BOOK REVIEWS

ADDITIVE NUMBER THEORY: THE CLASSICAL BASES
(Graduate Texts in Mathematics 164)

By MELVYN B. NATHANSON: 342 pp., DM.78.–, ISBN 0 387 94656 X
(Springer, 1996).

ADDITIVE NUMBER THEORY: INVERSE PROBLEMS AND THE GEOMETRY OF SUMSETS
(Graduate Texts in Mathematics 165)

By MELVYN B. NATHANSON: 293 pp., DM.78.–, ISBN 0 387 94655 1
(Springer, 1996).

Nathanson's monumental work, of which two volumes have appeared and are reviewed here, and a third is in preparation, is the first comprehensive textbook on additive number theory in 40 years (since Ostmann's [**2**]).

Additive number theory can be roughly split into two parts: 'classical', when we are interested in representations of integers as sums with nice summands, like primes or kth powers, and 'combinatorial', when we seek additive properties valid for large classes of sets, typically restricted by cardinality constraints. Volume I is devoted to the classical theory, and Volumes II and III to the combinatorial one. This distinction is not very sharp, and luckily Nathanson does not try to follow it rigidly. Historically, the combinatorial theory grew out of the classical one, and the books reflect this by including Schnirelmann's approach to the Goldbach conjecture in Volume I.

Waring's problem consists in showing that for each k, there is an m such that every positive integer is a sum of at most m positive kth powers, estimating the smallest possible value $g(k)$ of m, and estimating the smallest number $G(k)$ of summands that works with at most finitely many exceptions. The first approach to this, which led to Lagrange's four-squares theorem, Hilbert's proof of the existence of $g(k)$, and Wieferich and Kempner's result that $g(3) = 9$, is based on polynomial identities, like the familiar $(x_1^2 + x_2^2)(y_1^2 + y_2^2) = \ldots$. This, by now almost forgotten, method is discussed in the first three chapters.

Next comes the basic staple, the Hardy–Littlewood (or circle, or exponential, or trigonometric) method. Sums of type $f(t) = \sum_{n \leqslant N} e^{2\pi i a_n t}$ can be applied to deduce additive properties of the sequence (a_n); typically, the main problem is to estimate this sum. For the Waring problem, we put $a_n = n^k$. From the many existing methods, Nathanson explains only the simplest, oldest and weakest one, 'Weyl differencing'. This is my choice too; improvements (which yield better estimates for $g(k)$) are of interest probably to a much narrower readership, and can be found in several places, such as the recent excellent book by Montgomery [**1**].

The second part of Volume I is devoted to the Goldbach problem, or representations of integers as sums of primes. First we meet Schnirelmann's approach in a self-contained presentation, with the necessary sieve methods (Brun and Selberg) also carefully explained. As a dessert, Nathanson includes Romanov's theorem that the set of integers of the form $2^k + p$, where p is prime, has positive lower density, and Erdős' theorem that it misses complete infinite arithmetic progressions.

The centrepiece is, of course, Vinogradov's theorem that every large odd number is a sum of three primes, via Vaughan's simplified estimate for the corresponding exponential sum. The last two chapters are devoted to Chen's theorem that every large even number is of the form $p + r$, with p prime and r either prime or the product of two primes; the linear sieve, a central ingredient of this proof, is again explained in detail.

Volume II and the future Volume III treat the combinatorial part of the theory. This is an extensive and diffuse subject; for Volume II, Nathanson chose a well-defined part, the so-called 'inverse problems' whose systematization is mainly due to Gregory Freiman. Here the typical question is the following: suppose we know something about the sumset $A + B$ (often that it is small); tell us something about the summands. A classical example is Kneser's theorem: if A, B are finite sets in a commutative group and $|A + B| < |A| + |B|$, then there is a subgroup H such that $|A + B| = |A + B + H| = |A + H| + |B + H| - |H|$.

The crown jewel is Freiman's theorem that any set A of integers (or lattice points in a Euclidean space) such that $|A + A| \leqslant c|A|$ can be covered by a not much larger multidimensional arithmetical progression. This is a set of the form

$$\{a + x_1q_1 + \ldots + x_nq_n : 0 \leqslant x_i < l_i\},$$

and 'not much larger' means $\prod l_i < f(c)|A|$. This is presented in the reviewer's approach.

A key ingredient is Plünnecke's graph-theoretic method. In the thirties, Erdős proved that if B is a basis of order h for the integers (every positive integer is the sum of at most h elements of B), then for every set A we have

$$\sigma(A + B) \geqslant \sigma(A) + \sigma(A)\big(1 - \sigma(A)\big)/(2h),$$

where σ means Schnirelmann density; the same inequality is valid for the lower asymptotic density. In 1971, Plünnecke improved this bound to $\sigma(A)^{1-1/h}$, which is the correct order of magnitude. Strangely, this sensational improvement was ignored for a long time, together with the method, which is excellently suited for the study of cardinality questions of sumsets. A typical result easily obtained by Plünnecke's method is the following: if A is a set in any commutative group, $|A| = n$ and $|A - A| = cn$, then $|A + A| \leqslant c^2n$.

In both volumes, the choice of the topics is excellent. In Volume I this sometimes means sacrificing the sharpest known results in favour of something more transparent; in Volume II this compromise is not necessary. The tools that are outside the usual courses are explained, like the sieves in Volume I and Minkowski's theorems in Volume II, unless this would not be feasible, like the prime number theorem for arithmetical progressions. Sometimes this is slightly overdone, like the Appendix to Volume I on arithmetic functions or a proof of Schwarz' inequality.

Each chapter ends with a collection of exercises, mainly easy to intermediate. I would have welcomed a collection of more difficult exercises and unsolved problems.

