On Accountability and Hierarchy:
Appendix

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Notation and Definitions

The Basics:

Consider an *organization*, $\langle O, \rightarrow \rangle$, as a directed graph where nodes are elements of O and edges represent a binary review relation $\rightarrow$. We assume that $\rightarrow$ is irreflexive: for all $i \in O$ $\neg(i \rightarrow i)$. We also assume that O contains at least two elements and the organization's directed graph is weakly connected.

The members of O may have different specialties. There are $m \geq 1$ different specialties in the society. S denotes the set of all specialties in O. Let S_i denote the set of agent i's specialties; this set may be empty. We call S_i agent i's repertoire.

In addition, the following definitions will be used in the proofs. (Because Definitions 1-5 were given in the text they retain the numbers used there. Although the text provided the content of Definition 6, it was not listed as a formal definition there.)

Definition 1: Organization $\langle O, \rightarrow \rangle$ satisfies *universal accountability* if for every $i \in O$ there is a $j \in O$ such that $j \neq i$ and $j \rightarrow i$.

Definition 2: In organization $\langle O, \rightarrow \rangle$ set $C = \{i_1, \ldots, i_n\}$, $C \subset O$, will be called a *cycle* if $i_{k-1} \rightarrow i_k$ for all k such that $2 \leq k \leq n$ and $i_n \rightarrow i_1$. A review relation without cycles is called *acyclic*.

Definition 3: We will say that i is a *leader* in $\langle O, \rightarrow \rangle$ if there is no $j \in O$ such that $j \rightarrow i$.

Notation: In organization $\langle O, \rightarrow \rangle$ consider $i, j \in O$ such that $i \not\rightarrow j$ and $i \rightarrow k_1, k_1 \rightarrow k_2, \ldots, k_{t-1} \rightarrow k_t, k_t \rightarrow j$ we will say that i reviews j *indirectly* and denote it as $i \Rightarrow j$. If $i, j \in O$ are such that $i \rightarrow j$ or $i \Rightarrow j$ we will say that i reviews j *directly or indirectly* and denote it as $i \mapsto j$. We will also say that i and j are *connected by a review path.*

Definition 4: A group of at least two individuals, $G \subset O$ in an organization $\langle O, \rightarrow \rangle$ will be called an *oligarchy* if (1) for all $j \in O \setminus G$ there is an $i \in G$ such that $i \mapsto j$, (2) for all $i \in G$ and all $j \in O \setminus G$ it is not true that $j \rightarrow i$, and (3) for any $i \in G$, set $G \setminus \{i\}$ does not satisfy both (1) and (2).

Theorem 1: No review structure of any finite organization, $\langle O, \rightarrow \rangle$, with any distribution of specialties satisfies both acyclicity and universal accountability.

Proof: Assume, by contradiction, that $\langle O, \rightarrow \rangle$, with a finite O, that satisfies (1) acyclicity and (2) universal accountability. Take any $j_0 \in O$. From (2), there is $j_1 \neq j_0$ such that $j_1 \rightarrow j_0$. From (1) we have $\neg(j_0 \rightarrow j_1)$. From (2), there is $j_2 \neq j_1$ such that $j_2 \rightarrow j_1$ and, from (1), $\neg(j_1 \rightarrow j_2)$ and $\neg(j_0 \rightarrow j_2)$. And so on. Suppose $\{j_0, j_1, \ldots, j_t\}$ is the maximal set such that $j_k \rightarrow j_{k-1}$ for all k, $0 \leq k \leq t$. Since O is finite, such maximal set exists and is finite as well. Note that there is no $j \in O$ such that $j \rightarrow j_k$. If such j existed, the $\{j_0, j_1, \ldots, j_t\}$ set would not have been maximal. But this means that j_k is not reviewed by anyone which contradicts universal accountability. Finally, since the proof does not depend on the distribution of specialties, the theorem holds for all distributions of specialties. QED.

Corollary 1: In any organization $\langle O, \rightarrow \rangle$ with an acyclic review relation $\rightarrow$, every member of O who is not a leader is reviewed, directly or indirectly, by one of the organization's leaders.

