Web Appendix: *Modeling History Dependence in Network-Behavior Coevolution*

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1 *Siena*'s Continuous-Time Markov Model

In the network-analytic tradition, Snijders and colleagues (*op. cit.*) have advanced furthest in empirical modeling of dynamic, endogenous contagion and selection.[1] In *Siena*, N actors are connected by an observed, binary, potentially endogenous, and time-variant matrix, $\mathbf{x}$, of ties, $x_{ij,t}$. A vector of N observed, binary behaviors, $\mathbf{z}$, at time t has elements $z_{i,t}$. Additional exogenous explanators may exist at unit or dyadic level, $\mathbf{v}_{i,t}$ or $\mathbf{w}_{ij,t}$. Opportunities arise for actors to change their network ties, switching at most 1 tie on or off, at continuous-time fixed-rate, $\rho_{i,t}^{net}$, according to an exponential model. Likewise, opportunities to switch or leave unchanged the dichotomous behavior arise at rate $\rho_{i,t}^{beh}$.[2] When an opportunity to change network ties arrives for some i, she may choose to switch *on* or *off* any one of her N-1 ties. i makes these choices by comparing values of some objective function like:

$$f_i^{net}(\mathbf{x},\mathbf{x}',\mathbf{z})+\varepsilon_i^{net}(\mathbf{x},\mathbf{x}',\mathbf{z})=\sum\nolimits_h \left\{\beta_h^{net}\times \mathbf{s}_h^{net}(\mathbf{i},\mathbf{x},\mathbf{x}',\mathbf{z})\right\}+\varepsilon_i^{net}(\mathbf{x},\mathbf{x}',\mathbf{z}) \qquad (1)$$

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where $\mathbf{x}'$ is an alternative network under consideration, which can differ from the existing network, $\mathbf{x}$, only by changing one element of row i. Call $f_i^{net}(\cdot)$ the network-evaluation function. $s_h^{net}(\cdot)$ is some statistic, i.e., some function of the data, $\mathbf{x}, \mathbf{x}', \mathbf{z}$, that reflects i's objectives regarding network, $\mathbf{x}$, and behaviors, $\mathbf{z}$ (ideally, substantively-theoretically derived). The $\boldsymbol{\beta}_h^{net}$ to be estimated are the weights on these objectives. Assuming $\boldsymbol{\varepsilon}_i^{net}$ extreme-value distributed, independently across i and t, yields a multinomial-logit categorical-choice model. Similarly, when a chance to change behavior arrives, i compares values of an analogous objective function under alternative actions (here, binary): switch to 1 or 0 or leave unchanged. Formally, i compares $\mathbf{z}$ to $\mathbf{z}'$ given $\mathbf{x}$ and $\mathbf{z}_{j \neq i}$. Again, the behavior-evaluation function, $f_i^{beh}(\cdot)$, is the summed product of weights, $\boldsymbol{\beta}_h^{beh}$, and statistics, $s_h^{beh}(\cdot)$, and again assuming *i.i.d.* extreme-value stochastic components ($\boldsymbol{\varepsilon}_i^{beh}$), the logistic form emerges.[3]

The behavior and network objective-functions (and also the rate functions if desired) can include any of a number of commonly supposed social-network phenomena. For instance, importantly for our purposes, *covariate-related dissimilarity*, which is "defined by the sum of absolute covariate differences between i and the others to whom he is related" (p. 371):

$$\text{covariate-related dissimilarity: } s_i(\mathbf{x}) = \sum_j \mathbf{x}_{ij} |\nu_i - \nu_j|. \tag{2}$$

Entering $s_i(\mathbf{x})$ in the tie-formation equation with covariates ν_i and ν_j being i's and j's behaviors gives a behavioral-homophilic (or, rather, heterophilic) selection term.