The style of explanation is clear and detailed (sometimes too detailed for my taste). What I did not like was, sometimes, the order of presentation. The author seems to follow certain pedagogical principles, of which I have read many arguments in favour, and with which I strongly disagree, like ‘always proceed from the known to the unknown’ and ‘avoid indirect reasoning’. To illustrate this, take Linnik's theorem that $G(3) \leqslant 7$ with Watson's beautiful proof in Volume I, Section 2.3. The heart of this proof is the identity

$$p^3(4q^{18}+2r^{18})+6pq^6r^6(x^2+y^2+z^2)$$
$$=(pq^6+r^3x)^3+(pq^6-r^3x)^3+(pq^6+r^3y)^3+(pq^6-r^3y)^3+(pr^6+q^3z)^3+(pr^6-q^3z)^3.$$

Here two questions arise: (1) how to make this identity work (when is a number n represented in this form), and (2) why this identity? By the time this identity is written down, question 1 is already answered (by proving certain properties of a number n without motivation). If you have understood the proof, and compare it to the previous one where a simpler identity is applied and realize why this simpler identity fails here, you are able to answer question 2. I think that explicitly posing and answering these questions before the proof would facilitate understanding.

This is an excellent text for courses on various parts of additive number theory. (I have often realized the lack of and need for such a text; the monographs on subfields are, for various reasons, often not immediately suitable for classroom use.) It will probably be the number one choice for a long time.

It is also a valuable source for the researcher, including many results that are difficult to locate in the literature. What I miss is a survey of results outside the scope of the book. While the book does not aim to be a research monograph, the inclusion of further results (without proof) and references could have made it more useful.

References

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INTEGER FLOWS AND CYCLE COVERS OF GRAPHS
(Pure and Applied Mathematics 205)

By Cun-Quan Zhang: 379 pp., US$145.00, isbn 0 8247 9790 6 (Marcel Dekker, 1997).

An *integer flow* in a graph G is an orientation D of G together with an integer weighting of the edges of G so that for each vertex v of D, the sum of the weights on the edges entering v in D is equal to the sum of the weights on the edges leaving v. A *cycle cover* of G is a set of cycles of G such that each edge of G is contained in at least one cycle in the set. These concepts are the subject of two of the most enticing open problems in graph theory: a conjecture of W. T. Tutte that every bridgeless graph has an integer flow using only the weights $\pm1, \pm2, \ldots, \pm5$, and a conjecture of P. D. Seymour and G. Szekeres that every bridgeless graph has a cycle cover which covers each edge twice.

This book explores these and many other beautiful problems and results on flows and circuit covers. It assumes a knowledge of elementary graph theory but no familiarity with flows or cycle covers, and takes the reader to the forefront of research in these two topics. Work in this area has been active over a long period, and the book is of great value in gathering the many results together and describing the links between them. It is very well written, with neat proofs and a good use of figures to illustrate ideas. Results from other areas of graph theory which are used in the main text are proved in an appendix so that the book is self-contained. Elementary questions are explored through exercises at the end of each chapter, and hints to their solutions are given in another appendix.

This book is an excellent text for a postgraduate course on flows and cycle covers, and would be invaluable to anyone working in graph theory.

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RATIONAL CURVES ON ALGEBRAIC VARIETIES
(Ergeb. Math. Grenzgeb. (3) 32)

By JÁNOS KOLLÁR: 320 pp., DM.158.–, ISBN 3 540 60168 6 (Springer, 1996).

The theme running through this book is the bend-and-break technique for finding rational curves on algebraic varieties. Roughly, the method goes as follows. Suppose that X is a smooth projective variety, and that $f : C \to X$ is a non-constant morphism from a smooth projective curve C. If $p \in C$ and there exists a non-trivial family of deformations $f_t : C \to X$ for which $f_t(p)$ is a fixed point of X, then there will be some degeneration under which the image curve breaks up into components which include rational curves. This follows by an elementary argument: the assumptions imply that there is a smooth projective curve B and a rational map $F : C \times B - \to X$ which is constant on $\{p\} \times B$. It is easy to see that F cannot be a morphism, and so we need to blow up the surface $C \times B$ in order to resolve the indeterminacies. At least one of the exceptional curves is not contracted by F, and so yields a rational curve on X.

This technique was described by Mori in a paper from 1979, in which he proved a conjecture of Hartshorne characterizing projective spaces. The crucial innovation of that paper was a reduction modulo p argument to obtain the existence of rational curves. Suppose that X contains a curve $\bar{C}$ on which the canonical divisor class $K_X = -c_1(X)$ is negative, and that $f : C \to X$ denotes the morphism from the normalization C of $\bar{C}$. Mori realized that in characteristic p, one could compose f with the mth geometric Frobenius map $\phi_m : C_m \to C$ (a morphism of degree p^m), and that for m sufficiently large, the composite morphism $f \circ \phi_m$ would deform non-trivially, hence yielding rational curves. Moreover, the bend-and-break argument enables us to bound the degrees of such rational curves. If, however, we are in characteristic zero, Mori's method is to reduce modulo suitably large primes p and apply the previous argument. The bound on degrees of the rational curves found is independent of p, and then abstract machinery enables us to deduce the existence of such rational curves on the original variety. It is noteworthy that there is still no proof of this result purely in characteristic zero, despite its resemblance to the phenomenon of bubbling in symplectic geometry.

The idea was subsequently used by Mori to prove one of the foundational results of the *Minimal Model Program*, the so-called Theorem of the Cone for smooth projective varieties. Mori and Miyaoka used the method to produce an extremely useful criterion for a variety X to be uniruled (that is, dominated by a rational map $Y \times \mathbb{P}^1 - \to X$), namely, X is uniruled if and only if through a general point $x \in X$ there passes a curve C with $K_X \cdot C < 0$. Using other results of Miyaoka (also proved by reduction modulo p), this showed that a complex projective threefold being uniruled was equivalent to its having negative Kodaira dimension. The criterion also showed that *Fano varieties* (smooth projective varieties with $-K_X$ ample) in any dimension were uniruled. Kollár, Miyaoka and Mori then introduced the notion of a variety being *rationally connected* (any two general points may be connected by a rational curve), and deduced that n-dimensional Fano varieties in characteristic zero were rationally connected, the curves C involved all having $-K_X \cdot C \leqslant d(n)$, for some number $d(n)$ depending only on n. An argument going back to Fano then yields the fact that (in characteristic zero) there are only finitely many families of Fano varieties of a given dimension.