Proof: Consider organization $\langle O, \rightarrow \rangle$ and any $m_0 \in O$ who is not a leader. Since m_0 is not a leader, there is a $m_1 \in O$ such that $m_1 \rightarrow m_0$. If m_1 is a leader then m_0 is reviewed by a leader which proves the corollary. If m_1 is not a leader then there is m_2 such that $m_2 \rightarrow m_1$ and, again, if m_2 is a leader the corollary is proved and if m_2 is not a leader then there is m_3 such that $m_3 \rightarrow m_2$. Since $\rightarrow$ is acyclic, there is a finite review sequence $m_t \rightarrow m_{t-1} \rightarrow \cdots \rightarrow m_0$ which is maximal i.e., there is no $j \in O$ such that $j \rightarrow m_t$. But this means that m_t is a leader. The existence of the review path between m_t and m_0 proves that m_0 is (indirectly) reviewed by a leader and completes the proof of the corollary. QED.

Lemma 1: If $G \subset O$, in an organization $\langle O, \rightarrow \rangle$ is an oligarchy, then

(i) for any $i, j \in G$ $i \mapsto j$ if and only if $j \mapsto i$, and

(ii) G is the only oligarchy in $\langle O, \rightarrow \rangle$.

Proof: To prove part (i) let's assume, by contradiction, that $i, j \in G$ are such that $i \mapsto j$ and $j \not\mapsto i$. Let's define set $\{k^{\mapsto}\}$ as a set of all elements of O that are reviewed by k, i.e., $\{k^{\mapsto}\} = \{m \in O : k \mapsto m\}$.

Consider now $G^* = G \setminus \{i^{\mapsto}\}$. Since $i \mapsto j$, G^* is a proper subset of G. We will prove that G^* is an oligarchy, i.e., (1) everyone in $O \setminus G^*$ is reviewed by someone in G^*, and (2) no one in G^* is reviewed by someone in $O \setminus G^*$.

First, we prove that (1) holds. Take any $m \in O \setminus G^*$. Then either $m \in O \setminus G$ or $m \in G \cap \{i^{\mapsto}\}$. If $m \in O \setminus G$ then m is reviewed by either $n \in G^* = G \setminus \{i^{\mapsto}\}$ or by $n \in G \cap \{i^{\mapsto}\}$. In this second case, m is reviewed by i but since $i \in G^*$, this means that m is reviewed by an element of G^*. Thus, in both cases m is reviewed by someone in G^* which proves that (1) holds.

Second, we prove that condition (2) holds. Suppose, by contradiction, that (2) does not hold and there is $m \in G^*$ which is reviewed by $n \in O \setminus G^*$. Either (i) $n \in O \setminus G$ or (ii) $n \in G \cap \{i^{\mapsto}\}$. Condition (i) cannot hold since G is an oligarchy. If (ii) holds then $n \mapsto m$ where $n \in \{i^{\mapsto}\}$ means that $i \mapsto n$. But $n \mapsto m$ and $i \mapsto n$ imply that $i \mapsto m$. Hence $m \in \{i^{\mapsto}\}$ which contradicts $m \in G^*$ where $G^* = G \setminus \{i^{\mapsto}\}$ and completes the proof by contradiction.

Finally, since G is an oligarchy, G is the minimal set satisfying the two definitional properties of oligarchy, (1) and (2). Yet, we have proved that G^*, which is a proper subset of G, also satisfies (1) and (2). But this contradicts the minimality of G as the oligarchy and completes this part of the proof.

To prove (ii) let's assume, by contradiction, that G is not the only oligarchy in $\langle O, \rightarrow \rangle$ and there is some other oligarchy G^* such that $G \neq G^*$. Since $G \neq G^*$ it must be that either (a) $G^* \subset G$, or (b) there is a $j \in G^*$ such that $j \notin G$. If (a) holds then G is not the minimal set satisfying properties (1) and (2) of the definition of oligarchy. This means that G violates property (3) and, hence, is not an oligarchy. If (b) holds and there is a $j \in G^*$ such that $j \notin G$ then since G is an oligarchy, there is an $i \in G$ such that $i \mapsto j$ and such that $j \not\mapsto i$. Consider now oligarchy G^*. Either $i \in G^*$ or $i \notin G^*$. If $i \in G^*$ then since $j \in G^*$ and G^* is an oligarchy, from part (i) above we conclude that $i \mapsto j$ implies $j \mapsto i$ which contradicts $j \not\mapsto i$. If $i \notin G^*$ then since $i \mapsto j$, j is reviewed by i who is not in G^*, which contradicts the assumption that G^* is an oligarchy. QED.

