rights or go against the protection of these groups or against the provision of basic needs are coded as (-1). Examples of this: -clerk session vote number 270 in the 109th Senate: HR3010: To provide for appropriations for the Low-Income Home Energy Assistance Program. Coded as (+1) -clerk session vote number 295 in the 109th Senate: S1932 To replace title VIII of the bill with an amendment to section 214(c) of the Immigration and Nationality Act to impose a fee on employers who hire certain non-immigrants. Coded as (-1)

-Entitlements/Redistribution/Welfare: Captures actions in which goods are distributed to the general public or a smaller vulnerable group. Actions that fund public-interest projects, that protect public health, food stamps/SNAP, support agriculture, and support to federal employees and veterans are coded as (+1). Actions that are against funding, supporting or regulating public goods and infrastructure are coded as (-1). An example is Vote 27 of the 85th Senate: HR7221, which stipulates that the appropriation for feed and seed in disaster areas can only be used by states that have matched it with a 25% state appropriation is coded as (-1) because it denies funds to states that have not been able to match this amount. Examples Include: -Vote number 19 in the 85th House: HR6287 Cut unemployment compensation to federal employees. Coded as (-1). -Vote number 130 in the 85th House: HR12065 Temporary Unemployment Compensation. Coded as (+1)

-Socio-Cultural: Captures improvements for education, research and development, public parks, community centers, coded as (+1). Actions that would reduce or eliminate funding for these projects, limit access to them, or eliminate existing programs are coded as (-1) Examples include: -Vote 39 in the 85th Senate: HR. 6500. FISCAL 1958 APPROPRIATIONS FOR D.C. AMENDMENT TO INCREASE FUNDS FOR TEACHING PERSONNEL IN D.C. PUBLIC SCHOOLS. Coded as (+1) -Vote 40 in the 85th Senate: HR. 7441. FISCAL 1958 APPROPRIATIONS FOR AGRICULTURE DEPARTMENT. AMENDMENT TO ELIMINATE THE PROVISION LIMITING NATIONAL AVERAGE FOR CONSERVATION RESERVE PAYMENTS PER ACRE. Coded as (-1).

When the 5-factor collapses into the 4-factor, 3-factor and 2-factor arrangements, certain categories are grouped together (for example: Economic/Redistribution in the 4 factor or Social/Cultural/Civil Rights/Equality in the 2-factor). While there is certainly some overlap between certain categories (actions that promote redistribution of wealth to a historically oppressed group may also have some aspect of civil rights protection) these categories are coded based on the numeric values from the separate codes for each category. For example, if a bill is coded with a +1 for Economic policy but 0 for Distribution and Power, it will coded +1 based on the code for Economic Policy, not zero based on the codes for Distribution and Power. The non-zero value takes precedence in coding for all categories below the 5-factor categories. Similarly, where Civil Rights and Redistribution are both coded +1, then the combined Civil Rights/Redistribution will also be +1. If there are contradictory codes, and there is one predominant or primary category that can be assessed for the bill, the value of that category takes precedence.

Second Set

6-factor Economic Policy Distinguishes two general macroeconomic theories in U.S. politics – hands on government intervention in the economy, or hands-off government in the economy. I assume that all members of Congress would be pro-economic development. The code captures ideological differences on perspectives for U.S. economic development. A (+1) indicates pro government spending, increasing the budget, increasing taxes for big businesses, in addition to pro- government intervention in businesses and the economy. A (-1) indicates actions that decrease government spending, decrease the budget, decrease taxes for big businesses, and against government intervention in business and the economy.

Foreign Policy The code tries to generally capture two trends in foreign policy; however, it is not all encompassing. Perspectives on foreign policy change over time. The code attempts to capture "soft" and "hard" approaches to foreign policy. A (+1) indicates actions that are for soft U.S. intervention, diplomacy, and cooperation with other nations–e.g. Humanitarian assistance, trade regulation, and support for the United Nations. A (-1) indicates actions more representative of "hard" U.S. intervention–i.e. Against diplomatic measures, humanitarian assistance, but rather pro-military force. The code does not intend to capture for general actions taken to increase or decrease U.S. defense. Much like in economic policy–where I assume that members of Congress are for economic development and prosperity, and thus distinguish perspectives on economic policy–I assume that members of Congress support U.S. defense, especially at heightened periods of national security. In periods of national stability, however, there are different approaches to defense. Actions taken to decrease the defense budget, may be in effort to defund humanitarian assistance, more so than being against strong U.S. defense. General actions to increase or decrease the overall defense budget will be coded a 0, and rather captured in other categories such as Economic Policy. Actions pertaining to veterans will also be coded a 0, and rather captured in other codes such as Public Distribution. Veterans assistance does pertain to U.S. defense; however, it is not distinguished in different approaches to foreign policy. Actions that are unanimous and non-controversial in Foreign Policy–e.g. Senate votes to condemn terrorist attacks and sympathize with victims and their families–should be coded a 0, because they do not express ideological differences with respect to foreign policy.