The monograph under review represents the definitive account of this important theory. It starts with an extremely useful (albeit technical) chapter on Hilbert schemes and Chow varieties; this is needed later, to prove existence results for deformations of morphisms from curves, but in itself also represents a valuable reference for topics which are not well covered in the literature. In the subsequent chapters, the theory and its applications are developed hand-in-hand, and emphasis is placed on the contexts in which the applications should be viewed. The book is particularly impressive for the way that it proves results in appropriate generality whilst remaining readable. It will be used mainly by professional algebraic geometers (although mathematicians from other fields would gain from reading the introductions to chapters and subsections), and will clearly be the standard work of reference for the theory described. Previously, this area was fully covered only in a series of scattered research papers—it is indicative of this that the book's bibliography runs to nine full pages. We should be grateful to the author for having produced such an excellent and comprehensive account, in a reasonably concise and accessible form.

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BUILDINGS AND CLASSICAL GROUPS

By Paul Garrett: 373 pp., £55.00, isbn 0 412 06331 X
(Chapman and Hall, 1997).

The story of buildings and their applications to the study of groups is one of the most rewarding tales to read and to tell. Developed in the most part by Jacques Tits, buildings provide a powerful geometric tool for studying groups, like the classical groups, which act on such objects. For example, the language of buildings allows a beautiful description of the maximal compact subgroups of the classical groups over a local field (for example, groups like $\mathrm{SL}_n(\mathbb{Z}_p)$), a result which is presented in Garrett's book.

Eight years ago, two books ([**1**] and [**4**]) were published on the topic of buildings, which helped to demystify the subject for many mathematicians. The two books complemented each other nicely. Ronan's book [**4**] focused on the subject of buildings as interesting combinatorial objects in their own right, whilst Brown's book [**1**] tended more towards the interaction with groups admitting actions on buildings. Garrett's book falls more into Brown's camp, and the title reflects the fact that a significant part of the book is dedicated to a hands-on analysis of the classical groups.

At almost twice the length of Brown's book, Garrett's actually covers much of the same material, providing many details that will be appreciated by a newcomer to the subject. The book is thick with commentary, in the shape of numerous remarks laced through the text. Each chapter begins with a nice selection of pictures of buildings, which goes beyond the usual pictures of trees. There is an extensive bibliography, which unfortunately is not referred to in the text but nonetheless provides a good resource for those looking to explore some of Garrett's interests in the representation theory of p-adic reductive groups.

Garrett hints during the course of the book at some of the subtle differences that arise in non-split classical groups where the building depends on the nature of the field extension which splits the group. I believe that there is a place now for a more specialised treatment of the two volumes [**2**] and [**3**] of Bruhat and Tits, which would explain some of these tantalising comments. Nevertheless, this book joins those of Brown and Ronan as a good introduction to the subject.

References

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LEÇONS SUR LE THÉORÈME DE BEURLING ET MALLIAVIN

By Paul Koosis: 230 pp., C$33.65, isbn 2 921120 19 4
(CRM Publications, 1996).

Suppose that $\Lambda = \{\lambda_k\}$ is a sequence of complex numbers. The book under review is concerned with three related problems.

Problem A. Are the finite linear combinations of exponentials $e^{i\lambda_k t}$, for $\lambda_k \in \Lambda$, dense in $L_2(-L, L)$?

Evidently, there exists a number R, where $0 \leqslant R \leqslant \infty$, such that the answer is 'yes' for $L < R$ and 'no' for $L > R$. This number R is called the radius of totality (*rayon de totalité*) of Λ. The Theorem of Beurling and Malliavin determines R, via the zeros of certain entire functions of exponential type, in terms of a certain density of Λ. A good introduction and motivation for this area will be found in Levinson [**1**].

A nonconstant entire function $f(z)$ is said to be of exponential type if

$$|f(z)| \leqslant Ce^{A|z|},$$

where C and A are constants, and C depends on A. The lower bound of all possible A will be called the type of f.

Problem B. What can we say about the sequence of zeros of various classes of functions of exponential type?

Problems A and B are intimately related by duality. The particular form of Problem B which was solved by Beurling and Malliavin is as follows.

PROBLEM C. What are the conditions on a weight $W(x)$ on $\mathbb{R}$, where $W(x) \geqslant 1$, such that there exists an entire function f of arbitrarily small exponential type such that $W(x)|f(x)|$ is bounded on $\mathbb{R}$?

It is fairly clear that a necessary condition for this is

$$\int_{-\infty}^{\infty} \frac{\log W(x)}{1+x^2}\, dx < \infty.$$

Beurling and Malliavin showed (see page 140) that with the additional smoothness condition

$$|\log W(x) - \log W(x')| \leqslant \text{constant} \times |x - x'|, \quad \text{for } x, x' \text{ in } \mathbb{R},$$

the above condition is also sufficient for Problem C.

A consequence of this is the second result of Beurling and Malliavin (see page 141). Suppose that $\sum |\mathrm{Im}(1/\lambda_k)| = \infty$. Then $R = \infty$. Otherwise, $R = \pi \tilde{D}_{\Lambda'}$.

Here Λ' is the sequence of real numbers defined by

$$\lambda_k' = \frac{1}{\mathrm{Re}(1/\lambda_k)}, \quad \lambda_k \in \Lambda,\ \mathrm{Re}(\lambda_k) \neq 0,$$

and $\tilde{D}_{\Lambda'}$ is a rather complicated density of a real sequence (see page 58).