Lemma 3: For any distribution of specialties, if an organization is neither an oligarchy nor an autocracy then *every* pair of the members of O is connected by a review cycle.

Proof: Take any organization $\langle O, \rightarrow \rangle$ and consider its extended review form $\langle O, \mapsto \rangle$. Consider $C \subset O$ which is a cycle in $\langle O, \mapsto \rangle$. Given that $\mapsto$ relation is transitive, if C is a cycle then that for any $i, j \in C$ we have $i \mapsto j$ and $j \mapsto i$. We will say that C is a *maximal cycle* if there is no $C' \subset O$ such that $C \subset C'$ and C' is a cycle. Note the following simple fact: If $C \subset O$ is a cycle in $\langle O, \mapsto \rangle$, then (i) For all $i, j \in C$ $i \mapsto j$ and $j \mapsto i$; (ii) If C is a cycle then any $C' \subset C$ is also a cycle, and (iii) If C and C' are two maximal cycles then $C \cap C' = \emptyset$. Assume now that $C_1, \ldots, C_n$ is the set of all maximal cycles in $\langle O, \mapsto \rangle$. We will consider a contraction of $\langle O, \mapsto \rangle$ by representing each of the maximal cycles C_m by a single element k_m. More specifically, we will take the following transformation, Ct, of O:

$$Ct(k) = \begin{cases} k_i & \text{if } k \in C_i \quad \text{for some } i,\ 1 \leq i \leq n \\ k & \text{if } k \notin \cup C_i \end{cases}$$

Since each maximal cycle is an equivalence class, Ct contracts the entire equivalence class into one element. We can now consider $\langle Ct(O), \mapsto \rangle$, the contracted form of $\langle O, \mapsto \rangle$, as defined by transformation Ct. From the fact mentioned above we conclude that $\langle Ct(O), \mapsto \rangle$ contains no cycles. Moreover, given the definition of the *review* relation $\mapsto$, $\langle Ct(O), \mapsto \rangle$ is a strict partial order (i.e., $\mapsto$ is transitive and asymmetric.) Also, the property of being an oligarchy is preserved across the three different forms: G is an oligarchy in $\langle O, \rightarrow \rangle$ if and only if G is an oligarchy in $\langle O, \mapsto \rangle$ if and only if $Ct(G)$ is an oligarchy in $\langle Ct(O), \mapsto \rangle$.

Define $\max\{Ct(O)\}$ as the set of maximal elements of $Ct(O)$ under the strict partial order relation $\mapsto$, i.e., $\max\{Ct(O)\} = \{i \in Ct(O) : \neg\exists k \in Ct(O) \quad k \mapsto i\}$. Note that for any partial order the set of maximal elements has to be nonempty. By the definition of $\max(Ct(O))$ members of this set review everyone else in $Ct(O)$ and no one outside this set reviews anyone in it. This means that $\max\{Ct(O)\}$ satisfies the first two definitional properties of an oligarchy. If, in addition, $\max\{Ct(O)\} \neq Ct(O)$ then $\max\{Ct(O)\}$ must be an oligarchy, since it also satisfies the third definitional property of being the minimal set. If it were not the minimal set then we would have an $i \in \max\{Ct(O)\}$ such that $\max\{Ct(O)\} \setminus \{i\}$ reviews everyone outside this set and no one from outside this set reviews anyone in it. Hence there would have to be a $j \in \max\{Ct(O)\}$ such that $j \mapsto i$ and such that $i \not\mapsto j$. But this contradicts maximality of i. Thus the only way in which $\max\{Ct(O)\}$ can violate the definition of an oligarchy is when $\max\{Ct(O)\} = Ct(O)$.