RSiena estimates such models by simulated method-of-moments (s-MoM). To elaborate, the models' parameters are $\boldsymbol{\rho}$, the rates of events (or the parameters in the exponential models thereof), and $\boldsymbol{\beta}$, the parameters of the objective functions. The full parameter vector, $\boldsymbol{\theta}$, has dimensions k. As in any MoM estimator, one applies a statistic, $\mathbf{Z} = (\mathbf{Z}_1, ..., \mathbf{Z}_k)$, such that $\boldsymbol{\theta}$ is the solution of the k-dimensional moment equation: $E_{\boldsymbol{\theta}}\mathbf{Z} = \mathbf{z}$, where $\mathbf{z}$ is the sample outcome. Minimally, one needs for $\mathbf{Z}$ some statistics that respond in known way to the values of the parameters in question. (Sufficient sample-statistics would tend to optimize the moment-estimator efficiency, but sufficiency has not been established here.) Given such a statistic, we can specify as moment conditions, $\frac{\partial E_{\boldsymbol{\theta}}\mathbf{Z}_k}{\partial \theta_k} > 0$, for the MoM estimator.[4] E.g., for the rate from m to $m+1$, $\rho_{\mathbf{m}}$, a related (if insufficient) sample-statistic is the observed-change count from m to $m+1$, which tends to rise in ρ_m. Similarly, the sample value of the objective-function statistics should relate to $\boldsymbol{\beta}$. Estimated variance-covariances for the parameter estimates are numerically computed by the delta method.[5] These moment equations may seem simple, but their conditional expectations are not generally calculable explicitly. Siena uses stochastic-approximation methods: it simulates network-behavior outcomes according to the processes of the proposed model and estimates the parameters of that model by optimizing fit of simulated to observed sample-statistics.[6]

As a theoretical model and estimation strategy for actors' simultaneous tie-formation and behavioral choices, *Siena* is an impressive construct: not just *state of*, but the *entire corpus of*, the art. Yet, one notices also the many caveats stressed (Snijders 2001):

- "Although in our experience these equations mostly seem to have exactly one solution, they do not always have a solution" (p. 374).
- "This requirement [the minimal moment-conditions] is far from implying the statistical efficiency of the resulting estimator, but it confers a basic credibility to [...it and...] ensures the convergence of the stochastic approximation algorithm..." (p. 373).
- "...the method proposed here is not suitable for observations...too far apart in [...the number of intra-observational changes]. For such [...cases, dependence of one observation on the previous...] is practically extinguished, and it may be more relevant to estimate the parameters of the process [...separately]" (p. 374).
- "It is plausible that these estimators have approximately normal distributions, although a proof is not yet available" (p. 375).

This is a small subset of the stressed cautions, concerns, and comments noting various aspects of the estimation-strategy performance as unknown or maybe problematic, but we do not highlight them as criticism. *Siena* seems the most-sophisticated and best-developed tool capable of addressing coevolution, which we think is common and important in social science, and its approach to modeling network formation and behavioral choice shares our emphasis on affording address of a theoretically and substantively central empirical challenge for social science: the distinction and distinct estimation of common exposure, contagion, and selection in generating social outcomes that ubiquitously exhibit network/spatial association. Our point is instead to underscore how little is known regarding *Siena*'s performance as an estimator. Understandably given its complexity, little has been proven analytically about its properties; nor, also understandably given its computational demands and its specialized implementing software until *RSiena*'s recent advent, has its performance been explored much in Monte Carlo analysis. The next section's Monte Carlo evaluation may even be the first.

2 Estimation-Strategy Evaluation and Comparison

Next, we evaluate and compare the performance of our simple proposed time-lagged spatial-lag logistic-regression strategy and *Siena*'s simulated method-of-moments strategy for estimating models of network-behavior coevolution, i.e., with contagion and selection.

