## Supplementary Material

Reza Jahanbakhshi<sup>1</sup> and Cyrus K. Madnia<sup>1</sup><sup>†</sup>

<sup>1</sup>Department of Aerospace and Mechanical Engineering, State University of New York at Buffalo, Buffalo, NY 14260-4400, USA

The objectives of this supplementary section are to present movies of the complex flow field consisting of the turbulent structures, the flame, and the TNTI, and to examine the effect of the vorticity threshold used to detect the TNTI on the results of table 4 in the original paper.

## 1. Movies from the simulations

The movie files are created from the simulation data corresponding to case M02Q81Z05 in table 1 of the original paper. These files are:

Movie # 1: In this movie a 3D view of the DNS data during the self-similar stage is shown. The gray surfaces are isosurfaces of a positive value of discriminant of velocity gradient tensor representing the vortical structures in the flow and the red surface is the flame.

Movie # 2: This movie is a streamwise cut through of the shear layer viewed from the lower stream. The gray surfaces are isosurfaces of a positive value of discriminant of velocity gradient tensor, the red surface is the flame, and the orange translucent surfaces are the TNTIs.

Movie # 3: This movie is the instantaneous mixture fraction contours in a x-y plane. Dark blue is the oxidizer stream and dark red is the fuel stream. The white lines correspond to the detected TNTIs and the black line correspond to the flame.

## 2. Effect of TNTI threshold on the entrained mass flux

Table 1 shows the effect of the TNTI threshold on the contribution of different terms in equations 4.12 and 4.13 of the paper to average entrained mass flux. Five different threshold levels are examined in this table, and cases M02NR and M02Q81Z05 are presented for the comparison. The vorticity thresholds, which correspond to the inflection points in figure 4 of the paper, are  $\omega_{th}/\omega_{rms} = 0.10$  and 0.13 for cases M02NR and M02Q81Z05 respectively. As can be seen in table 1, the threshold chosen to detect the TNTI affects the value of the contribution of different terms to total mass flux,  $\mathcal{E}_{\mathcal{N}}$ . However, the trends of heat release effect on the contribution of different terms are not dependent on  $\omega_{th}$ . For example regardless of the threshold value chosen to compare the two cases, as the level of heat release increases the contribution of vortex stretching term,  $\mathcal{E}_{\mathcal{II}}$ , to entrainment increases. In the original paper, the inflection points of the plateau regions in figure 4 of the paper is used to detect the TNTI for each case. It is previously

$\omega_{th}/\omega_{rms}$	Case	$\left< \dot{m}^{\prime\prime} \right> / \left( \rho_0 \Delta U \right)$	$\mathcal{E}_{inv}$	$\mathcal{E}_{vis}$	$\mathcal{E}_{\mathcal{I}\mathcal{I}}$	$\mathcal{E}_{\mathcal{I}\mathcal{I}\mathcal{I}}$	$\mathcal{E}_{\mathcal{IV}}$	$\mathcal{E}_{\mathcal{V}}$	$\mathcal{E}_{\mathcal{VI}}$	$\mathcal{E}_{\mathcal{VII}}$
0.008	M02NR M02Q81Z05	$0.0106 \\ 0.0019$	$\begin{array}{c} 6 \\ 55 \end{array}$	$94 \\ 45$	$\begin{array}{c} 6\\ 38 \end{array}$	$\begin{array}{c} 0 \\ -2 \end{array}$	$\begin{array}{c} 0 \\ 19 \end{array}$	$\frac{94}{35}$	$\begin{array}{c} 0 \\ 1 \end{array}$	$\begin{array}{c} 0\\9\end{array}$
0.034	M02NR M02Q81Z05	$0.0093 \\ 0.0015$	14 63	$\frac{86}{37}$	14 41	$\begin{array}{c} 0 \\ -7 \end{array}$	0 29	86 19	$\begin{array}{c} 0 \\ 4 \end{array}$	$\begin{array}{c} 0\\ 14 \end{array}$
0.072	M02NR M02Q81Z05	$0.0085 \\ 0.0014$	$\begin{array}{c} 25\\ 69 \end{array}$	$\frac{75}{31}$	$\begin{array}{c} 25\\ 43 \end{array}$	$0 \\ -10$	$\begin{array}{c} 0 \\ 36 \end{array}$	$\frac{75}{8}$	$\begin{array}{c} 0 \\ 6 \end{array}$	$\begin{array}{c} 0 \\ 17 \end{array}$
Inflection points in Fig. 4 of paper	M02NR M02Q81Z05	$0.0078 \\ 0.0014$	34 90	66 10	$\begin{array}{c} 34 \\ 61 \end{array}$	$0 \\ -12$	$\begin{array}{c} 0\\ 41 \end{array}$		$\begin{array}{c} 0 \\ 7 \end{array}$	$\begin{array}{c} 0\\ 20 \end{array}$
0.181	M02NR M02Q81Z05	$0.0066 \\ 0.0012$	$\begin{array}{c} 54 \\ 109 \end{array}$	$46 \\ -9$	54 78	$0 \\ -13$	$\begin{array}{c} 0\\ 44 \end{array}$	$46 \\ -37$	$\begin{array}{c} 0 \\ 8 \end{array}$	$\begin{array}{c} 0\\ 20 \end{array}$

TABLE 1. Effect of TNTI threshold,  $\omega_{th}$ , on the average entrained mass flux,  $\langle \dot{m}'' \rangle$ , and contribution of different terms of equations 4.12 and 4.13 of the paper to this flux,  $\mathcal{E}_{\mathcal{N}} = 100 \times \langle \dot{m}''_{\mathcal{N}} \rangle / \langle \dot{m}'' \rangle$ .  $\omega_{rms}$  is the root mean square of vorticity on the mixing layer midplane,  $\rho_0$  is the reference value of density, and  $\Delta U$  is the velocity difference of upper and lower streams.

shown by da Silva *et al.* (2014) that the inflection point corresponds to the minimum of the PDF of vorticity magnitude, which gives the minimum-volume threshold. As is shown in the original paper the volume corresponding to the interface is small compared to the total volume of the flow field, i.e. irrotational region plus turbulent region. Thus the vorticity threshold that corresponds to minimum-volume is the best choice for detecting the TNTI in our analysis. The threshold values corresponding to the inflection points are in the range  $0.1 \leq \omega_{th}/\omega_{rms} \leq 0.13$  for different cases of the present study.

## REFERENCES

DA SILVA, C. B., TAVEIRA, R. R. & BORRELL, G. 2014 Characteristics of the turbulent/nonturbulent interface in boundary layers, jets and shear-free turbulence. In *Journal of Physics: Conference Series*, vol. 506, p. 012015. IOP Publishing.