Note now that for any $i, j \in \max(Ct(O))$ we cannot have $i \mapsto j$. If we had $i \mapsto j$ then we would either have (a) $j \not\mapsto i$ or (b) $j \mapsto i$. If (a) were to hold then j would not have been a maximal element. If (b) were to hold then we would have $i \mapsto j$ and $j \mapsto i$. But this means that $\{i, j\} \subset C_i$, i.e., i and j would have to belong to the same maximal cycle in $\langle O, \mapsto \rangle$. But this contradicts the assumption that i and j are different elements of $Ct(O)$. Hence, for all $i, j \in \max\{Ct(O)\}$ we must have $i \not\mapsto j$. If, however, $i \not\mapsto j$ and $j \not\mapsto i$ then the graph of $\langle Ct(O), \mapsto \rangle$, given that $\max\{Ct(O)\} = Ct(O)$, is not connected. This is not possible since the graphs of $\langle O, \rightarrow \rangle$ and $\langle O, \mapsto \rangle$ are both weakly connected and weak connectivity of $\langle O, \rightarrow \rangle$ implies weak connectivity of its contracted form. We, thus, conclude that $\max\{Ct(O)\}$ consists of a single element. From the first part of the proof we know that an absence of oligarchy in $\langle O, \rightarrow \rangle$ implies that $\max\{Ct(O)\} = Ct(O)$. If, however, $\max\{Ct(O)\} = Ct(O)$ and $\max\{Ct(O)\} = \{i\}$ it means that all elements of organization $\langle O, \mapsto \rangle$ belong to a single cycle or, equivalently, every pair of elements of O is connected by a review cycle. Finally, since the proof does not depend on the distribution of specialties, the lemma holds for all distributions of specialties. QED.

Definition 5: A direct review $i \to j$ exhibits a *Weberian tension* if there is some specialty s such that $s \in S_j$ but $s \notin S_i$. If no such s exists then we say that the review of j by i is free of Weberian tension.

Theorem 3: Consider any O where everyone has exactly one specialty. For any such O there exists a review structure that is free of Weberian tensions if and only if everyone in O has the same specialty.

Proof: Sufficiency is obvious. Regarding necessity we prove the converse statement: if there are $k > 1$ specialties in O then Weberian tensions must exist.

Because everybody in O has exactly one specialty, the set of k specialties induce a partition of O. Consider, without loss of generality, S_1, the subset of people with specialty 1. Because the review structure is weakly connected, in S_1 there must be at least one agent, say j, who either is reviewed by someone outside S_1 or who reviews somebody outside S_1. But if j is reviewed by anyone not in S_1 there is a Weberian tension because the reviewer lacks specialty 1. Similarly, if j reviews anyone outside S_1 then j is reviewing an agent with a different specialty, which again implies that a Weberian tension exists. Thus if there is more than one specialty in O, Weberian tensions must exist. QED

Definition 6: We say that i is the *autocrat* of O, given a particular review structure, if (i) nobody in O reviews i and (ii) everybody else in O is reviewed, directly or indirectly, by i.

Definition 7: We will call $C \subseteq O$ a *class of specialists* if for any two $i, j \in C$ there is a $k \in C$ such that $S_i \subseteq S_k$ and $S_j \subseteq S_k$.

Proposition 1: A review structure is free of Weberian tension only if nobody is reviewed by someone who does not belong to the same class of specialists.

Proof: Suppose i and j do not belong to the same class of specialists and i reviews j. Since they aren't in the same class of specialists, $S_j \not\subseteq S_i$. Given the definition of Weberian tension, this immediately implies that there is a Weberian tension in i's reviewing of j. QED.

Theorem 4: In the set of review structures that have an autocrat there exists a review structure that is free of Weberian tension if and only if O has just one class of specialists.

