J. Fluid Mech. (1971), *vol.* 50, *part* 1, *pp.* 206–208
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REVIEW

Heat and Mass Transfer in Boundary Layers. 2nd edition. By S. V. PATANKAR and D. B. SPALDING. Intertext Books, 1970. 255 pp. £6.

This book holds two surprises for a reader who knows only the title. First, it deals only with calculation of boundary layers and not with experimental results, and this is acknowledged in the subtitle to this second edition: 'A general calculation procedure'. Second, although the emphasis is naturally on turbulent flow there is very little discussion of actual assumptions for the Reynolds stresses; the book is mainly a presentation of, and a users' guide for, two versions of the computer program developed by the authors and their colleagues for solving sets of coupled quasi-linear parabolic equations (which arise when the shear stress and other transport rates in a thin shear layer are expressed in terms of effective diffusivities). All types of boundary condition can be accommodated, so the programs can cope with thin shear layers other than true boundary layers, and the number of equations is effectively unlimited. The only Reynolds stress assumption the authors offer to justify the title of the book is the mixing-length form of the eddy-viscosity hypothesis (coupled with a constant turbulent Prandtl number). A new section on "More recent work" gives references to newer Reynolds stress assumptions but the reader is not given any information about how to incorporate them in the program. Both this section and the discussion of alternative numerical methods in the introduction contain statements which seem to me likely to mislead the reader; I shall return to this point later.

The programs described in the book are based on fully implicit finite-difference equations, in 'conservation' form, using co-ordinates x and ω, the latter being the stream function normalized to unity at the edge of the flow, so that in a growing shear layer the constant-ω lines are *not* streamlines. Coefficients of the quasi-linear equations are usually evaluated at the upstream station and not improved by iteration. Each equation is solved separately at a given forward step, a tri-diagonal matrix being inverted by successive substitution as in the usual Crank–Nicholson or fully implicit methods. There is no comparison of the accuracy of the 'conservation' scheme with the more usual schemes and there are no accuracy checks in the programs. The newer version of the program contains an 'upwind difference' modification for the case of large velocity *normal* to the constant-ω lines; at this point in the book the notation has become rather complicated but I think that in a typical turbulent boundary layer the modification would be implemented only when $d\delta/dx$ exceeded something like 0·15. Grid points adjacent to the boundaries are dealt with by introducing suitable 'slip' values at the boundaries and the neglect of the momentum flux in the half interval next to the inner boundary has been rectified in the newer version of the program. Also, the authors are now more cautious about the use of the van Driest formula in the viscous sublayer, used in the once and for all integrations to provide the law of the wall for the inner boundary condition, and they recommend direct appeal to experiment where possible.

In general the description of the numerical analysis and the programs (which are listed in Fortran) is very thorough, although the notation is (perhaps necessarily) complicated and in places (perhaps unnecessarily) non-standard and too little time is devoted to the rejected alternatives for the various choices of numerical treatment. The basic numerical scheme seems sound and the criticisms that occur to me are really matters of opinion. Undoubtedly the programs are very versatile, probably more so than any other program for solving equations of boundary-layer type, and the authors' colleagues and other workers have used them with a very wide range of Reynolds stress hypotheses.

How do the programs and the mixing-length model compare with other programs and other turbulence models? I think the authors' statements on these subjects, which form a small but obviously important part of the book, tarnish their achievement. The only general comparison of calculation methods for turbulent boundary layers is that made at the 1968 Stanford meeting (1969, *J. Fluid Mech.* **36**, 481). The results of 25 methods applied to 16 test cases were assessed by an independent 'evaluation committee' and the methods were divided into three groups of decreasing accuracy (in the early stages of planning the meeting the organizers were urged not to attempt an individual ranking). The three methods using basically similar mixing-length formulae were placed in the second group (which argues a certain consistency of performance, although the evaluation committee emphasized that there was no sharp break between the first and second groups). Now on p. 86 the authors say "In the event, the experimental data proved to be insufficiently reliable to distinguish between the serious contenders for the accuracy prize: however, it can be said that the predictions of the authors' mixing-length method were in as good agreement with the data as these deserved". I do not think this statement would give an unprepared reader a fair idea of the results of the Stanford meeting (or the data) and it falls far short of the reasoned analysis of those results which the authors are entitled to make.

The discussion on pp. 85–88 contains several other dubious statements, both about the mixing-length formula and about other calculation methods, which cannot be dealt with in a short review. Connoisseurs of the art of selective quotation should read p. 290 of Sivasegaram & Whitelaw (1971, *Aero Quart.* **22**, 274) from which one sentence is paraphrased at the top of p. 88 of this book.

However, despite its title, the main contribution of this book is not the use of a particular turbulence model but a pair of computer programs which are claimed to be very flexible and fast. Their wide use proves the claim of flexibility, but the claim about relative speed seems to be considerably exaggerated. The section on p. 12 entitled "A summary of the arguments governing the choice of a theory of the turbulent boundary layer" contains the following sentence. "As will be shown below, the new method appears to be between 100 and 1000 times as fast as the cross-stream-integration method; and conventional marching procedures are so slow and clumsy as not, it appears, to be in the race at all". The authors do not exactly *say* that marching is much worse than cross-stream integration, although I think this is what most readers would infer, and readers would certainly infer that the programs described in this book were orders of magnitude faster than their competitors at the time the book went to press in 1969. This is simply not borne out by computing times quoted by the entrants at the Stanford meeting (though some of these may also have been optimistic!). The better grid methods all take very roughly the same time – a cross-stream-integration program was apparently fastest – while the method of integral relations is perhaps half an order of magnitude faster than the best. The same would apply in laminar flows. Incidentally, the reason why the authors' program is not particularly fast, in spite of the trouble they have taken to simplify the integration procedure, seems to be that the streamwise step has to be kept rather small, a maximum of $0{\cdot}25\delta$ for the Stanford runs (I cannot find *any* figures for step length in the present book).

In conclusion, it seems to me that the authors' statements amount to a misleading impression of the relative merits of the authors' numerical method and those of other workers and of the applicability of the Prandtl mixing-length formula. This is sad, because the numerical method is flexible and represents a good compromise between speed and accuracy, so that it can be a useful tool in the hands of those who know more about heat and mass transfer in boundary layers than is recorded between the covers of this book.

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