UK household structure and Infectious Disease Transmission: Supplementary Information

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1 Mathematical Description of the model

We use a simple epidemic model, with each member of the population either susceptible, infectious or removed [1, 2]. In addition to the standard 'mass action' (or random-mixing) route of transmission, individuals are placed in households of varying sizes, within which there is an additional strength to transmission. This is described mathematically by the equations below:

$$\begin{aligned}\frac{d}{dt}H_{x,y,z}(t) &= \gamma\left[-yH_{x,y,z}(t) + (y+1)H_{x,y+1,z-1}(t)\right] \\ &\quad + \tau\left[-xyH_{x,y,z}(t) + (x+1)(y-1)H_{x+1,y-1,z}(t)\right] \\ &\quad + \beta I(t)\left[-xH_{x,y,z}(t) + (x+1)H_{x+1,y-1,z}(t)\right] \ .\end{aligned} \tag{1.1}$$

$H_{x,y,z}$ is the proportion of households with x susceptible individuals, y infectious individuals and z recovered individuals. β is the rate constant for 'mass action' transmission and τ is the rate constant for within-household transmission; γ is the recovery rate and hence $1/\gamma$ is the average infectious period. The quantity $I(t)$ is the proportion of infected individuals in the population, and is given by

$$I(t) = \frac{\sum_{x,y,z} yH_{x,y,z}(t)}{\sum_{x,y,z}(x+y+z)H_{x,y,z}(t)} \ . \tag{1.2}$$

Our model parameters are $\beta = \frac{6}{5}\gamma$, $\tau = \frac{2}{3}\gamma$; with the precise recovery rate γ not affecting any results presented in the paper. The values of β and τ are chosen to match two observables associated with pandemic influenza outbreaks: delivering a 40% chance of transmission between any two members of a household and, when the aggregate household size distribution for GB is used, a basic reproductive ratio r_0 of close to 2. The qualitative results are not affected by moderate changes to these parameter values. Throughout vaccination is modelled as initially placing individuals in the recovered rather than susceptible class at the start of an epidemic.

2 Calculation of r_0 and relationship to R_0

The classic definition of the basic reproductive ratio is "the average number of secondary cases produced by an average infectious individual in a totally susceptible population" and is written as R_0. For simple unstructured models, R_0 can be calculated as β/γ and the early growth of infected cases is given by:

$$I(t) = I(0)\exp\left([R_0 - 1]\gamma\right). \tag{2.1}$$

When populations are structured (either by age, risk groups or into households) the calculation of the basic reproductive ratio becomes more complex and the relationship between the verbal definition and the early growth rate no longer holds [3, 4]. To alleviate the confusion this causes, we set R_0 to be the value of the basic reproductive ratio as given by the verbal definition; whereas r_0 is defined in terms of the early growth rate. We note that R_0 and r_0 agree at the invasion threshold ($r_0 = R_0 = 1$); invasion of a pathogen is only possible when $R_0 > 1$ in which case $r_0 > 1$ as well. Throughout we use r_0 as our measure of epidemic potential, as its connection to the early growth of infection means that it has a clearer relationship to observable quantities from an epidemic. In particular, we determine r_0 as:

$$r_0 := (1 + \text{Early growth rate of infection})\,\gamma\,, \tag{2.2}$$

where the early growth rate is itself determined numerically using

$$\text{Early growth rate of infection} = \frac{1}{T}\ln\left(\frac{I(t_0+T)}{I(t_0)}\right)\,, \tag{2.3}$$

and where the initial level of infection is extremely small ($I(0) \approx 10^{-20}$), t_0 is sufficiently large to remove any transient dynamics, and T is sufficiently small that the number of susceptibles have not been significantly depleted and hence there is negligible non-linear behaviour.

3 Urban wards and r_0

Figure S1 shows the population density of the aggregate of all wards with basic reproductive ratios $r_0 \geq r$, for a given cutoff value r. The general upwards trend of this graph demonstrates that higher r_0 values are on average associated with higher population densities, which occur in the main urban centres. This re-enforces the observed geographical patterns seen in Figure 2a of the main text, where wards with high r_0 are seen primarily in the large urban concurbations.