3 The Data-Generating Process and the Monte-Carlo-Simulation Scenarios

We follow Snijders (2001) to specify a data-generating process (DGP) replicating a *Siena* model of coevolution with the behaviors of N actors contagious through a network of ties generated by behavioral homophily. The DGP first creates T vectors, one for each interobservational period, of cumulative probabilities per *microstep* (intraobservational simulation period) of network-change events from a negative-exponential distribution with hazard-rate ρ^{net}. A parallel procedure produces T vectors of cumulative behavior-event probabilities using hazard-rate ρ^{beh}. Given these probabilities of events at each microstep, the DGP draws the steps in which an actor considers a change in behavior and the steps where an actor will make a network change. When an event occurs, each i is equally likely (a uniform(N) random-draw) to be chosen to consider change.[7] For an i selected to act on her network ties, a multinomial-logit form, $\frac{exp(f(\mathbf{x}_k))}{\sum_{j \neq k,i} exp(f(\mathbf{x}_j))}$, with $f(\mathbf{x}_k)$ given by i's network objective-function evaluated for a tie to k, gives the probability i changes her k^{th} tie. A draw from a multinomial distribution with this vector of probabilities (i's objective function evaluated at the current network matrix and behavior vector) then determines which of i's ties is changed. In our DGP, the objective function is *covariate-related similarity* (Ripley and Snijders (2010) monadic covariate effect #39, p. 66): the sum of centered similarity scores, $sim(v_{ij})$, between i and the j's to whom i is currently tied. Using i's and j's behavior last period, this metric gives the behavioral-homophily effect in network-tie formation. The coefficient (in this multinomial-logit network-tie equation) on covariate-related similarity is set to 1. Analogously for a behavior event: an i drawn to consider changing behavior uses her behavioral-objective function, $g(\mathbf{x})$, in logistic form, $\frac{exp(g(\mathbf{x}))}{1-exp(g(\mathbf{x}))}$, to compare the utility at the current values of the network matrix and behavior vector of switching or keeping behavior. In our DGP, $g(\mathbf{x})$ is given by the *average similarity effect*, defined as the average of centered similarity scores, with behavior again serving as the basis on which similarity is measured. This makes behavior of i depend more on the behaviors of j's with whom i is similar in behavior. The coefficient on average similarity in this logit behavior-equation is set to 1.

Using this DGP, we generated 100 trials each of 8 different scenarios: varying the number of actors $N \in \{30, 50\}$, the number of observed periods, $T \in \{5, 11\}$, and the rates of event occurrence, $\rho_{net} = \rho_{beh} \in \{1, 5\}$. We set the vector of first-period behaviors to 1 (0) for the first (second) $\frac{1}{2}N$, and the initial network (spatial-weights matrix) such that each actor is connected to the next actor with the same behavior, wrapping at the end. I.e., ones are in the upper first-minor, elements $(i, i+1)$, except for the $\frac{1}{2}N^{th}$ and the N^{th} rows, where ones are in the 1^{st} and $\left(\frac{1}{2}N+1\right)^{st}$ columns. Thus 1 connects to 2, 2 to 3, etc., but the $\frac{1}{2}N^{th}$ connects back to the 1^{st}; that lower-right block diagonal then repeats that upper-left.

4 Monte-Carlo Simulations: Parameter-Estimation Performance

Tables 1 and 2 report Monte Carlo explorations of *Siena* and our simple spatial-logistic strategy. We must re-emphasize first that the models differ: *Siena*'s estimation model mirrors the true DGP; our spatial-logistic simplification is a different model, differently parameterized. **The coefficient magnitudes are *not* directly comparable.** Nevertheless, we start by briefly discussing parameter-estimate performance before moving to the more-important implied estimates of *effects* (responses to common counterfactuals).

