Phonology **33** (2016). Supplementary materials

Input Optimisation: phonology and morphology

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Supplementary materials

Appendix: statistical methods

The principal statistical tool used here is χ^2 (Pearson's chi-square test). What it allows us to do is test whether some distribution of items is significantly different from what's expected. For example, imagine we have a fair coin and throw it ten times and it comes up heads seven times. Is the coin fair? Here the expectation is that we'd get heads half the time, but in this instance we get somewhat more than that. For a linguistic example, imagine we expect words beginning with labials to occur just as often as words beginning with dorsals. In some sample of speech or text, we find 40 words beginning with labials and 60 words beginning with dorsals. Is this distribution significantly different from what we expect?

The χ^2 value can be calculated straightforwardly, as in (34).

(34) $$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

Here O_i is the observed value for some cell and E_i is the expected value for that cell. In the coin example above, there are two cells: heads and tails. Given that it is a fair coin and we throw it ten times, we expect five in each cell. We then get (35).

(35) $$\frac{(7-5)^2}{5} + \frac{(3-5)^2}{5} = \frac{4}{5} + \frac{4}{5} = 1.6$$

For the linguistic example above, we expect 50 in each cell, and we have (36).

(36) $$\frac{(40-50)^2}{50} + \frac{(60-50)^2}{50} = \frac{100}{50} + \frac{100}{50} = 4$$

These χ^2 values are compared against the χ^2 distribution to determine if these departures from what is expected are significant. The χ^2 distribution is defined in terms of a normal distribution. A χ^2 distribution for one degree of freedom, like the examples above, is simply the squared normal, as illustrated in Fig 1. Specifically, to determine whether a χ^2 value indicates a distribution significantly different from what is expected, we compare the χ^2 value to the distribution to see what percentage of the distribution falls to the right of the value. If that is less than 0.05 of the total, then the distribution is significantly different from expected.

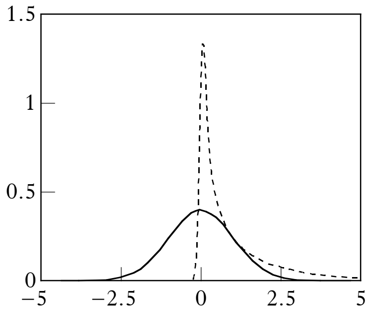

Figure 1

χ^2 distribution (dashed line) superimposed on a normal distribution (solid line).

For the coin example, the distribution is not significantly different from expected ($\chi^2(1,\ N = 10) = 1.600$, $p = 0.206$). For the linguistic example, however, the distribution is significantly different from expected ($\chi^2(1,$ $N = 100) = 4.000$, $p = 0.046$).

In most of the examples in the paper, I am explicitly comparing two distributions. For example, we might have the distribution for words beginning with dorsals *vs.* words beginning with labials, and wish to know whether it is significantly different from a distribution of ten word-final labials and twelve word-final dorsals. In these cases, I define one of the distributions as the expected distribution and test whether the other is significantly different. (Another possibility is to test whether the entire four-way distribution departs from what is expected, but this asks a different question, whether the dimensions that define the grid are independent, and thus examines the specific values across all four cells.)

In these cases, I always define the distribution with the fewer tokens as the distribution to be tested, and define the expected probabilities in terms of the distribution with more tokens. In the case at hand, this means we would test the observed distribution of ten and twelve against the expected proportions of 0.4 and 0.6 ($\chi^2(1,\ N = 22) = 0.273$, $p = 0.602$). (To calculate the expected proportions here, we simply divide the occurring values by the total: $0.4 = (40/(40+60))$; $0.6 = (60/(40+60))$.) This is a more stringent test than doing it the other way around, i.e. testing 40 and 60 against 0.45 and 0.55 ($\chi^2(1,\ N = 100) = 1.200$, $p = 0.273$).

Consider now the data treated in Table IV in the paper. The first comparison given is between non-prenominal tokens (6785 + 970 + 118 + 115 = 7988) and prenominal tokens (11136 + 950 + 304 + 140 = 12530). The second prenominal distribution has more tokens, so it is the distribution to be tested against: 0.889, 0.076, 0.024, 0.011. Expected values for the non-prenominal distribution are calculated by multiplying these values by the total for the non-prenominal distribution, as in (37).

(37)

$\acute{\sigma}\,\breve{\sigma}$	$0.889 \times 7988 =$	7099.3
$\breve{\sigma}\,\acute{\sigma}$	$0.076 \times 7988 =$	605.6
$\acute{\sigma}\,\grave{\sigma}$	$0.024 \times 7988 =$	193.8
$\grave{\sigma}\,\acute{\sigma}$	$0.011 \times 7988 =$	89.3

Results of χ^2 tests are presented in standard APA format, as above. However, following linguistic practice, in addition to observed values (O), I also give expected values (E) and observed/expected ratios (O/E).