Public Distribution Captures actions in which goods, whose costs are collected from either a small group or the general public, are distributed to the general public. It does not include actions that distribute goods to a smaller group, such as the low income community. A (+1) includes actions that fund public-interest projects such as infrastructure, general public K-12 education–that is not specified as helping a smaller group such as low income, and furthering research and development. It also includes efforts to increase regulations that protect the environment and/or public health. Efforts to assist farmers and regulate agriculture–e.g. government funding to rotate crops– is also included, because they protect long-term food supply for the general public. Actions taken to increase the wages, benefits, and work conditions for federal employees, including veterans, are also included, because they work for the public good. Disaster relief is also included, because although the disaster may be specific to one community at any given time, actions taken to increase disaster relief in one specific area prepares the nation for aid in any given part of the nation in the future, and can thus be thought of as a public good. A (-1) includes actions that are against funding, supporting, or increasing regulations for the public good. Actions related to active military personnel and military related research and technology should be coded a 0, because the distinction between different kinds of funding within defense is coded for in Foreign Policy.

Redistribution Unlike Public Distribution, where the goods are distributed to the general public, Redistribution includes actions where costs are collected from a larger majority or a smaller group–e.g. the wealthy elite–and the benefits are distributed to a smaller group, especially low income or historically oppressed groups. A (+1) includes actions such as SNAP/food stamps, housing assistance for the poor, disability assistance. It also includes actions that try to increase the taxes on the small elite, and redistribute to low-income communities. A (-1) includes actions that are against redistributing goods to low income or other historically oppressed groups. It also includes actions taken to defend the wealth of the elite minority–e.g. tax breaks for the wealthy–against redistribution of wealth.

Power Captures actions taken to increase or decrease federal government power. A (+1) includes actions that strengthen the power of the federal government. It includes actions that defend public interest over individual liberty, including business endeavors. A (-1) includes actions that decrease the power of the federal government. It includes actions that defend individual liberty over public interest. It does not, however, include defense of historically oppressed minorities, criminal or terrorist rights. Although these are protections of individual liberty, there are different conceptions of who is considered a "person" throughout U.S. history. The protection of historically oppressed groups is coded for in Civil Rights. Terrorist protections are accounted for in Foreign Policy and Civil Rights. Treatment of terrorists changes over time in relation to national security.

Civil Rights Includes protection of historically oppressed groups. A (+1) includes actions that protect groups including but not limited to racial and ethnic minorities, women, sexual and gender minorities, indigenous communities, undocumented immigrants, criminals and terrorists. A (-1) includes actions that are against the protection of historically oppressed communities.

4-factor Economic Policy Same as in the 6-factor Foreign Policy Same as in the 6-factor Public Distribution/Power Encompasses all (+1) from Public Distribution and Power as a (+1), and both categories' (-1) as (-1). Although there are differences between the two, there is reason to believe that there is overlap between both categories. If there are contradictory codes-e.g. an action that is a (-1) in Public Distribution and a (+1) in Power–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Public Distribution and (+1) in Power– the action should be coded with the non-zero value.

Civil Rights/Redistribution Encompasses all (+1) from Civil Rights and Redistribution as a (+1), and both categories' (-1) as (-1). Although there are differences between the two, there is reason to believe that there is overlap between both categories. If there are contradictory codes-e.g. an action that is a (-1) in Civil Rights and a (+1) in Redistribution–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Civil Rights and (+1) in Redistribution– the action should be coded with the non-zero value.

4-factor Economic Policy/ Power Encompasses all (+1) from Economic Policy and Power as a (+1), and both categories' (-1) as (-1). Although there are differences between the two, there is reason to believe that there is overlap between both categories. If there are contradictory codes-e.g. an action that is a (-1) in Economic Policy and a (+1) in Power–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Economic Policy and (+1) in Power– the action should be coded with the non-zero value.

Foreign Policy Same as in the 6-factor Distribution Encompasses all (+1) from Public Distribution and Redistribution as a (+1), and both categories' (-1) as (-1). Although there are differences between the two, there is reason to believe that there is overlap between both categories. If there are contradictory codes-e.g. an action that is a (-1) in Public Distribution and a (+1) in Redistribution–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Public Distribution and (+1) in Redistribution– the action should be coded with the non-zero value.