On the network-selection side, *Siena*'s estimates of the homophilic-selection parameter, β_h, show little to no bias at small-T at either N or $\boldsymbol{\rho}$. On the behavior-contagion side, at the lower event-rate, β_c shows a sizable (25%) deflation or negative bias in the smallest sample ($N = 30, T = 5$), but improvement to just -6% to -9% bias in the larger samples. At this lower event-rate, β_h (oddly) develops an appreciable -10% to -13% bias in the larger-T samples. At the higher event-rate of Table 2, the same pattern of biases emerges, much more severely: β_h essentially unbiased at low-T, but -50% to -60% biased at $T = 11$; β_c horribly biased (-62%) at small-T and -N, improving to just badly (-22% to -32%) so in the larger samples. The "correct" coefficient values in our spatial-logistic simplification are unknown, so we cannot evaluate bias directly (although the notably smaller magnitudes *per se* are not alarming because these parameter estimates would reflect a dampening from the event rates). However, patterns in the relative magnitudes of parameter estimates across scenarios may be informative. At Table 1's lower-rate, we see something similar to *Siena*'s pattern: the larger-T scenarios seem to dampen the estimates, slightly for the contagion and more notably for the selection parameter, and especially at lower N. This may suggest that these are properties of coevolution-model estimation rather than of the estimators. Table 2 suggests that the difficulties raised by high rates of intraobservational event-occurrence—i.e., by large amounts of interobservational change in networks and behaviors—may be debilitating: parameter magnitudes plummet, in some cases past zero. This reminds of Snijders' important caveat: this method "...is not suitable for observations...too far apart in the sense of the [...total number of changes between observations]. For such observations the dependence of [...this observation on the previous...] is practically extinguished..." (p. 374).

Regarding efficiency, the standard deviations of the estimates across trials in Table 1, proportionately (i.e., considered in ratio to mean of the corresponding parameter-estimate), reveal an appreciable advantage of our spatial-logistic simplification. This is particularly notable on the behavioral-contagion side, where *Siena* seems to have greater difficulty. At Table 2's higher rate, standard deviations exceed mean parameter-estimates, sometimes greatly, in all but one of eight cases for *Siena* and in five cases for the simple logits. Root mean-squared error (RMSE) is only calculable for *Siena*; it (reassuringly) shows improvements with sample size, but again raises alarms by its magnitudes in Table 2.

As for standard-error accuracy, at low event-rates (Table 1), either strategy seems reasonably accurate; ratios of actual to estimated estimate-variability (overconfidence) range from .82 to 1.07. Even at high rates (Table 2), our simple strategy remained passably honest about its huge parameter-estimate variability: overconfidence ranging .83-1.145. *Siena*'s standard errors were badly skewed across trials, occasionally exploding, necessitating report of medians instead. The overconfidence scores may not therefore be comparable, but, ranging .71-2.03, they seem to underscore further the unreliability at high event-rates.[8]

In many ways, the results for *power* (technically: share of trials in which standard t-tests both parameters rejected at .05 level) are the most (depressingly) telling. At lower event-occurrence rates, power grows with N and T, naturally and reassuringly, and to quite appreciable size in the high-N, high-T case, although one could certainly wish for better, especially from *Siena*, which, after all, is the exactly correctly specified empirical model for this DGP. Notwithstanding that fact, though, our simpler time-lagged spatial-lag logistic strategy dominates in power, especially in the small-T samples. At the higher event-occurrence rate, on the contrary, neither strategy could detect both the contagion and the homophily actually present, even though these unit magnitudes are rather large substantively and, in fact, are the only systematic source of variation in the outcomes. In sum, regarding power as seen also for bias, efficiency, and standard-error accuracy: either estimator loses traction badly in scenarios of high interobservational event-occurrence. Neither seems able to produce revealing estimates of coevolutionary processes from observations too sparsely dotted over much higher-frequency changes in network connections and node behavior. At lower intraobservational event-occurrence rates, perhaps either could do so, but to evaluate that, we need to consider the estimates in terms of their implied *comparable* substantive effects.

5 Monte-Carlo Simulations: Effect-Estimates Performance

To evaluate the relative performance of *Siena* and our simple logit, we must calculate estimated effects on behavior or tie-formation by each estimator of some common hypothetical.

We consider the following hypothetical regarding contagion. If all i's network partners behave in one way (all 0 or 1), what are the odds that i will choose the network-consistent over the network-inconsistent behavior? In the *Siena* DGP, we can get these odds thus: if all i's ties are initially to dissimilar behavior-types (so her average-similarity score is 0) and i switches her behavior to match her network partners, her average-similarity will go to 1, and the corresponding odds of going from inconsistent to consistent behavior, assuming i is chosen to act, are $exp(\beta_{beh})$ to 1 (≈ 2.714). In the simple-logit model, if i's network partners switch their behavior from all-0 to all-1, then for i, behavior 1 likewise goes from being network inconsistent to network consistent, and the odds of choosing behavior 1 gives the equivalent contagion effect, here as the spatial lag variable goes from 0 to 1.

