Principal axes of inertia (PAI) and moments of inertia (MI) of the cross-sectional imaging (CSI)

We use to show the rectangular coordinate system of the CSI’s COM. The CSI is consisted of finite surface area element of volume ().  stand for moments of inertia (MI) on axis respectively, and  stands for product of inertia (PI), which is expressed by the equation:

 (1)

where stands for area element,  for its grey value, and  for its position coordinate.

Let the coordinate system rotate  around the CSI’s COM. A new coordinate system will be formed: . The relation between surface area element coordinates  and those of  is:

 (2)

 stand for the MI of axis  respectively, which is expressed as:

 (3)

Substitute Eq 2(b) into 3(a), we will get:

 (4)

Substitute 2(a) into 3(b), we will get:

 (5)

Add Eq 4 and Eq 5, we will get:

 (6)

Eq 6 shows that when CSI rotates around its COM, its MI is invariable, which means Eq 6 is indeterminate. To indeterminate equation, we can set up an equation as follows:

 (7)

Substitute Eq 4 and Eq 5 into Eq 7, we will get:

 (8)

Since , , substitute these relations into Eq 8, we will get:

 (9)

Let

,

and we will get:

 (10)

By Eq 10, we will get:

 (11)

By Eq 1 and Eq 11, we will get:

 (12)

Divide both sides of Eq 12 by , and we will get:



The inverse function of Tangent is:

 (13)

Eq 13 shows that by only one rotation, we can position the CSI.