The statements and proofs of the above results can be found by the persistent and interested reader. They provide the motivation for what is, unfortunately, a rather complicated subject. Koosis takes the reader to the frontier of what is known, introducing techniques of Hörmander and Khabiboulline as well as his own. To follow him through all this demands a degree of commitment which your reviewer did not wholly possess. This is nevertheless an important area, as is shown by the quality of the people who have worked and still work in it.

The first two chapters are devoted, respectively, to the classes of Levinson and Cartwright of functions of exponential type and their zeros. The Cartwright class consists of those functions of exponential type for which

$$\int_{-\infty}^{\infty} \frac{\log^+ |f(x)|}{1+x^2}\, dx < \infty.$$

The Levinson class is a little larger, and the sets of zeros of this class were completely characterised by Levinson (see Chapter 1). Much of Chapter 2 is devoted to the additional conditions that must be satisfied by Λ for it to be a zero set of a function of the Cartwright class.

In Chapter 3, techniques involving subharmonic and other functions are introduced. Some of these methods have independent interest. The results of Beurling and Malliavin are stated in Chapter 4 and proved in Chapter 5.

I would recommend this book for libraries and keen students in the area.

Reference

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THE JAMES FOREST
(LMS Lecture Note Series 236)

By Helga Fetter and Berta Gamboa de Buen: 255 pp., £27.95 (LMS Members' price £20.96), ISBN 0 521 58760 3 (Cambridge University Press, 1997).

When R. C. James introduced James' space J in 1950 and James' tree JT in 1974, these were first seen as pathological examples. The space J was introduced to show that there are non-reflexive Banach spaces which do not contain c_0 or ℓ_1, as well as producing a Banach space which is isomorphic to its second dual space without being reflexive. It was conjectured by Banach that a separable Banach space with a non-separable dual space must contain a subspace isomorphic to ℓ_1. The space JT is a counter-example to this conjecture. But as the 'examples J and JT' became the 'theory' of quasi-reflexive spaces and the vast array of binary and other trees, it became clear that these constructions represented fundamental ideas which are required knowledge for any sort of understanding of the structure of Banach spaces. With over 100 papers already published using these ideas, it was, until now, a daunting task to try to learn these techniques. Even more formidable was the fact that this area incorporates most of the deep tools from the geometry of Banach spaces.

The book under review is the first comprehensive and systematic treatment of the ideas surrounding the Banach spaces J and JT. It is greatly enhanced by the careful breakdown and setting out of those techniques which are common to the many diverse applications of the material. What used to be a stumbling block in this area has now become its strength: that is, by taking the time to read this book, one gets for free an introduction to many topics from the geometry of Banach spaces. For example, while learning about the richness of James' space J, we are also introduced to: finite representability, isometries, super-reflexivity, the Banach–Saks property, summing bases, extreme points, pre-duals, and spreading models. To see that J is primary, we must analyse projections on J and study the approximation properties and the basic techniques for passing from weaker forms to stronger forms of the approximation properties. We also get an introduction to type and cotype while learning that the dual of J has cotype 2. In studying JT, we learn the basic techniques for 'treeing' a Banach space, and how to analyse bounded operators on trees, the fixed point property and normal structure, the Kadec–Klee property, and the important topic of Weakly Compactly Generated (WCG) spaces. Finally, the reader is pointed in the direction of further study, including: James' space can be renormed to be a Banach algebra, the automorphisms of J and $\mathrm{GL}(J^n)$, local unconditional structure, Tsirelson's space, and even the Gowers–Maurey constructions.

Banach space theory is an area with an appalling lack of books (for example, all analysts would benefit from a (good) book on the approximation property). With much of the field relegated to 'folklore', it has been nearly impossible for outsiders to access and apply the very powerful tools which are routinely used within the field. Bernard Beauzamy alludes to this fact in his prologue to the book: 'The powerful tools which have been created [in Banach space theory] over the last fifty years should have more applications to other fields, such as, for instance, Operator Theory, Harmonic Analysis, Numerical Analysis, Economics.' The book under review is much more than a view of the magnificence of 'The Forest'. In fact, it is a path through 'the trees' which carefully and systematically introduces the reader to almost all of the deep tools from the geometry of Banach spaces. Although it remains to been seen whether this book will have such a profound impact on other branches of mathematics, it is clear that it is an invaluable resource within the field for students and experts alike.

University of Missouri PETER CASAZZA

BEST APPROXIMATION BY LINEAR SUPERPOSITIONS (APPROXIMATE NOMOGRAPHY)
(Translations of Mathematical Monographs 159)

By S. YA. KHAVINSON: 175 pp., US$69.00, ISBN 0 8218 0422 7 (American Mathematical Society, 1997).

Consider the real-valued function $f(x, y) = x \cdot y$ on $\mathbb{R}^2$. Can f be represented (on the unit square, say) as $f(x, y) = g(x) + h(y)$?

The reader may want to take a minute to convince himself that the answer is negative. So, how closely can f be approximated by such functions? (Answer: $1/4$.) Does a best approximation exist? (Yes.) And what if we are willing to consider sets smaller than the square?

Such problems, motivated by Hilbert's 13th problem and its solution by Arnold and Kolmogorov, are studied in this book.

Let X be a closed subset of $X_1 \times X_2 \times \cdots \times X_k$ (compact factors). Let D denote the set of all functions

$$f(x) = g_1(x_1) + g_2(x_2) + \cdots + g_k(x_k) \quad (x = (x_1, x_2, \ldots, x_k) \in X)$$

with $g_i \in C(X_i)$ $(1 \leqslant i \leqslant k)$. (Here, $C(X)$ is the sup-norm Banach space of real-valued continuous functions on X.)

Clearly, D is contained in $C(X)$. The following problems are considered in the book.