Proof: If there is one class of specialists, consider the member of this class who has the maximal set of specialties. A review structure where this member directly reviews everyone else in O is free of Weberian tensions. To prove that the existence of more than one class of specialists implies the existence of Weberian tensions consider a case in which there are only two classes of specialists C and C'. (If there are more than two classes of specialists the same logic applies.) Suppose $i \in C$ and $j \in C'$ are members of the two classes who have the maximal number of specialties. Given that $S_i \setminus S_j \neq \emptyset$ and $S_j \setminus S_i \neq \emptyset$ then if i is an autocrat then $i \mapsto j$ must exhibit Weberian tension. The same is true if j were an autocrat. And, lastly, if k, who is neither i nor j, is an autocrat then either $k \mapsto i$ or $k \mapsto j$ must exhibit Weberian tension. QED.

The following three facts are useful in the proof of Theorem 5.

Fact 1: If $\langle O, \rightarrow \rangle$ does not exhibit Weberian tension then for any $j_1 \rightarrow j_2, \ldots, j_{m-1} \rightarrow j_m$, where $j_1, \ldots, j_m \in O$, $S_{j_1} \supseteq S_{j_2} \supseteq \ldots \supseteq S_{j_m}$.

Proof: Suppose, by contradiction, that one of the conjectured subset relations does not hold. Without loss of generality, assume that $S_{j_1} \supseteq S_{j_2}$ does not hold. But if $S_{j_1} \not\supseteq S_{j_2}$ then there is $s_p \in S_{j_2}$ such that $s_p \notin S_{j_1}$. But, since $j_1 \rightarrow j_2$, this implies the existence of Weberian tension and completes the proof by contradiction.

Fact 2: If $\langle O, \rightarrow \rangle$ does not exhibit Weberian tension then for any cycle $j_1 \rightarrow j_2, \ldots, j_{m-1} \rightarrow j_m, j_m \rightarrow j_1$, where $j_1, \ldots, j_m \in O$, $S_{j_1} = S_{j_2} = \ldots = S_{j_m}$.

Proof: From Fact 1 we get $S_{j_1} \supseteq S_{j_m}$ and $S_{j_m} \supseteq S_{j_1}$ which implies that all the sets must be equal.

Fact 3: If $i, j \in O$ are such that $S_i \subseteq S_j$ then i and j belong to the same class of specialists.

Proof: Immediately follows from the definition of a class of specialists.

Theorem 5: There exists a review structure that is free of Weberian tension if and only is the set of classes of specialists is non-separable.

Proof: First we will prove that non-separable classes of specialists implies the existence of a review structure which is free of Weberian tensions. Consider any class of specialists C and for any $i, j \in C$ define the review relation over C as follows: $i \rightarrow j$ if and only if $S_i \supseteq S_j$. From Facts 1-3 we know that such a review structure is free of Weberian tensions. Defining the review relations the same way over all classes of specialists we get a review structure over the entire O. Given that classes of specialists are non-separable, the graph of the review relation on O is connected which means that an O with this review structure is an organization.

Second, we will prove that if a review structure is free of Weberian tensions then the set of classes of specialists is non-separable. Equivalently we will prove that separability of classes of specialists implies the existence of Weberian tensions. Suppose then that classes of specialists can be separated into two sets S and S' such that $S \cap S' = \emptyset$. Since $S \cup S' = O$ and O's review structure is a connected graph, there is $i \in S$ and $j \in S'$ such that $i \rightarrow j$. But $S_i \nsubseteq S_j$ and $S_j \nsubseteq S_i$. If it were not the case then from Fact 3, i and j would belong to the same class of specialists and, hence, they would both be in either S or S'. Yet $S_i \nsubseteq S_j$ and $S_j \nsubseteq S_i$ imply the existence of a Weberian tension in $i \rightarrow j$, thus completing the proof that separability implies Weberian tensions. QED

Theorem 6: If O has multiple classes of specialists then the following properties hold for all review structures that are free of Weberian tension.

(i) Every such review structure is an oligarchy.

(ii) If a top specialist i is reviewed then all of i's reviewers must be top specialists from i's class of specialists.

(iii) There exists a group that contains all of the top specialists in O whose members

(1) review everyone outside the group and

(2) are not reviewed by anyone outside that group.