Civil Rights Same as in 6-factor

3-factor Economic Policy Same as in 6-factor

Distribution/Power/Civil Rights Encompasses all (+1) from Public Distribution, Redistribution, Power, and Civil Rights as a (+1), and all four categories' (-1) as (-1). Although there are differences between the four, there is reason to believe that there is overlap between the four categories. If there are contradictory codes–e.g. an action that is a (-1) in Redistribution and a (+1) in Power–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Redistribution and (+1) in Public Distribution and (+1) in Power– the action should be coded with the non-zero value.

Foreign Policy Same as in 6-factor

3-factor Economy/Distribution/Power Encompasses all (+1) from Economic Policy, Public Distribution, Redistribution, and Power as a (+1), and all four categories' (-1) as (-1). Although there are differences between the four, there is reason to believe that there is overlap between the four categories. If there are contradictory codes–e.g. an action that is a (-1) in Redistribution and a (+1) in Power–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Redistribution and (+1) in Public Distribution and (+1) in Power– the action should be coded with the non-zero value.

Civil Rights Same as in 6-factor

Foreign Policy Same as in 6-factor

2-factor Economy/ Distribution/ Power Encompasses all (+1) from Economic Policy, Public Distribution, Redistribution, and Power, as a (+1), and all four categories' (-1) as (-1). Although there are differences between the four, there is reason to believe that there is overlap between the four categories. If there are contradictory codes–e.g. an action that is a (-1) in Redistribution and a (+1) in Power–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Redistribution and (+1) in Public Distribution and (+1) in Power– the action should be coded with the non-zero value.

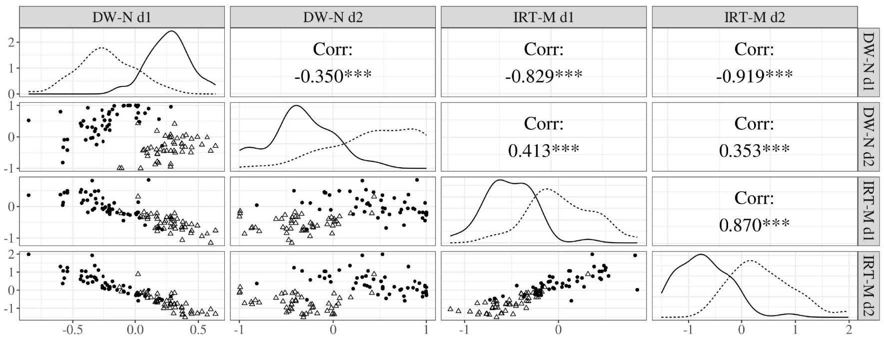
Civil Rights Same as in 6-factor

2-factor Economy/Public Distribution/ Power Encompasses all (+1) from Economic Policy, Public Distribution, and Power as a (+1), and all three categories' (-1) as (-1). Although there are differences between the three, there is reason to believe that there is overlap between the three categories. If there are contradictory codes–e.g. an action that is a (-1) in Public Distribution and a (+1) in Power–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Economic Policy, (+1) in Public Distribution and (+1) in Power– the action should be coded with the non-zero value.

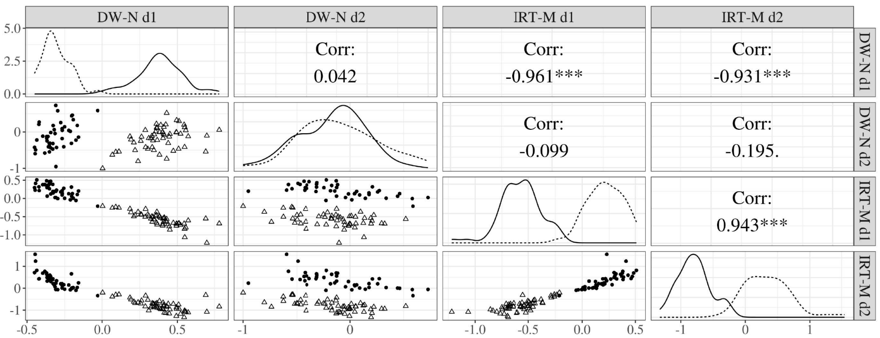
Civil Rights/Redistribution Encompasses all (+1) from Civil Rights and Redistribution as a (+1), and both categories' (-1) as (-1). Although there are differences between the two, there is reason to believe that there is overlap between the two categories. If there are contradictory codes–e.g. an action that is a (-1) in Redistribution and a (+1) in Civil Rights–the action should be coded 0. If there are combinations of 0s and values–e.g. (0) in Civil Rights and (+1) in Redistribution - the action should be coded with the non-zero value.