(i) When is $D = C(X)$? (In that case, X is said to be basically embedded in $X_1 \times X_2 \times \cdots \times X_k$.)
(ii) When is D dense in $C(X)$?
(iii) When is D closed in $C(X)$?
(iv) Given f in $C(X)$, what is the distance of f from D?
(v) Is this distance attained by some element of D?
(vi) Is there an algorithm to find the nearest element?

This readable book is the first systematic presentation of the theory. (An earlier survey article [**4**] appeared in 1988.) It applies methods of functional analysis, topology, approximation theory and other fields.

The author avoids the presentation of proofs of some deeper and more difficult results such as the characterization due to Marshall and O'Farrell of extreme annihilating measures, and Sternfeld's characterization of dimension by linear superpositions (even though a relatively short proof by Levin [**1**] is now available).

Some recent developments of the theory are omitted: for example, Skopenkov's characterization of $\mathbb{R}^2$ basically embeddable continua [**3**], and the study of one-dimensional X_i (in particular, dendrites [**5**]) and hereditarily indecomposable X_i [**2**].

I regard this book by Khavinson as a solid introduction to the theory of linear superpositions.

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WAVELETS: CALDERÓN–ZYGMUND AND MULTILINEAR OPERATORS (Cambridge Studies in Advanced Mathematics 48)

By Yves Meyer and Ronald Coifman: 314 pp., £40.00, isbn 0 521 42001 6 (Cambridge University Press, 1997).

Harmonic analysis began as the study of the Fourier transform and operators relating to it. One of the most interesting phenomena that arises in the subject is the Heisenberg uncertainty principle, which says loosely that no function may be simultaneously localised in time and in frequency. This creates difficulties, for instance, when studying the action of a Fourier multiplier operator on an L^p-space. The operator is most naturally defined in terms of frequency, but the space is defined in terms of time.

In the 1950s, Calderón and Zygmund began the systematic study of singular integral operators. These operators were modelled on the Hilbert transform, which is given by convolution with $1/x$. While this is a convolution operator and hence a Fourier multiplier, it was studied by Calderón and Zygmund in terms of its kernel, which displays some cancellation and also has a reasonable amount of decay. Implicit in all the arguments was the limited way in which it is possible to localise simultaneously in time and frequency.

Singular integrals were a 'continuous' theory. Wavelets made it discrete. Wavelets are self-similar bases of L^2, and may be thought of as bases whose elements are simultaneously localised in time and frequency to the extent that this is possible. These bases also have nice properties with respect to L^p-spaces. In the 1970s and 1980s, practitioners using wavelets produced revisionist (and, in some ways, simplified) versions of singular integral theory which allowed them to attack previously inaccessible problems. Wavelets seemed to be liberating harmonic analysis from cancellation and the Fourier transform.

In the 1990s, among pure mathematicians, wavelets sometimes have a bad reputation. Because of their rapid acceptance by those working on applications, the power of wavelets in operator theory is often overlooked. That is why the book under review is especially valuable. Previously available only in French, it includes discussions of the $T(1)$ and $T(b)$ theorems, the theory of Cauchy integrals on curves, and the theory of paraproduct and paradifferential operators. Even its omissions (the John–Nirenberg theorem is neither stated nor exploited) attest to the ingenuity and elegance of the arguments.

This edition of the book has a few technical weaknesses. The index is extremely sparse. As an example, Carleson's lemma does not appear in the index, though its proof is one of the most popular passages. The bibliography also has its quirks. I lost a five-pound bet when I assured someone that I would be able to locate the part of the text where a certain paper by Bourgain was referred to. But these quibbles do not represent a serious obstacle to the reader.

The best way of stressing the importance of this work is to say that it was conceived as the manifesto of a radical revolutionary movement, and even before its publication in English, it had become a time-honoured classic.

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LIFTING SOLUTIONS TO PERTURBING PROBLEMS IN C*-ALGEBRAS
(Fields Institute Monographs 8)

By TERRY A. LORING: 165 pp., US$44.00, ISBN 0 8218 0602 5
(American Mathematical Society, 1997).

If some elements in a C*-algebra approximately satisfy a given set of relations, can we find nearby elements that satisfy the relations exactly? As an illustration of how such a vague question can be made precise, consider the following problems about the algebras M_n of complex $n \times n$ matrices, in which the relations are those of being commuting contractions.

Question 1. Given $\varepsilon > 0$, is there a $\delta > 0$ such that for each positive integer n, and elements X, Y in M_n with $\|X\| \leqslant 1$, $\|Y\| \leqslant 1$ and $\|XY - YX\| \leqslant \delta$, there exist A, B in M_n with $\|A - X\| \leqslant \varepsilon$, $\|B - Y\| \leqslant \varepsilon$ and $AB = BA$?

Question 2. Given $\varepsilon > 0$, is there a $\delta > 0$ such that for each positive integer n, and element X in M_n with $\|X\| \leqslant 1$ and $\|XX^* - X^*X\| \leqslant \delta$, there exists A in M_n with $\|A - X\| \leqslant \varepsilon$ and $AA^* = A^*A$?

The answer to Question 1 is No, as was shown by a short but ingenious construction of Choi's [**1**]. Question 2 is much harder, and it has recently been proved by Lin [**5**] that the answer is Yes. Loring's book is devoted to setting up powerful techniques for proving results like this. In fact, Loring proves (Corollary 19.2.8) a generalisation of Lin's theorem in which M_n is replaced by an arbitrary C*-algebra of stable rank one, using a simplified (but still not exactly simple) approach due to Friis and Rørdam [**2**].

The title of the book makes it sound like a narrowly specialised monograph, but in fact Loring deals, in a beautifully organised and systematic manner, with a wide swathe of modern C*-algebra theory, covering such topics as generators and relations, multipliers and corona algebras, extendibility, lifting, projectivity and semiprojectivity. One of the most impressive results (Theorem 14.1.4) is that if $\mathscr{R}$ is a finite, bounded set of relations generating a universal C*-algebra A, then $\mathscr{R}$ is stable if and only if A is semiprojective. In a short review there is no space, alas, even to define the terms in that statement: read the book to find out!