(iv) Every oligarchy that is free of Weberian tension includes at least one top specialist of every class of specialists.

(v) The smallest oligarchies consist of exactly one top specialist from every class of specialists.

Proof: (i) By Theorem 2, there are only three possible types of review structures: autocracy, oligarchy, and democracy. Given that there are multiple classes of specialists, Theorem 4 tells us that no autocracy can be free of Weberian tensions. Next, consider democracies. Let i and j be top specialists from two distinct classes of specialists. Thus $S_i \not\subseteq S_j$ and $S_j \not\subseteq S_i$. But in democracy every two individuals are connected by a review relation, i.e., there are $i_1, \ldots, i_n$ such that $i \to i_1, \ldots, i_n \to j$. From Fact 1, if there are no Weberian tensions we must have $S_i \supseteq S_{i_1} \supseteq \ldots \supseteq S_{i_n} \supseteq S_j$. But this contradicts $S_i \not\subseteq S_j$ and implies that there must be Weberian tensions. This rules out democracy. Since the review structure is neither an autocracy nor a democracy, it must be an oligarchy.

(ii) Consider any class of specialists. Top specialists, i.e., those with the maximal number of specialties in this class, must all have the same set of specialties, and everyone with this set of specialties must be a member of this group. If a member of this group is reviewed by someone outside the group, he must be reviewed either (1) by someone whose set of specialties is larger, which is not possible since all group members have the maximal number of specialties, or (2) he is reviewed by someone who does not have all of his specialties, which implies a Weberian tension. Hence, if a top specialist is reviewed, he must be reviewed by another member of the class of specialists with the same set of specialties.

(iii) Take $G \subset O$ containing all top specialists and all leaders in O. Note that no one outside G reviews any $i \in G$, for if there were $j \in O \setminus G$ such that $j \to i$ then either i is not a leader or i is reviewed by someone with fewer specialties, hence creating a Weberian tension. Finally, since all leaders belong to G, Corollary 1 implies that everyone in O is reviewed by someone in G.

(iv) Suppose an oligarchy did not include any top specialist from some class of specialists C. Since an oligarchy controls everyone outside it, the top specialists in C would be controlled by members of the oligarchy who lack at least one specialty of those top specialists of C which contradicts the absence of Weberian tensions.

(v) If we take any other review structure then from (iii) we know that it has to include at least one top specialist from each class of specialists. The smallest such set consists of a single top specialist in each class. QED

Theorem 7: For any organization with c classes of specialists the following holds.

(i) If the review structure is an autocracy, or an oligarchy with separable classes of specialists, then it has at least $c-1$ Weberian tensions.

(ii) If a review structure is a democracy then there are no Weberian tensions if agents' repertoires are homogeneous and at least c Weberian tensions if repertoires are heterogeneous.

Proof: We will prove Part (i) for autocracies and oligarchies separately.

Assume, first, that we have an autocracy with c classes of specialists. If the autocrat is a top specialist in one of the classes then his review of top specialists in the other $c-1$ classes creates $c-1$ Weberian tensions. If the autocrat is not a top specialist in any class of specialists then his review of top specialists in all c classes creates c Weberian tensions.

Assume now that the review structure is an oligarchy with c classes of specialists $C_1, \ldots, C_c$ which are separable, i.e., $C_1 \cap \ldots \cap C_c = \emptyset$. Assume, without loss of generality, that $i_k^* \in C_k$ for $1 \leq k \leq c$ is a top specialist in C_k and that the set $\{i_1^*, \ldots, i_c^*\}$ is an oligarchy. Consider now C_1 and $C_2 \cup \ldots \cup C_c$. Since $C_1 \cap (C_2 \cup \ldots \cup C_c) = \emptyset$ and the review graph of any organization is connected, there is $i \in C_1$ and $j \in C_2 \cup \ldots \cup C_c$ such that either $i \to j$ or $j \to i$. Denote by S_m a set of specialties of m. Since $S_i \setminus S_j \neq \emptyset$ and $S_j \setminus S_i \neq \emptyset$ both $i \to j$ and $j \to i$ must exhibit Weberian tensions. Without loss of generality we can assume that $j \in C_2$ and consider the following two sets: $C_1 \cup C_2$ and $C_3 \cup \ldots \cup C_c$. Since $(C_1 \cup C_2) \cap (C_3 \cup \ldots \cup C_c) = \emptyset$ and the review structure is connected there is $i' \in C_1 \cup C_2$ and $j' \in C_3 \cup \ldots \cup C_t$ such that either $i' \to j'$ or $j' \to i'$. But exactly the same reasoning as in the step above each of the two reviews must exhibit Weberian tension. Clearly, this reasoning can be continued until all classes of specialists are exhausted. Since each step in this construction generates at least one Weberian tension and we have $c-1$ steps total, the review structure in question has at least $c-1$ Weberian tensions. This completes the proof of Part (i) for the case of oligarchies with separable classes of specialists.