4 Outlying wards

Finally we present here a graphical representation of the demographic and associated social features that lead to relative inefficiency of childhood vaccination. This analysis is based on the socio-demographic ACORN classification for all postcodes within each ward.

The definition of all ACORN types is presented in table S1. To each the comparison and highlight key features, we have defined our own aggregations of these standard classifications as follows:

- 'Student' is numbers 20 and 23;
- 'Multiple Adult' is numbers 3, 5 and 7;
- 'Prosperous' is numbers 1–36, excluding 3, 5, 7, 20 and 23;

- 'Moderate' is numbers 37–43;
- 'Hard Pressed' is numbers 44–56.

Figures S2 and S3 compare the UK average with the nine exceptional wards (highlighted in figure 4 of the main text) in which random vaccination outperforms childhood vaccination.

Figure S2 displays the UK population average within our five aggregated classifications (top row) together with four exceptional wards where students (red bar) dominate the population. This large number of students leads to many large households of childless young adults and explains the failure of childhood vaccination to achieve appropriate selective targeting. Figure S3 shows the remaining five exceptional wards, which are generally dominated by an older demographic (blue bar), whose offspring are too old to be registered as dependent children, but who remain in larger adult households either due to old age or a rural setting.

We note that unclassified postcodes (black bar) include both business addresses (that do not contribute to ward population) and also halls of residence or other institutional accommodation, which may also appear in census data as large childless households. In some cases this may also be a contributing factor, but one that is hard to quantify.

References

[1] RM Anderson and RM May. *Infectious diseases of humans*. 1992.

[2] M J Keeling and P Rohani. *Modeling Infectious Diseases in Humans and Animals*. Jan 2007.

[3] O Diekmann, J Heesterbeek, and J Metz. On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations. *Journal of Mathematical Biology*, Jan 1990.

[4] O Dieckmann and JAP Heesterbeek. *Mathematical Epidemiology of Infectious Diseases: Model Building, Analysis and Interpretation*. 2000.

[5] Office for National Statistics. 2001 census: Commissioned table C0844. ESRC/JISC Census Programme.