For a comparable homophilic-selection effect of behavioral-similarity on network ties, we ask: what are the odds that i will choose to connect to another actor who behaves similarly over to one who behaves dissimilarly? In the *Siena* model, if all i's ties are between dissimilar behavioral types (average-similarity 0), choosing to connect to a similar behavior-type increases covariate-(behavior)-related similarity from 0 to 1, and the odds of forming such a tie (relative to choosing a tie with a dissimilarly behaving actor) are $exp(\beta_{net})$ to 1. In the simple logit model, an indicator variable turns on (off) when a potential network partner behaves similarly (dissimilarly), so the relevant odds calculation is straightforward.

Several issues remain. First, the *Siena* effects described above assume that i is chosen to make a behavioral or network change, but not all actors will be selected in that DGP. With rate of event-occurrence set to 1, the probability an actor i is selected during an interobservational period is about .63 (the negative-exponential cumulative-distribution evaluated at 1). The selection-adjusted odds-ratio is $.63 \times 2.714 = 1.71$. Second, the logit models are dynamic in a way the *Siena* DGP is not. Specifically, the logit parameter-estimates determine transition probabilities for a first-order Markov chain. Accordingly, the comparable odds-ratios would derive from the steady-state (stationary) distribution of the Markov chain. Finally, even with these adjustments, the logit models are still misspecified, especially the network model, because the true DGP only allows actors to make one change at a time, a restriction the simple logits do not impose. Consequently, the logit model will likely underestimate the size of the relevant selection effects. Many ties that would have formed among similarly behaving actors absent this restriction, will not be formed in the *Siena* DGP.

Table 3 compares these behavioral-homophilic selection and behavioral contagion effects using the estimates from the lower event-occurrence rate scenarios of Table 1. We provide the mean effect-estimate and the standard deviation and root-mean-squared-errors (RMSE) for these estimates. The relative efficiency we saw in estimating the structural parameter estimates transfers to the effects estimates. While the mean *Siena* effect-estimates frequently exhibit less bias, the simple logistic strategy outperforms the *Siena* estimates across the board in RMSE terms, often by large margins. In the small-sample case (N=30, T=5), e.g., the RMSE from the simple-logit model for behavior-homophilic selection-effect is a little over $\frac{1}{5}$ the size of corresponding RMSE calculated from the *Siena* estimates.

6 Summarizing the Monte Carlo Results

We can conclude generally on several points. First, data from contexts with higher event-rates, i.e., where intraobservational changes in networks and behavior are likely to have been great, seem unamenable to reliable estimation by either strategy of coevolutionary processes, to say the very least. At lower rates, either estimator reports reasonably honestly about the certainty of its estimates. *Siena* seems essentially unbiased in lower-T samples but suffers some downward or deflationary bias in its estimates at larger T (oddly), even in these better-suited low-rate conditions. This is true in the effects calculations as well. The bias in the spatial-logistic effect-estimates also roughly parallel the estimate-magnitude's decreasing with T, suggesting a similar (strange) "large-T bias" there. On the other hand, the simpler spatial-logistic strategy has somewhat of an edge in efficiency and, thereby, in power, with this advantage growing more-noticeable with lower T and smaller samples. The same holds for the effects estimates. The upshot of all this seems to be: neither strategy can offer much hope of learning anything reliable in almost any regard about coevolution when event-rates are high–which may be discernable by high amounts of change in networks and/or behaviors between observational periods that seem substantively far apart in that actors could have undertaken many actions in the interim. At low event-rates, conversely, both strategies work generally acceptably and roughly comparably well, with an efficiency advantage and simplicity perhaps favoring the logistic strategy.

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Notes

[1]Wasserman (1980b,a), Leenders (1997) presage. Bayesian latent-space approaches to longitudinal networks (Hoff et al. 2002; Hoff and Ward 2004; Hoff and Westveld 2007) may also relate.