To prove Part (ii) note first that if all members of O have the same set of specialties then there are no Weberian tensions no matter what the review structure. In particular, democratic review structures (with homogeneous repertoires) have no Weberian tensions. If repertoires are heterogeneous, consider the case of c classes of specialists and take the set of top specialists, $\{i_1, \ldots, i_c\}$, where each agent belong to a different class. Since in democracy there is a review path between any two individuals, there is a cycle $i_1 \mapsto i_2 \ldots \mapsto i_c \mapsto i_1$. Each direct or indirect review in this cycle must involve a Weberian tension. Since there are c reviews in the cycle the the minimal number of Weberian tensions is c. QED

Proposition 2: If some members of O acquire specialties which are new to the organization, i.e., those which are not in S, and otherwise O remains unchanged then for all review structures there are at least as many hard Weberian tensions in the new organization as in the old organization.

Proof: Consider the following two systems $\mathbb{S} = \langle O, \rightarrow, \{S_1, \ldots, S_n\}\rangle$ and $\mathbb{S}' = \langle O', \rightarrow', \{S'_1, \ldots, S'_n\}\rangle$ which describe the same organization, i.e., $O = O'$ and $\rightarrow = \rightarrow'$, and only differ with respect to the set of agent's repertoires. More specifically, all agents retain their specialties but some agents acquire new specialties, i.e., there is a $t > 1$ such that $S_1 \subset S'_1, \ldots, S_t \subset S'_t$ and $S_{t+1} = S'_{t+1}, \ldots, S_n = S'_n$.

We will prove now that if a review relation, say of j by i, exhibits a hard Weberian tension in $\mathbb{S}$, then it does so in the new system $\mathbb{S}'$. Since the review of j by i exhibits a hard Weberian tension in $\mathbb{S}$, then (i) there must be a specialty s such that $s \in S_j$ but $s \notin S_i$ and (ii) high quality information regarding j's performance is not available. There are the following four possible cases: (1) $S_i = S'_i$ and $S_j = S'_j$; (2) $S_i = S'_i$ and $S_j \subset S'_j$; (3) $S_i \subset S'_i$ and $S_j = S'_j$; and (4) $S_i \subset S'_i$ and $S_j \subset S'_j$. In case (1) the hard Weberian tension between i and j in $\mathbb{S}$ obviously persists in $\mathbb{S}'$. Similarly, in case (2) since $s \in S'_j$ and $s \notin S'_i$ still holds, the hard tension between i and j remains unchanged. In case (3) since S'_i adds to S_i specialties that did not previously exist in the organization, $s \in S'_j$ and $s \notin S'_i$ continues to hold and so does the hard tension between i and j. Similarly, in case (4) since S'_i adds to S_i specialties that did not previously exist in the organization and so does S'_i to S_j, $s \in S'_j$ and $s \notin S'_i$ continues to hold and so does the hard tension between i and j. This concludes the proof of the claim.

Since all hard tensions in $\mathbb{S}$ remain in $\mathbb{S}'$, the minimal number of hard Weberian tensions in $\mathbb{S}'$ cannot be smaller than in was in $\mathbb{S}$. This number, obviously, can be larger if the new specialties acquired by members of O create new Weberian tensions for which high quality information is not available. QED