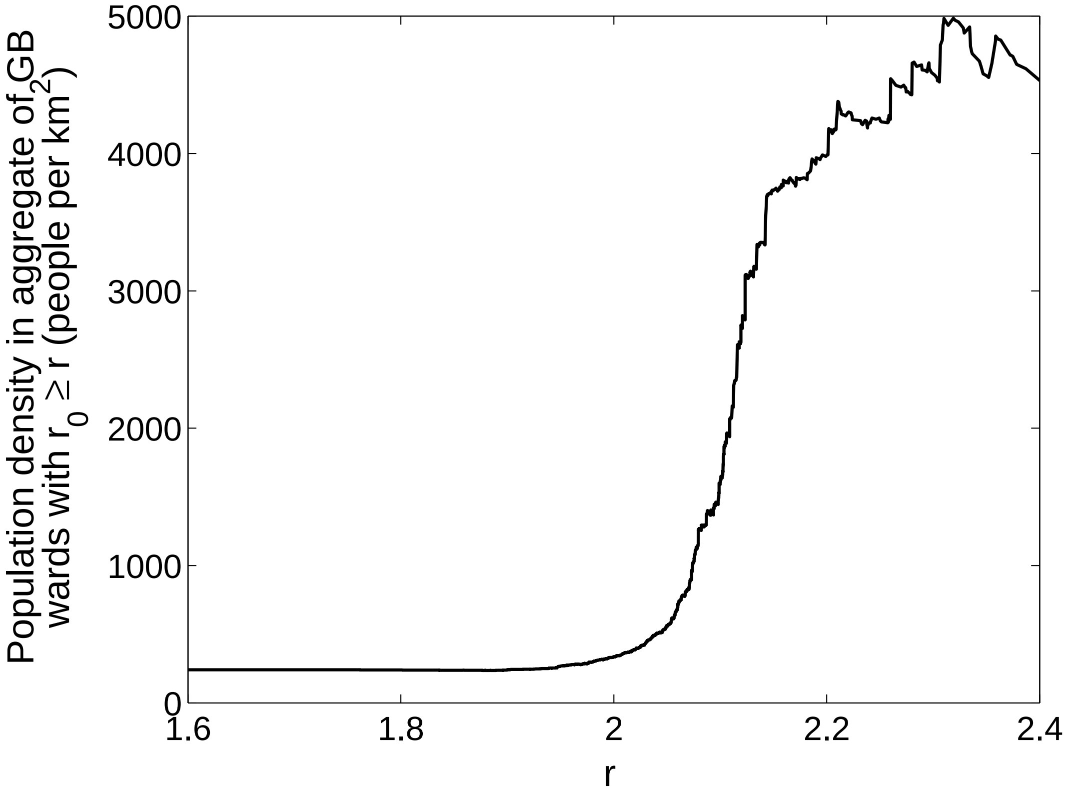

Figure S1: General scaling of r_0 with population density. For a cutoff value r, we plot the average population density for wards in GB with a basic reproductive ratio $r_0 > r$. The graph starts at the GB mean population density, as all wards are above the cutoff. As our focus narrows to those sections of GB with higher r_0 values, so the population density in the area under consideration rises sharply, showing that larger r_0 values are associated with more densely populated areas.

Wealthy Achievers	Wealthy Executives	01 – Affluent mature professionals, large houses 02 – Affluent working families with mortgages 03 – Villages with wealthy commuters 04 – Well-off managers, larger houses
	Affluent Greys	05 – Older affluent professionals 06 – Farming communities 07 – Old people, detached houses 08 – Mature couples, smaller detached houses
	Flourishing Families	09 – Larger families, prosperous suburbs 10 – Well-off working families with mortgages 11 – Well-off managers, detached houses 12 – Large families & houses in rural areas
Urban Prosperity	Prosperous Professionals	13 – Well-off professionals, larger houses and converted flats 14 – Older Professionals in detached houses and apartments
	Educated Urbanites	15 – Affluent urban professionals, flats 16 – Prosperous young professionals, flats 17 – Young educated workers, flats 18 – Multi-ethnic young, converted flats 19 – Suburban privately renting professionals
	Aspiring Singles	20 – Student flats and cosmopolitan sharers 21 – Singles & sharers, multi-ethnic areas 22 – Low income singles, small rented flats 23 – Student Terraces
Comfortably Off	Starting Out	24 – Young couples, flats and terraces 25 – White collar singles/sharers, terraces
	Secure Families	26 – Younger white-collar couples with mortgages 27 – Middle income, home owning areas 28 – Working families with mortgages 29 – Mature families in suburban semis 30 – Established home owning workers 31 – Home owning Asian family areas
	Settled Suburbia	32 – Retired home owners 33 – Middle income, older couples 34 – Lower income people, semis
	Prudent Pensioners	35 – Elderly singles, purpose built flats 36 – Older people, flats

Moderate Means	Asian Communities	37 – Crowded Asian terraces 38 – Low income Asian families
	Post Industrial Families	39 – Skilled older family terraces 40 – Young family workers
	Blue Collar Roots	41 – Skilled workers, semis and terraces 42 – Home owning, terraces 43 – Older rented terraces
Hard Pressed	Struggling Families	44 – Low income larger families, semis 45 – Older people, low income, small semis 46 – Low income, routine jobs, unemployment 47 – Low rise terraced estates of poorly-off workers 48 – Low incomes, high unemployment, single parents 49 – Large families, many children, poorly educated
	Burdened Singles	50 – Council flats, single elderly people 51 – Council terraces, unemployment, many singles 52 – Council flats, single parents, unemployment
	High Rise Hardship	53 – Old people in high rise flats 54 – Singles & single parents, high rise estates
	Inner City Adversity	55 – Multi-ethnic purpose built estates 56 – Multi-ethnic, crowded flats

Table S1: ACORN map. Copyright CACI Limited 2003.