There is a marked algebraic flavour to much of the book, and Loring has been careful to separate out those sections that are purely ring-theoretic in nature. Algebraists are beginning to discover some of the rich structure that nonunital rings can possess, and they should find much of interest here. But be warned that the key concept of a σ-unit is defined both for rings and for C*-algebras, and the two definitions differ (for C*-algebras). When reading a section of the book, you need to be aware of whether it is a ring-theoretic or a C*-algebraic section.

Loring's book should be accessible to graduate students who have read Murphy's book [**6**]. But they should be prepared to work hard—the book is written in the elliptical style pioneered by Pedersen [**7**]. Also, some editing to reduce the number of misprints and wrong cross-references would have been beneficial. The index is inadequate (but the bibliography is very complete and useful). Finally in this list of grumbles, the Introduction is not really an introduction at all, but an overview of selected highlights from the book, which is not easily absorbed on its own, and is best read *after* the rest of the book.

But I do not want to finish on a negative note. Terry Loring has done a superb job in assembling a mass of powerful machinery which, quite apart from providing the tools for tackling perturbation problems, gives a coherent conceptual framework for proving such fearsomely technical results as Glimm's theorem on type I C*-algebras [**3**] and Kasparov's Technical Theorem [**4**]. Every operator algebraist will want a copy of this book.

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TECHNIQUES IN FRACTAL GEOMETRY

By Kenneth Falconer: 256 pp., £24.95, isbn 0 471 95724 0 (John Wiley, 1997).

So, you are interested in fractals? Perhaps you have seen the pretty computer-generated pictures, and want to know more about how they come about. Perhaps fractals are being used in your own field of study, and you want to learn the mathematics behind the theory, so you can treat 'fractal dimension' as more than just a metaphor for a degree of irregularity. Then this book is not for you; instead, you will want to read one of the mathematically-based introductory treatments of fractal geometry: for example, [**5**], [**3**] or [**1**].

Now, a year later, you have done that. But you want to learn more about this theory. Theoretical papers in your own field refer you to research-level mathematical papers or books. Or you consider taking up research in the mathematical field of fractal geometry yourself. But when you try to read these papers and books, you find that you are still an 'outsider'. Examples of texts of this kind are [**4**], [**8**] and [**7**].

For example, you may be reading an interesting paper or book, and at a crucial juncture the author cites 'the renewal theorem' (or 'the ergodic theorem', 'the martingale theorem' or 'the thermodynamic formalism'), and you have no idea what he is talking about. This is the way research-level mathematics is done: a mathematician speaks primarily to readers who know the important results of the field in question. In the worst case, he may use a result (with or without naming it) but provide no reference for it at all. In other cases, a little better, he will cite as a reference an entire book ('Feller, Volume 2' [**6**] and 'Dunford and Schwartz' [**2**] are popular for this). In the best case, the author will provide a page-number reference to a text where the result is discussed and proved; but even then you (the hapless student) end up with large portions of another book to read.

Here is where mid-level texts like the one under review are essential. The author has collected from the various sources the techniques and methods that are useful in the mathematical study of fractal geometry. For example, a chapter on 'the renewal theorem' explains the background of the result, provides a proof (that may be omitted on first reading), and exhibits some of the ways the theorem can be used in the study of fractals.

Other techniques used nowadays in fractal geometry are discussed in the same way. A list of chapter headings will provide an idea of the topics included: Mathematical background; Review of fractal geometry; Some techniques for studying dimension; Cookie-cutters and bounded distortion; The thermodynamic formalism; The ergodic theorem and fractals; The renewal theorem and fractals; Martingales and fractals; Tangent measures; Dimension of measures; Some multifractal analysis; Fractals and differential equations.

For the benefit of the student, there is an exercise list at the end of each chapter.

The book has a wide range of interest levels. For example, concerning the background required in measure theory, the author says: 'technicalities of measure theory are played down, with the existence of "intuitively obvious" properties of measures taken for granted'. For the chapters near the beginning of the book, this is perhaps enough. But for those near the end of the book, I doubt that anyone will get through without a firm grasp of measure theory. On the other hand, students with substantial mathematical backgrounds will be able to speed through much of the first half of the book in no time, but there are later portions of the book that should be of interest to them as well.

I expect that there are very few people who will read the book through from beginning to end, but there are many who will find a few chapters of the book at a level appropriate for fruitful and interesting study.

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POINCARÉ AND THE THREE BODY PROBLEM
(History of Mathematics 11)

By June Barrow-Green: 272 pp., US$49.00, isbn 0 8218 0367 0
(American Mathematical Society, 1996).

CELESTIAL ENCOUNTERS: THE ORIGINS OF CHAOS AND STABILITY

By Florin Diacu and Philip Holmes: 233 pp., US$24.95, £19.95,
isbn 0 691 02743 9 (Princeton University Press, 1997).

Nunquam praescriptos transibunt sidera fines—Nothing exceeds the limits of the stars. This was the epigraph with which Poincaré chose to identify his (supposedly) anonymous entry in the international prize competition sponsored by King Oscar II of Sweden and Norway to mark his 60th birthday in 1889. Both the books being reviewed serve to demonstrate the profound influence of celestial mechanics, and in particular the work of Poincaré, on the development of not only the theory of dynamical systems, but also many other areas which are now regarded as central to pure mathematics.

According to Aleksandrov: 'To the question of what is Poincaré's relationship to topology, one can reply in a single sentence: he created it.' However, Poincaré regarded himself as a very pragmatic mathematician, and he was primarily interested in topology not for its own sake, but because he regarded it as a powerful tool for solving problems in celestial mechanics and other branches of classical mathematics. From this perspective it is not far-fetched to regard modern topology as having been created by the study of the three body problem.