[2]Although *Siena* can accommodate richer parameterizations, both $\boldsymbol{\rho}$ are held constant across i but allowed to differ arbitrarily by t here. These rates of intra-observational event-occurrence can vary freely, so the assumption of one i making one 1-unit change at a time is inconsequential. The strong assumption (we also make) of conditional independence of the choices does remain though.

[3]*Siena* 4.0 also has a *gratification* function, which we ignore, that considers alternative networks in changes, not levels. This allows different effects for switching ties on *vs.* off. *Gratification* is otherwise identical to the text's *objective*, so it merely adds a third multinomial-logit to the model.

[4]In fact, a quadratic of the moment condition must hold also, giving a (typically more efficient) generalized MoM, G-MoM, estimator, $\mathbf{a}'\left(\frac{\partial E_{\boldsymbol{\theta}}\mathbf{Z}_k}{\partial \theta}\right)\mathbf{a} > 0 \;, \quad \forall \mathbf{a}$.

[5]Conditioning on previous observations rather than using those moment equations can greatly reduce the problem dimensionality, which grows combinatorially in M and $\boldsymbol{\rho}$.

[6]See Snijders (2001) and Ripley and Snijders (2010) for estimation-procedure details and options.

[7]Had we instead parameterized the rate, i's probability of being chosen to act would be weighted by the ratio of hazard-function value for i, ρ_i, to the average ρ at that step.

[8]We also conducted (not reported) test-size analyses for behavioral contagion ($H_0: \beta_c = 0$) and behavior-homophilic selection ($H_0: \beta_s = 0$) in the lower-rate case. Both estimators yielded accurately-sized tests; i.e., with contagion, homophily, or both stripped from the DGP, the estimators rejected 5% of the 0.05-level tests that those coefficients were zero.

Table 1: Monte Carlo Results for $\boldsymbol{Rate}_{net} = \boldsymbol{Rate}_{beh} = 1$

Sample: N=30, T=5				**Sample: N=30, T=11**			
Parameter	**Result**	**SIENA**	**Simple Logit**	**Parameter**	**Result**	**SIENA**	**Simple Logit**
Network Selection	Mean	0.998	0.464	Network Selection	Mean	0.870	0.385
	S.D.	0.561	0.197		S.D.	0.305	0.113
	RMSE	0.561	–		RMSE	0.332	–
	Mean S.E.	0.662	0.193		Mean S.E	0.319	0.121
	Overconfidence	0.848	1.020		Overconfidence	0.956	0.933
Behavior Contagion	Mean	0.744	0.752	Behavior Contagion	Mean	0.918	0.75
	S.D.	0.770	0.648		S.D.	0.437	0.459
	RMSE	0.811	–		RMSE	0.445	–
	Mean S.E.	0.882	0.604		Mean S.E	0.533	0.444
	Overconfidence	0.874	1.070		Overconfidence	0.818	1.03
Power		*0.03*	*0.19*	*Power*		*0.30*	*0.34*
Sample: N=50, T=5				**Sample: N=50, T=11**			
Parameter	**Result**	**SIENA**	**Simple Logit**	**Parameter**	**Result**	**SIENA**	**Simple Logit**
Network Selection	Mean	0.996	0.473	Network Selection	Mean	0.899	0.405
	S.D.	0.400	0.156		S.D.	0.237	0.1
	RMSE	0.400	–		RMSE	0.258	–
	Mean S.E.	0.385	0.149		Mean S.E	0.244	0.093
	Overconfidence	1.040	1.050		Overconfidence	0.969	1.070
Behavior Contagion	Mean	0.942	0.801	Behavior Contagion	Mean	0.928	0.768
	S.D.	0.599	0.473		S.D.	0.466	0.343
	RMSE	0.602	–		RMSE	0.472	–
	Mean S.E.	0.622	0.465		Mean S.E	0.441	0.347
	Overconfidence	0.962	1.020		Overconfidence	1.060	0.991
Power		*0.29*	*0.41*	*Power*		*0.57*	*0.62*

