On the Mechanism of Turbulent Drag Reduction with Super-Hydrophobic Surfaces Supplementary Materials

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1. Verification of Numerical Methods

A number of numerical tests were performed to verify the accuracy of the Lattice Boltzmann (LB) methods employed in the present study. These include comparisons of LB DNS predictions in turbulent channel flow with no-slip walls with pseudo-spectral DNS, comparisons of LB DNS predictions in turbulent open channel flow with experiments, assuming flat, shear-free boundaries at the free surface, and comparisons of LB DNS results with experiments and analytical solutions in laminar channel flow with Super-Hydrophobic (SH) walls, assuming flat, shear-free boundaries for the gas/liquid interfaces on the SH walls.

The simulations of turbulent channel flow with no-slip walls were performed in channels of size $5h \times 2.5h \times 2h$ in the streamwise (x), spanwise (y), and wall-normal (z) directions, respectively, for both LB DNS and pseudo-spectral DNS, where h denotes the channel half height. A uniform grid, with a grid size of $\Delta^+ \approx 2$ in all three directions, was used in the LB DNS. The pseudo-spectral DNS was performed on a uniform Fourier grid in the streamwise and spanwise directions, with resolutions of $\Delta_x^+ \approx 8$ and $\Delta_y^+ \approx 4$, respectively, and a Chebyshev grid, with a resolution of $0.07 \lesssim \Delta_z^+ \lesssim 4.5$, in the wall-normal direction. In both these simulations, a constant flow rate, corresponding to a bulk Reynolds number of $Re_b \equiv U_b h/\nu = 3600$, was maintained in the channel, where U_b and ν represent the bulk flow velocity and kinematic viscosity, respectively. The skin-friction coefficient predicted by LB DNS was within 1% of the skin-friction coefficient predicted by pseudo-spectral DNS, and within 4.5% of the value from Dean's correlation (Dean 1978), as shown in figure 1(a). The profiles of the mean velocity, turbulence intensities, and Reynolds stresses predicted by LB DNS were within 1% of the results from pseudo-spectral DNS, as shown in figures 1(b-e), while the rms pressure fluctuations were within 4% of the pseudo-spectral results. The discrepancy in rms pressure arises from the compressibility effects and the cubic non-linear error terms in LBM (Qian & Orszag 1993; Lammers *et al.* 2006). The magnitude of the error observed in the present studies is comparable to that reported by other investigators (Lammers *et al.* 2006; Bespalko, Pollard & Uddin 2012) in LB DNS of turbulent channel flow with no-slip walls.

To verify the accuracy of the specular reflection algorithm used for imposing the slip boundary conditions (Succi 2001; Benzi *et al.* 2006), LB DNS was performed in turbulent open channel flow at a bulk Reynolds number of $Re_b \equiv U_b H/\nu = 4000$ ($Re_{\tau_0} = u_{\tau_0} H/\nu \approx 250$) and Froude number of $Fr \equiv U_b/\sqrt{gH} = 0$, with flat, shear-free boundary conditions

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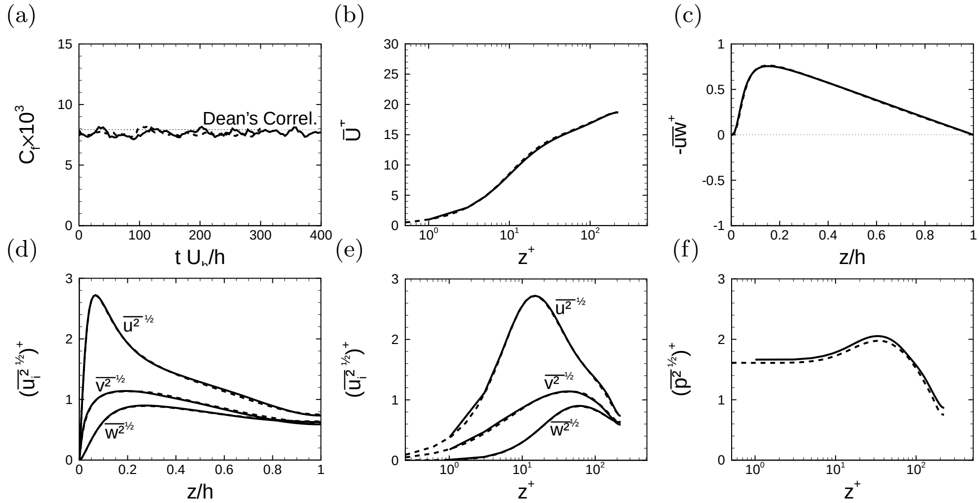