The book by June Barrow-Green focusses on Poincaré's prize memoir. The book places the work in its historical context, analyses its mathematical content, and looks at its influence on other mathematicians. After a brief introduction to the history of the problem, there is a fascinating chapter which considers Poincaré's work before 1889. At this early stage of his career he had already produced a considerable body of important work. For example, his four-part memoir on 'curves defined by differential equations' initiated the study of the qualitative theory of differential equations, and broke new ground in the way it focussed on global rather than local results and used geometrical rather than analytical methods. It also contained some wonderful results, including an analysis of the possible types of singularity and the Poincaré index theorem both for the plane and for a surface of arbitrary genus; it introduced the concept of limit cycles and also showed how to prove their existence by constructing what we now call a Poincaré mapping. He also gave his definition of stability in terms of recurrence, and proved a number of stability theorems using it. As well as all this, he also introduced the concept of rotation number, characteristic exponent and invariant integral, and thus had in place many of the building blocks he was to use with such skill in the prize memoir.

An amusing chapter follows, which sheds much light on the personalities of many of the leading mathematicians of the time, and gives details of the problems Mittag-Leffler had in assembling a prize jury of eminent mathematicians to set the questions and judge the outcome. As Sonya Kovalevskya wrote to him: 'how could one hope that four famous mathematicians Weierstrass, Hermite, Cayley and

Tschebychev would *ever* agree on the merits of a memoir. I believe it is certain that each of the four would refuse to become part of the jury as soon as he learned the names of the other three.' In the end, the jury consisted of just Hermite, Weierstrass and Mittag-Leffler himself. Apart from the question on the n-body problem, they set three other questions, all of which could have been attempted by Poincaré, which leads Barrow-Green to speculate that the questions had in fact been chosen with Poincaré in mind. By the closing date of the competition, 12 entries had been received, of which only three were regarded as significant. The jury quickly agreed that Poincaré should win the prize; however, the paper which finally appeared in *Acta Mathematica* differed substantially from the version that had actually won the prize almost two years earlier. The original manuscript, although quite long, often lacked detail in the wealth of new results it presented. In response to requests from the jury for more detail, Poincaré expanded the memoir by adding a number of substantial explanatory *Notes*, which nearly doubled its length. However, he later became aware of a much more serious error concerning his treatment of doubly asymptotic solutions (or what he later called homoclinic orbits). He had believed that the stable and unstable manifolds would coincide, and that in these circumstances the three body problem would be stable; but at a very late stage—after some copies of the erroneous manuscript had been printed and privately circulated—Poincaré realised that he had not considered all the possibilities, and that generically there would be a homoclinic tangle. Thus, far from being stable, such an orbit would be chaotic. Over the next few months, Poincaré worked frantically to correct his error and produce a new version which also incorporated much of the material from the *Notes* into the main body of the text.

The heart of Barrow-Green's book is a careful analysis of the differences between these two versions, based on comparing a copy of the original printed version of the manuscript, annotated by Poincaré himself, with that eventually published in *Acta*. She then goes on to examine how these ideas were developed further in Poincaré's magnificent three-volume book *Les méthodes nouvelles de la méchanique céleste*. This analysis is a work of great scholarship, which reveals the development of Poincaré's thoughts on the subject. However, I should have liked more details of the mathematics at this point, as I often had to consult the published memoir to understand the details. I fear that readers who do not already have a good grasp of the subject matter would find these chapters fairly hard going, which is a pity given the accessibility of the rest of the book. Barrow-Green next considers Poincaré's related work after 1889, including his formulation of the 'last geometric theorem' on fixed points of area-preserving maps of the annulus, which was proved shortly after his death by Birkhoff. The last two chapters consider Poincaré's influence on the work of Birkhoff and Hadamard, and ends with a brief discussion of Kolmogorov–Arnold–Moser theory. It is understandable that in these last chapters the book limits its attention to the influence Poincaré had on these authors, but it would be of great interest to trace further the influence of Poincaré's work in this area on 20th century mathematics as a whole.

The book *Celestial encounters* by Diacu and Holmes casts its net a little wider in this direction. It shows the way in which work on celestial mechanics, by Poincaré in particular, influenced the development of the whole of dynamical systems theory, including the more pure mathematical areas such as symbolic dynamics. It assumes a fairly limited previous mathematical knowledge, but uses simple mathematical concepts and diagrams to try and explain the key mathematical ideas and techniques. It also tries to convey something of the personality of and influences on the mathematicians involved. The book aims to describe the history of celestial mechanics in the style of James Gleick's *Chaos*, but at the same time to provide a solid description of the underlying mathematics. On the whole, it succeeds admirably in its task of providing a dramatic narrative without compromising on the mathematics. Although its style will not appeal to everyone, it manages to convey a real flavour of what it is like to be a mathematician, and a sense of the mixture of hard work and excitement that is involved in studying mathematics.

In their very different styles, these two books do much to demonstrate the impact that the n-body problem has had on modern mathematics. The book by Diacu and Holmes should encourage those who know only a little about the subject to read further, while that of Barrow-Green should inspire readers to look again at one of the classic works of modern mathematics.

University of Southampton JAMES VICKERS

ALGEBRAIC COMPLEXITY THEORY
(Grundlehren der Mathematischen Wissenschaften 315)

By PETER BÜRGISSER, MICHAEL CLAUSEN and M. AMIN SHOKROLLAHI: 618 pp., DM.188.–, ISBN 3 540 60582 7 (Springer, 1997).

Horner's rule evaluates $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ using n multiplications and n additions. In 1954, Ostrowski asked if this is optimal. Although he settled the question only for $n \leqslant 4$, he made an important contribution by proposing an appropriate model of computation for such algebraic problems, essentially straight line algorithms. (The optimality of Horner's rule was settled by Pan in 1966.) Later, this model was augmented by computation trees to include such problems as gcds of polynomials.