APPENDIX D: RESULTS FOR SENATE

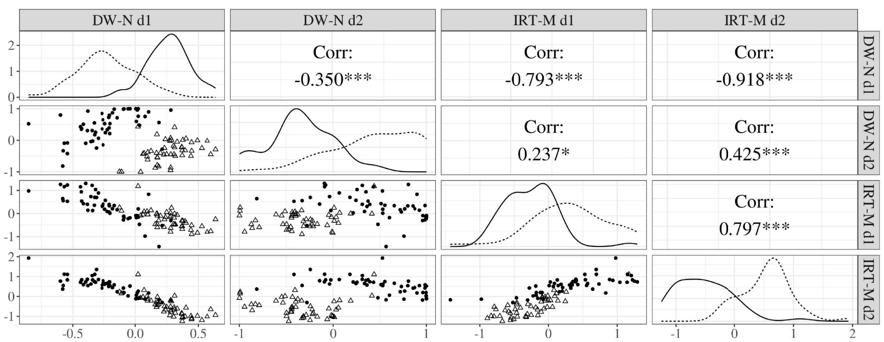
FIGURE 6. Correlations between IRT-M and DW-NOMINATE ideal points in the Senate



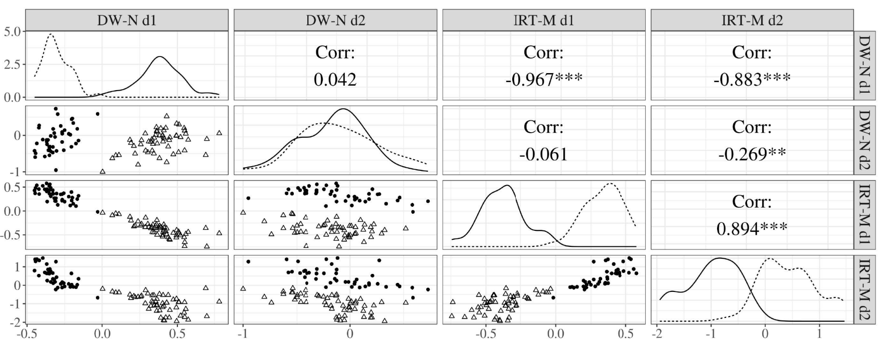
(a) 85th Senate, coding rule A



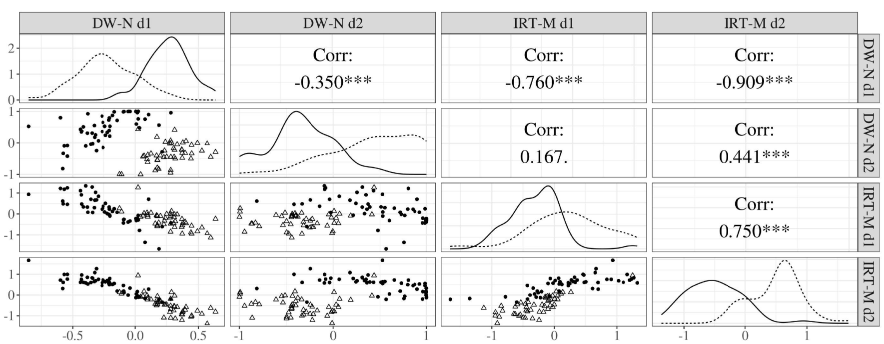
(b) 109th Senate, coding rule A



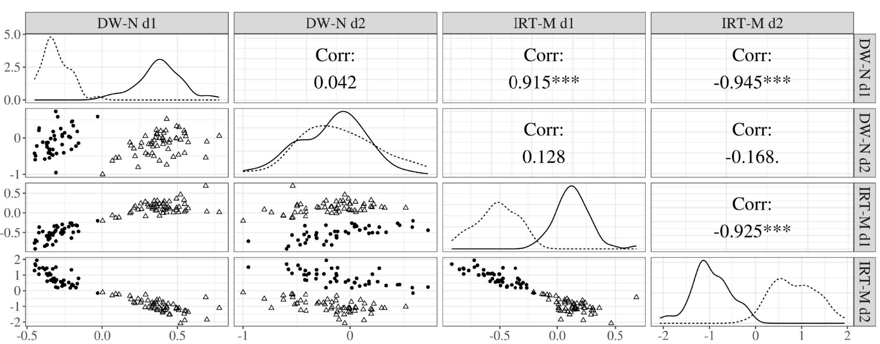
(c) 85th Senate, coding rule B



(d) 109th Senate, coding rule B



(e) 85th Senate, coding rule C



(f) 109th Senate, coding rule C

Note: Each row/column within each subfigure is one of the latent dimensions estimated either by Nominate or IRT-M. The bottom triangle of each subfigure displays scatterplots with each pair of dimensions on each axis. The diagonal contains density plots for each pair of dimensions. The top triangle contains Spearman correlation coefficients for each pair of dimensions.