FIGURE 1. Verification of LB DNS in turbulent channel flow with no-slip walls: (a) skin friction coefficient; (b) mean velocity profile; (c) Reynolds shear stress; (d-e) turbulence intensities; (f) rms pressure fluctuations; ——, LB DNS of turbulent channel flow at $Re_b = 3600$ ($Re_{\tau_0} \approx 223$); – – –, pseudo-spectral DNS of turbulent channel flow at $Re_b = 3600$ ($Re_{\tau_0} \approx 222$); ····, Dean's correlation (Dean 1978) at $Re_b = 3600$ ($Re_{\tau_0} \approx 228$).

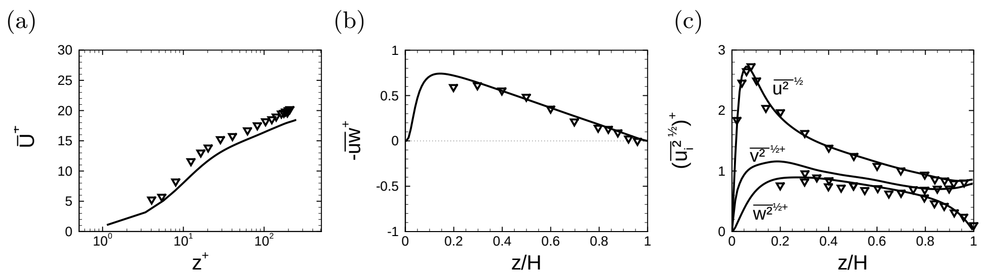

FIGURE 2. Verification of LB DNS in turbulent turbulent open channel flow: (a) mean velocity profile; (b) turbulence intensities; (c) Reynolds shear stress; ——, LB DNS of turbulent open channel flow at $Re_b = U_bH/\nu = 4000$ ($Re_{\tau_0} = u_{\tau_0}H/\nu \approx 250$); ∇, experiments of Komori *et al.* (1993) for an open channel flow at $Re_b = U_bH/\nu = 3000$ ($Re_{\tau_0} = u_{\tau_0}H/\nu \approx 160$).

on the free surface, where H and g denote the full height of the channel, and gravitational acceleration, respectively. These simulations were also performed in a channel of size $5H \times 2.5H \times H$, in the streamwise, spanwise, and wall-normal directions, respectively, and employed a uniform grid, with a grid resolution of $\Delta^+ \approx 2$ in all three directions. A constant flow rate was maintained in the open channel during the simulations. The LB DNS results were compared to the experiments of Komori *et al.* (1993) in open channel flow at $Re_b = U_bH/\nu = 3000$ ($Re_{\tau_0} = u_{\tau_0}H/\nu \approx 160$) and $Fr \approx 0$. Good agreement was observed between the predictions of LB DNS and experiments for the profiles of mean velocity, turbulence intensities, and turbulent Reynolds stresses, as shown in figures 2(a-c).

Finally, to verify the accuracy of LB DNS in the presence of SH walls, a series of simulations were performed in laminar channel flow with longitudinal arrays of slip/no-slip stripes on one wall, and the results were compared to the analytical solution of Philip (1972) and experimental measurements of Ou & Rothstein (2005) in laminar channel flow with SH longitudinal Micro-Grooves (MG) on one wall. A constant flow rate was imposed in the SH channel during the coarse of the simulations, corresponding to a bulk Reynolds

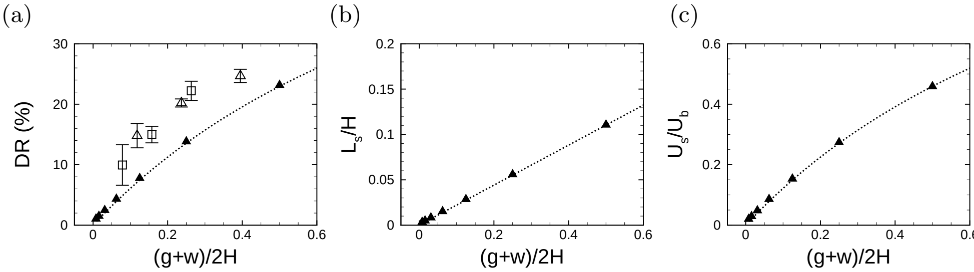

FIGURE 3. Verification of LB DNS in laminar channel flow with a superhydrophobic wall: (a) percent DR; (b) ratio of slip length to channel height; (c) ratio of slip velocity to bulk velocity; ▲, LB DNS at $Re_b = U_b H/\nu = 100$, longitudinal slip/no-slip stripes on one wall, $g/w = 1$; □, △, experiments of Ou *et al.* (2005) in laminar flow, longitudinal MG on one wall, $g = w = 20\mu m$, and $g = w = 30\mu m$, respectively; ····, analytical solution of Philip (1972), longitudinal slip/no-slip stripes on one wall, $g/w = 1$.