As another example, consider the number of multiplications needed to multiply a pair of 2×2 matrices. The obvious method uses 8, and a natural conjecture is that this is necessary. Strassen showed in 1969 that 7 suffice. Significantly, his algorithm works over any ring, and he applied it recursively to multiply $n \times n$ matrices in $O(n^{2.81})$ arithmetic operations. For 10 years there was no improvement, although progress was made on the nature of algorithms for quadratic and bilinear forms, for example, avoidance of division. Starting around 1979, a sequence of surprising and very beautiful results led to the current 'world record' of $O(n^{2.38})$, due to Coppersmith and Winograd. This development is one of the jewels of the area and of the book under review.

Algebraic complexity has grown out of such concerns, alongside other areas of computational complexity that use machine models of computation or boolean circuits; see, for example, Papadimitriou, *Computational complexity* (Addison-Wesley, 1994). The book under review, which concentrates on minimizing the number of arithmetic operations (of various kinds) rather than the amount of storage, is a timely addition to the literature. Apart from the introductory chapter, it is organized in five parts which are further subdivided into chapters. Each chapter ends with a notes section giving historical background, a guide to the literature and topics not covered in the main text.

Part I covers fundamental algorithms, concentrating on upper bounds. The focus is on polynomial arithmetic, including the rôle of the FFT, the use of Huffman coding for multiplying several polynomials, and algorithms for formal power series. Problems that fit into the computation tree model are also considered. The implications of using a non-uniform model of computing are discussed; there are polynomial time *non-uniform* algorithms for NP-complete problems, that is, a different algorithm can be used for each size of the problem. This involves VC-dimension and results from computational geometry.

Part II defines the two models of computation used, and goes on to discuss various lower bounds, including the substitution method of Pan and transcendence degree methods.

Part III deals with lower bounds based on methods from algebraic geometry; a version of Bézout's theorem is proved. This leads to Strassen's degree bound, which gives a lower bound on the multiplicative complexity of collections of rational functions in terms of the (logarithm of the) degree of their graph. There are also methods for proving high complexity for specific polynomials. In contrast to transcendence degree methods, these apply to polynomials with algebraic coefficients. The Milnor–Thom bound on the number of connected components of semi-algebraic subsets of $\mathbb{R}^n$ is used to derive a general lower bound for membership problems for such sets. Degree arguments are not so useful for additive complexity ($X^n - 1$ requires only one addition for arbitrary n), and results on the number of non-degenerate real solutions of systems of polynomial equations are put to effective use.

Part IV deals with problems of low degree, beginning with linear forms; this includes the DFT and related issues from graph theory (superconcentrators). There is also a study of generalized DFTs, that is, algebra isomorphisms $\mathbb{C}G \to \bigoplus_{i=1}^{h} \mathbb{C}^{d_i \times d_i}$ for a finite group G.

The rest of this part is essentially self-contained. There is a detailed study of quadratic maps on finite-dimensional vector spaces and especially bilinear maps, the latter being of special interest since they represent problems such as matrix multiplication. The computational model is further refined to so-called bilinear algorithms. These ideas are developed further by phrasing them in the language of tensors. A bilinear map $\phi : U \times V \to W$ corresponds to a unique tensor $t \in U^* \otimes V^* \otimes W$. The rank of the map, or tensor, is the least r for which there are $u_1, \ldots, u_r \in U^*$, $v_1, \ldots, v_r \in V^*$ and $w_1, \ldots, w_r \in W$ such that $t = \sum_{i=1}^{r} u_i \otimes v_i \otimes w_i$. Rank, in this sense, is a more manageable notion than multiplicative complexity (to which it is within a factor of 2), since it is invariant under various transformations, for example, an action of S_3. These notions are further refined by considering an equivalence relation on tensors and forming a semi-ring equipped with extra structure that corresponds to algorithmic aspects.

Matrix multiplication is studied in detail, leading to the upper bound $O(n^{2.39})$ (slightly weaker than the current 'world record'). A sequence of fascinating constructions is presented which, roughly speaking, allow us to obtain algorithms for matrix multiplication from ones for different but related problems. (More colourfully, one could say that 'incorrect' algorithms are 'repaired' in such a way that the gain far exceeds the cost incurred.) The importance of matrix multiplication is reinforced by relating it, in complexity terms, to many other problems.

Refinements of the substitution method are used to obtain lower bounds for the complexity of bilinear maps defined by multiplication in associative algebras. A highlight is the lower bound of $2n-t$ for the complexity of an algebra of dimension n with t maximal two-sided ideals; this unifies many previous results.

The rank of bilinear maps over finite fields is a special case: for example, linear block codes can be used for lower bounds. Polynomial multiplication and matrix multiplication are considered. Furthermore, the rank of the multiplication map for $\mathbb{F}_{q^n}$ over $\mathbb{F}_q$ for fixed q is shown to be linear by the use of an interpolation algorithm on algebraic curves together with a result on such curves with many rational points. Finally in this part, there is a study of the rank and border rank of various types of tensors.

Part V is somewhat different in flavour; it studies a non-uniform algebraic analogue of **P** versus **NP** introduced by Valiant in the 1970s. In particular, the rôles of the determinant and the permanent are studied, so that the clear tractability of the former and the (almost certain) intractability of the latter are set in context.

To sum up, this is a very carefully written book that covers a large area. The authors take care to prepare the reader for more specialist results (for example, the Morse–Sard theorem); the reader needs a reasonable background in algebra, geometry and, to some extent, number theory. Most of the material is available in papers, but there are also some new approaches. The authors state that the book is intended both as a textbook and as a reference book; they have certainly met this aim, and have made an invaluable contribution in presenting this growing field so well. There is plenty of scope for further work in the area, but the reader should be warned that some innocent looking problems have been open for a long time!

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