Table 2: Monte Carlo Results for $\boldsymbol{Rate}_{net} = \boldsymbol{Rate}_{beh} = \mathbf{5}$

Sample: N=30, T=5				**Sample: N=30, T=11**			
Parameter	**Result**	**SIENA**	**Simple Logit**	**Parameter**	**Result**	**SIENA**	**Simple Logit**
Network Selection	Mean	1.026	0.088	Network Selection	Mean	0.424	-0.007
	S.D.	1.845	0.096		S.D.	0.457	0.574
	RMSE	1.845	–		RMSE	0.735	–
	Mean S.E.	0.907*	0.096		Mean S.E	0.646*	0.537
	Overconfidence	2.034	1.000		Overconfidence	0.707	1.070
Behavior Contagion	Mean	0.383	-0.074	Behavior Contagion	Mean	0.68	0.065
	S.D.	1.802	0.769		S.D.	0.698	0.055
	RMSE	1.905	–		RMSE	0.768	–
	Mean S.E.	1.283*	0.684		Mean S.E	0.881*	0.061
	Overconfidence	1.405	1.123		Overconfidence	0.792	0.911
Power		*0*	*0*	*Power*		*0*	*0.01*
Sample: N=50, T=5				**Sample: N=50, T=11**			
Parameter	**Result**	**SIENA**	**Simple Logit**	**Parameter**	**Result**	**SIENA**	**Simple Logit**
Network Selection	Mean	0.982	0.085	Network Selection	Mean	0.504	0.082
	S.D.	1.165	0.065		S.D.	0.406	0.036
	RMSE	1.165	–		RMSE	0.641	–
	Mean S.E.	0.916*	0.070		Mean S.E	0.518*	0.044
	Overconfidence	1.272	0.933		Overconfidence	0.784	0.826
Behavior Contagion	Mean	0.778	-0.005	Behavior Contagion	Mean	0.681	0.023
	S.D.	0.827	0.621		S.D.	0.747	0.494
	RMSE	0.856	–		RMSE	0.812	–
	Mean S.E.	1.200*	0.543		Mean S.E	0.810*	0.435
	Overconfidence	0.689	1.145		Overconfidence	0.922	1.135
Power		*0*	*0.02*	*Power*		*0.01*	*0.03*

*: Median standard error estimate reported.

Table 3: Monte Carlo Simulation Results for Comparable Effects (**True Effect** = **1.72**)

Sample: N=30, T=5				**Sample: N=30, T=11**			
Parameter	**Result**	**SIENA**	**Simple Logit**	**Parameter**	**Result**	**SIENA**	**Simple Logit**
Network Selection CF	Mean	2.068	1.512	Network Selection CF	Mean	1.580	1.410
	S.D.	1.836	0.320		S.D.	0.491	0.172
($0 \rightarrow 1$)	RMSE	1.869	0.381	($0 \rightarrow 1$)	RMSE	0.510	0.353
Behavior Contagion CF	Mean	1.763	1.800	Behavior Contagion CF	Mean	1.732	1.735
	S.D.	1.621	0.922		S.D.	0.740	0.577
($0 \rightarrow 1$)	RMSE	1.622	0.926	($0 \rightarrow 1$)	RMSE	0.740	0.577
Sample: N=50, T=5				**Sample: N=50, T=11**			
Parameter	**Result**	**SIENA**	**Simple Logit**	**Parameter**	**Result**	**SIENA**	**Simple Logit**
Network Selection CF	Mean	1.865	1.539	Network Selection CF	Mean	1.598	1.462
	S.D.	0.874	0.268		S.D.	0.392	0.145
($0 \rightarrow 1$)	RMSE	0.886	0.323	($0 \rightarrow 1$)	RMSE	0.410	0.294
Behavior Contagion CF	Mean	1.951	1.761	Behavior Contagion CF	Mean	1.778	1.716
	S.D.	1.353	0.598		S.D.	0.854	0.438
($0 \rightarrow 1$)	RMSE	1.373	0.600	($0 \rightarrow 1$)	RMSE	0.856	0.438