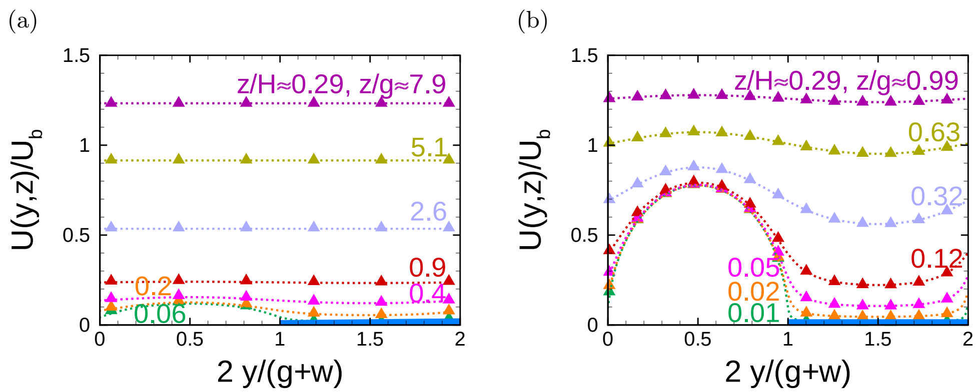

FIGURE 4. Verification of LB DNS in laminar channel flow with a superhydrophobic wall: (a,b) spanwise variation of the streamwise velocity at different wall-normal locations: (a) $(g+w)/2H = 0.036$, $g/w = 1$ (2.8%DR), (b) $(g+w)/2H = 0.29$, $g/w = 1$ (15.6%DR); ▲, LB DNS at $Re_b = U_b H/\nu = 100$, longitudinal slip/no-slip stripes on one wall, $g/w = 1$; ····, analytical solution of Philip (1972), longitudinal slip/no-slip stripes on one wall, $g/w = 1$; ——, locations of the no-slip surfaces.

number of $Re_b = U_b H/\nu = 100$. A uniform grid, with a resolution of $\Delta/H \approx 0.004$ was used for these studies. The predictions of LB DNS for the Drag Reduction (DR), slip length, and slip velocity were found to be in agreement with the analytical solution of Philip (1972), as shown in figure 3(a-c). Here DR is defined as $DR = (C_f^0 - C_f)/C_f^0$, where C_f is the average skin-friction coefficient on the two channel walls, one of which has the slip/no-slip pattern, and the other is a regular no-slip wall, and C_f^0 is the skin-friction coefficient in a channel with no-slip walls at the same Re_b. Experiments (Ou & Rothstein 2005) show similar trends in DR, but the magnitudes of DR are higher, by 5-10%, compared to the analytical solution of Philip (1972) and the present LB DNS results. The deviations of experimental data from the analytical solution of Philip (1972) have been attributed to the presence of surface curvature in the experiments (Ou & Rothstein 2005). The presence of the SH surface also gives rise to a spanwise variation in the mean streamwise velocity, which persists up to a distance of $z \approx g$ from the SH walls. LB DNS predicts these variations in good agreement with the analytical solution of Philip (1972), as shown in figures 4(a,b). Unlike turbulent flows, these spanwise variations do not lead to a secondary mean flow in laminar flow regime.

2. Estimation of DR for DNS of Türk *et al.* (2014)

The DNS of Türk *et al.* (2014) were performed under an imposed constant pressure gradient. The authors report the increase in the bulk velocity in the presence of SH walls, compared to the base turbulent channel flow with no-slip walls, with an imposed constant pressure gradient. To estimate the DRs for the DNS studies of Türk *et al.* (2014) in the present study, the DR is computed using the relation $DR = (C_f^0 - C_f)/C_f^0 = 1 - (U_{b_0}/U_b)^2$. We note, however, that this expression assumes a different Re_b in the SH channel compared to the base flow, and thus overestimates the magnitude of DR compared to the standard definition of DR, for which C_f and C_f^0 are evaluated at the same Re_b.

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