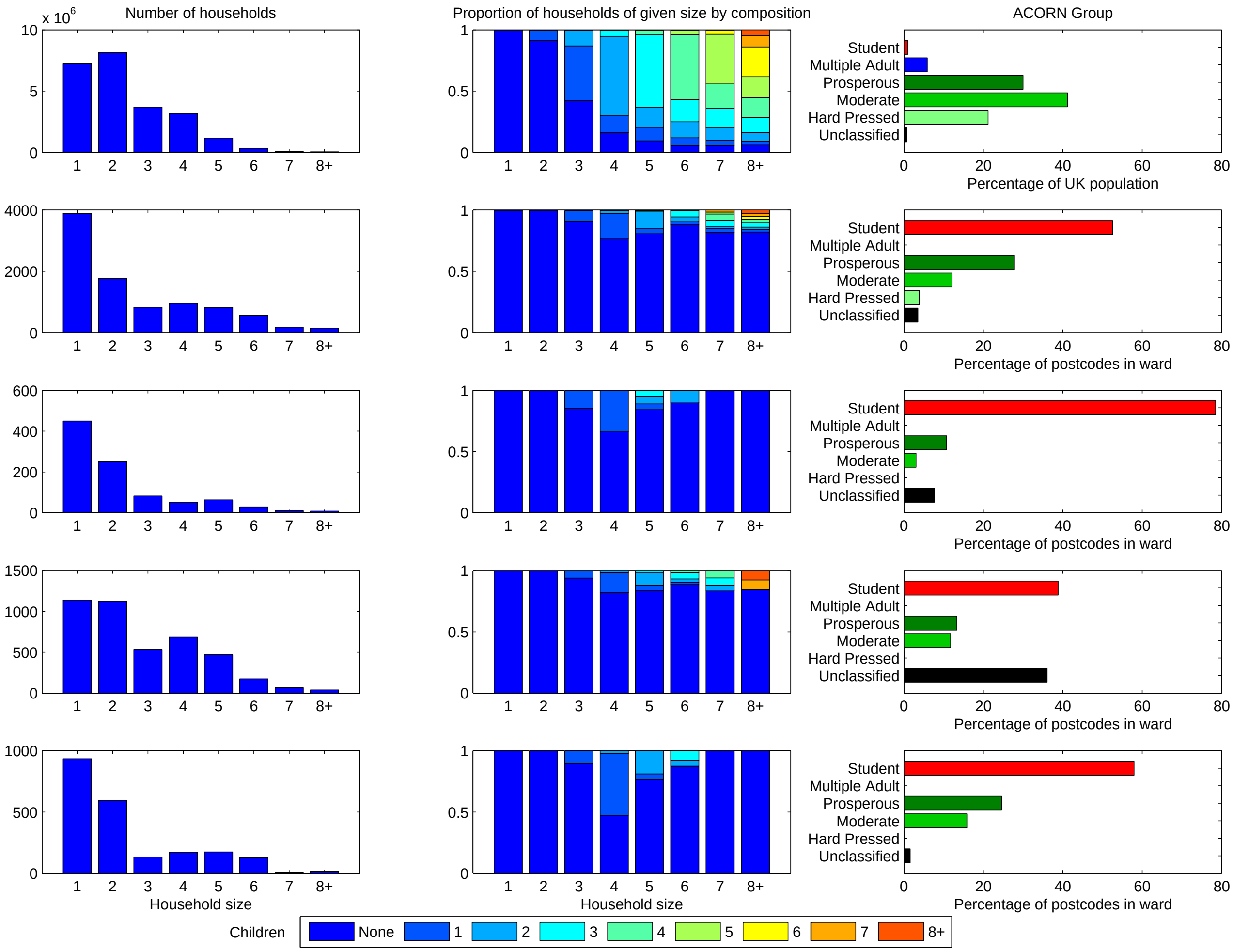

Figure S2: Demographics for wards in which heterogeneous individual vaccination outperforms child vaccination. The top row shows the GB average and the next four rows show wards in which students dominate. The first two columns show the number of households of various sizes within each ward, and the distribution of dependent children within each household size: these can be compared to Figure 1a and b in the main text. The right-hand column shows the proportion of postcodes falling within our five groups based on the ACORN classification. (Data source for first two columns: 2001 Census Commissioned Table [5]. Crown copyright. 2003. Crown copyright material is reproduced with the permission of the Controller of HMSO.)

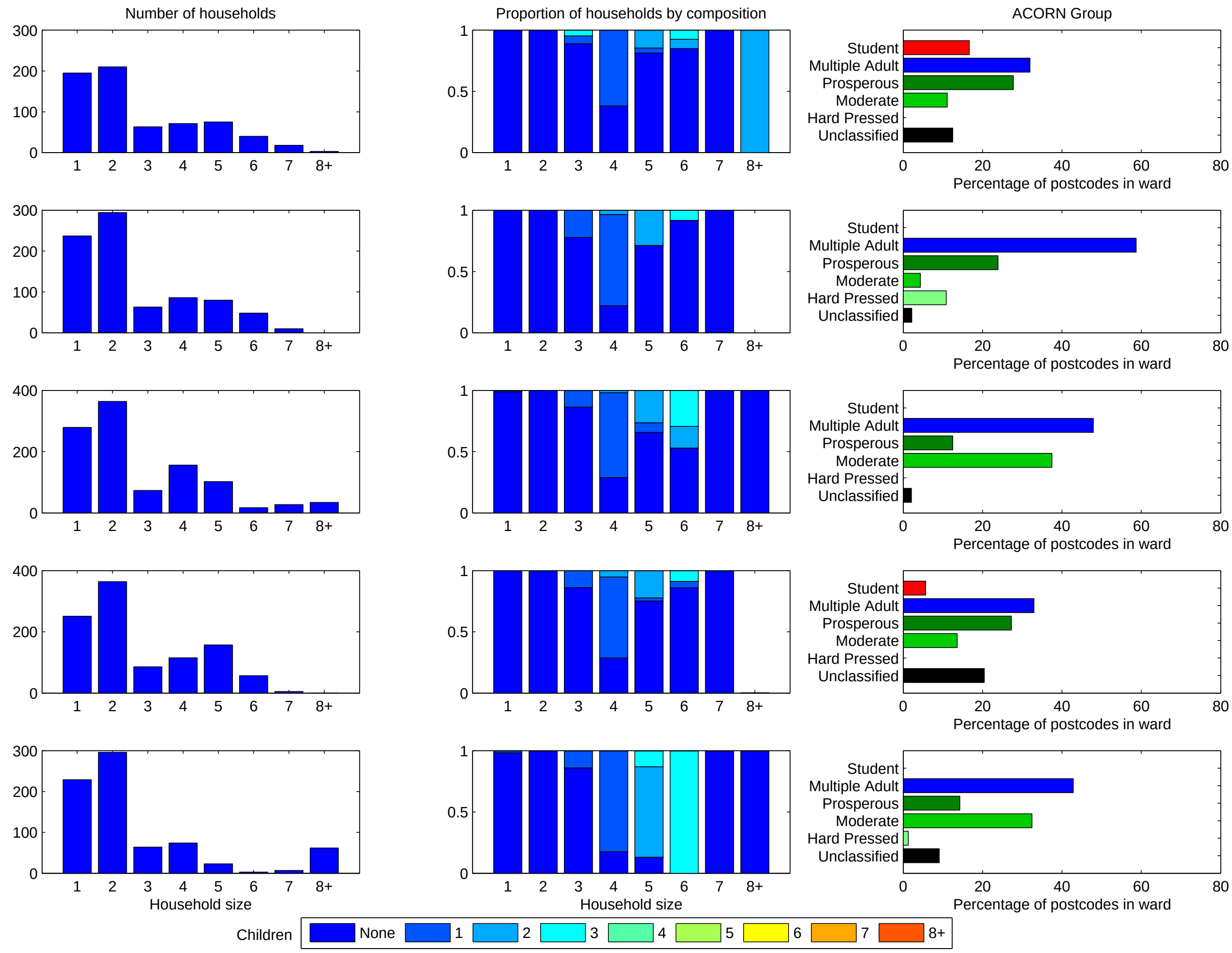

Figure S3: Demographics for wards in which heterogeneous individual vaccination outperforms child vaccination, showing wards in which older large childless households dominate. The first two columns show the number of households of various sizes within each ward, and the distribution of dependent children within each household size. The right-hand column shows the proportion of postcodes falling within our five groups based on the ACORN classification. (Data source for first two columns: 2001 Census Commissioned Table [5]. Crown copyright. 2003. Crown copyright material is reproduced with the permission of the Controller of HMSO.)